CHAPTER 0
Chapter 0

The Language of Mathematics

In this introductory chapter, we present some preliminaries which will be used throughout the present work. Throughout the study \((X, \tau), (Y, \sigma)\) and \((Z, \eta)\) (or simply \(X, Y\) and \(Z\)) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset \(A\) of \(X\), \(\text{Cl}(A), \text{Int}(A)\) and \(A^c\) denote the closure of \(A\), the interior of \(A\) and the complement of \(A\) respectively. The following sections facilitate to recollect the definitions fundamental for the present study.

- Various forms of open sets
- Various forms of mappings
- Various forms of spaces
- Various forms of sets in Bitopological spaces

0.1 Various forms of open sets

Definition 0.1.1 A subset \(A\) of a space \((X, \tau)\) is called

(i) semi-open [43] if \(A \subseteq \text{Cl}(\text{Int}(A))\).
(ii) pre open [52] if \( A \subseteq \text{Int}(\text{Cl}(A)) \).

(iii) \( \alpha \)-open [54] if \( A \subseteq \text{Int}(\text{Cl}(\text{Int}(A))) \).

(iv) \( \beta \)-open [1] (= semi-pre open [2]) if \( A \subseteq \text{Cl}(\text{Int}(\text{Cl}(A))) \).

(v) regular open [60] if \( A = \text{Int}(\text{Cl}(A)) \).

The complement of the above mentioned sets are called their respective closed sets. The family of all \( \alpha \)-open sets (resp. semi-open sets, pre-open sets, \( \beta \)-open sets and regular open sets) of \((X, \tau)\) is denoted by \( \alpha O(X) \) (resp. \( SO(X), PO(X), \beta O(X) \) and \( RO(X) \)) and the family of all \( \alpha \)-closed sets (resp. semi-closed sets, pre-closed sets, \( \beta \)-closed sets and regular closed sets) of \((X, \tau)\) is denoted by \( \alpha C(X) \) (resp. \( SC(X), PC(X), \beta C(X) \) and \( RC(X) \)). The collection of all closed (resp. clopen) subsets of a space \((X, \tau)\) will be denoted by \( C(X) \) (resp. \( CO(X) \)). We set \( C(X, x) = \{ V \in C(X) \mid x \in V, \text{ for } x \in X \} \). Similarly we define \( CO(X, x) \).

The set of all open sets containing the point \( x \) is denoted by \( O(X, x) \). Similarly \( \alpha O(X, x) \) denote the set of all \( \alpha \)-open sets containing the point \( x \) respectively.

The intersection of all semi-closed (resp. pre closed, \( \alpha \)-closed and semi-pre closed) sets containing a subset \( A \) of \((X, \tau)\) is called the semi-closure (resp. pre closure, \( \alpha \)-closure and semi-pre closure) of \( A \) and is denoted by \( sCl(A) \) (resp. \( pCl(A), \alpha Cl(A) \) and \( spCl(A) \)). The \( \alpha \)-interior of a subset \( A \) of \( X \), denoted by \( \alpha \text{Int}(A) \) is defined to be the union of all \( \alpha \)-open sets contained in \( A \). If \( A \subseteq B \subseteq X \) then \( \alpha Cl_B(A) \) and \( \alpha Int_B(A) \) denote the \( \alpha \)-closure of \( A \) relative to \( B \) and \( \alpha \)-interior of \( A \) relative to \( B \) respectively. For a topological space \((X, \tau)\), the subset \( A \) of \((X, \tau)\) is \( \alpha \)-closed (resp. semi-closed) if and only if \( \alpha Cl(A) = A \) (resp. \( sCl(A) = A \)).

**Definition 0.1.2** A subset \( A \) of a space \((X, \tau)\) is called

(i) generalized closed (briefly \( g \)-closed) [44] if \( Cl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is open in \((X, \tau)\). 

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(ii) semi-generalized closed (briefly $sg$-closed) [8] if $s\text{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open in $(X, \tau)$.

(iii) generalized semi-closed (briefly $gs$-closed) [4] if $s\text{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.

(iv) $\alpha$-generalized closed (briefly $\alpha g$-closed) [51] if $\alpha\text{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.

(v) generalized $\alpha$-closed (briefly $g\alpha$-closed) [51] if $\alpha\text{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha$-open in $(X, \tau)$.

(vi) generalized pre-regular closed (briefly $gpr$-closed) [32] if $p\text{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $(X, \tau)$.

(vii) generalized semi-pre-closed (briefly $gsp$-closed) [20] if $sp\text{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.

(viii) generalized pre-closed (briefly $gp$-closed) [48] if $p\text{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.

(ix) $\hat{g}$ (=$\omega$-closed) [61,64] if $\text{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open in $(X, \tau)$.

(x) $^{*}g$-closed set [66] if $\text{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\omega$-open in $(X, \tau)$.

(xi) $^{#}g$-semi-closed set [67] (briefly $^{#}gs$-closed) if $s\text{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $^{*}g$-open in $(X, \tau)$.

(xii) locally closed [10] if it can be represented as the intersection of an open and a closed set.

(xiii) $\alpha$-locally closed [33] if it can be represented as the intersection of an $\alpha$-closed and an $\alpha$-open set.
(xiv) sg-locally closed [36] if it can be represented as the intersection of a sg-closed and a sg-open set.

(xv) \( \hat{g} \)-locally closed [65] if it can be represented as the intersection of a \( \hat{g} \)-closed and a \( \hat{g} \)-open set.

The complement of the above mentioned closed sets from (i) to (xi) are called their respective open sets. The class of \( g \)-closed (resp sg-closed, \( \alpha g \)-closed, \( \beta g \)-closed, \( gs \)-closed, \( \alpha g \)-open, \( \beta g \)-open) sets is denoted by \( GC(X) \) (resp \( SGC(X) \), \( \alpha GC(X) \), \( \beta GC(X) \), \( GPRC(X) \), \( GPRC(X) \), \( GSPC(X) \), \( GPC(X) \), \( \hat{G}C(X) \), \( \alpha GC(X) \), \( \beta GC(X) \), \( \#GSC(X) \)). The class of all \( g \)-open (resp sg-open, \( \alpha g \)-open, \( \alpha g \)-open, \( \beta g \)-open, \( \#gs \)-open) sets is denoted as \( GO(X) \) (resp \( SGO(X) \), \( \alpha GO(X) \), \( \beta GO(X) \), \( \#GO(X) \)). The class of locally closed (resp \( \alpha \)-locally closed, sg-locally closed, \( \hat{g} \)-locally closed) sets is denoted as \( LC(X) \) (resp \( \alpha LC(X) \), \( SGLC(X) \), \( \hat{GLC}(X) \)).

### 0.2 Various forms of mappings

**Definition 0.2.1** A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is called

(i) semi-continuous [43] if \( f^{-1}(V) \) is semi-closed in \( (X, \tau) \) for every closed set \( V \) in \( (Y, \sigma) \).

(ii) pre-continuous [52] if \( f^{-1}(V) \) is pre-closed in \( (X, \tau) \) for every closed set \( V \) in \( (Y, \sigma) \).

(iii) \( \alpha \)-continuous [53] if \( f^{-1}(V) \) is \( \alpha \)-closed in \( (X, \tau) \) for every closed set \( V \) in \( (Y, \sigma) \).

(iv) \( \beta \)-continuous [1] if \( f^{-1}(V) \) is \( \beta \)-closed in \( (X, \tau) \) for every closed set \( V \) in \( (Y, \sigma) \).
(v) $g$-continuous [7] if $f^{-1}(V)$ is $g$-closed in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

(vi) $sg$-continuous [62] if $f^{-1}(V)$ is $sg$-closed in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

(vii) $gs$-continuous [17] if $f^{-1}(V)$ is $gs$-closed in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

(viii) $ga$-continuous [18] if $f^{-1}(V)$ is $ga$-closed in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

(ix) $\alpha g$-continuous [18] if $f^{-1}(V)$ is $\alpha g$-closed in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

(x) $gpr$-continuous [32] if $f^{-1}(V)$ is $gpr$-closed in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

(xi) $gsp$-continuous [20] if $f^{-1}(V)$ is $gsp$-closed in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

(xii) $gp$-continuous [3] if $f^{-1}(V)$ is $gp$-closed in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

(xiii) $\widehat{g}$-continuous [64] if $f^{-1}(V)$ is $\widehat{g}$-closed in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

(xiv) $#gs$-continuous [67] if $f^{-1}(V)$ is $#gs$-closed in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

(xv) strongly continuous [56] if $f^{-1}(V)$ is clopen in $(X, \tau)$ for every subset $V$ in $(Y, \sigma)$.

(xvi) perfectly continuous [56] if $f^{-1}(V)$ is clopen in $(X, \tau)$ for every open set $V$ in $(Y, \sigma)$.  

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(xvii) LC-continuous [31] if $f^{-1}(V)$ is locally-closed in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

(xviii) contra-continuous [21] if $f^{-1}(V)$ is closed in $(X, \tau)$ for every open set $V$ in $(Y, \sigma)$.

(xix) contra-sg-continuous [22] if $f^{-1}(V)$ is sg-closed in $(X, \tau)$ for every open set $V$ in $(Y, \sigma)$.

(xx) contra-$\alpha$-continuous [34] if $f^{-1}(V)$ is $\alpha$-closed in $(X, \tau)$ for every open set $V$ in $(Y, \sigma)$.

(xxi) contra $\alpha g$-continuous [34] if $f^{-1}(V)$ is $\alpha g$-closed in $(X, \tau)$ for every open set $V$ in $(Y, \sigma)$.

(xxii) slightly continuous[35] if $f^{-1}(V)$ is closed in $(X, \tau)$ for every clopen set $V$ in $(Y, \sigma)$.

**Definition 0.2.2** A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called

(i) $\alpha$-irresolute [46] if $f^{-1}(V)$ is $\alpha$-open in $(X, \tau)$ for every $\alpha$-open set $V$ in $(Y, \sigma)$.

(ii) $\alpha g$-irresolute [18] if $f^{-1}(V)$ is $\alpha g$-closed in $(X, \tau)$ for every $\alpha g$-closed set $V$ in $(Y, \sigma)$.

(iii) $\# gs$-irresolute [67 if $f^{-1}(V)$ is $\# gs$-closed in $(X, \tau)$ for every $\# gs$-closed set $V$ in $(Y, \sigma)$.

**Definition 0.2.3** A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called

(i) $\alpha$-closed (resp. $\alpha$-open) [53] if the image of every closed (resp. open) set in $(X, \tau)$ is $\alpha$-closed (resp. $\alpha$-open) set in $(Y, \sigma)$.

(ii) pre-$\alpha$-closed (resp. pre-$\alpha$-open) [19] if the image of every $\alpha$-closed (resp. $\alpha$-open) set in $(X, \tau)$ is $\alpha$-closed (resp $\alpha$-open) in $(Y, \sigma)$. 

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(iii) generalized semi-closed (briefly gs-closed) (resp. gs-open) [16] if the image of every closed (resp. open) set in $(X, \tau)$ is gs-closed (resp. gs-open) in $(Y, \sigma)$.

(iv) semi-generalized closed (briefly sg-closed) (resp. sg-open) [16] if the image of every closed (resp. open) set in $(X, \tau)$ is sg-closed (resp. sg-open) in $(Y, \sigma)$.

(v) pre-$\#gs$-closed (resp. pre-$\#gs$-open) [67] if the image of every $\#gs$-closed (resp. $\#gs$-open) set in $(X, \tau)$ is $\#gs$-closed (resp. $\#gs$-open) in $(Y, \sigma)$.

**Definition 0.2.4** A map $f : (X, \tau) \to (Y, \sigma)$ is called

(i) an $\alpha$-homeomorphism [18] if $f$ is bijective, $\alpha$-irresolute and pre-$\alpha$-closed.

(ii) a semi-generalized homeomorphism (briefly sg-homeomorphism) [17] if $f$ is bijective, sg-open and sg-continuous.

(iii) a generalized semi-homeomorphism (briefly gs-homeomorphism) [17] if $f$ is bijective, gs-open and gs-continuous.

(iv) an $\alpha$-quotient [38] if $f$ is surjective, $\alpha$-continuous and $f^{-1}(V)$ is an open set in $(X, \tau)$ implies $V$ is an $\alpha$-open set in $(Y, \sigma)$.

(v) a quotient map [38], provided a subset $U$ of $(Y, \sigma)$ is open in $(Y, \sigma)$ if and only if $f^{-1}(U)$ is open in $(X, \tau)$.

(vi) an $\alpha^*$-quotient map [38] if $f$ is $\alpha$-irresolute and $f^{-1}(V)$ is an $\alpha$-open set in $(X, \tau)$ implies that $V$ is an open set in $(Y, \sigma)$.

**0.3 Various forms of spaces**

**Definition 0.3.1** A topological space $(X, \tau)$ is called

(i) a $T_b$ space [16] if every gs-closed set in it is closed.
(ii) an $\alpha T_b$ space [19] if every $\alpha g$-closed set in it is closed.

(iii) an Ultra Hausdorff or clopen $T_2$ [59] if for each pair of distinct points $x$ and $y$ in $X$ there exist $U \in CO(X, x)$ and $V \in CO(Y, y)$ such that $U \cap V = \emptyset$.

(iv) an Ultra $T_1$ or clopen $T_1$ [59] if for each pair of distinct points $x$ and $y$ in $X$ there exist clopen sets $U$ and $V$ containing $x$ and $y$ respectively such that $y \notin U$ and $x \notin V$.

(v) a locally indiscrete space [14] space if every open subset is closed.

(vi) an Alexandra space [13] if arbitrary intersection of open sets is open.

(vii) a door space [23] if every set is either open or closed.

0.4 Various forms of sets in Bitopological spaces

Definition 0.4.1 A subset $A$ of a bitopological space $(X, \tau_1, \tau_2)$ is called

(i) $\tau_1\tau_2$-open[40] if $A \subseteq \tau_1 \cup \tau_2$

(ii) $\tau_1\tau_2$-closed[40] if $A^c \subseteq \tau_1 \cup \tau_2$.

Definition 0.4.2 (40) Let $A$ be a subset of $(X, \tau_1, \tau_2)$. Then $\tau_1\tau_2$-$\text{Cl}(A)$ denotes the $\tau_1\tau_2$-closure of $A$ and is defined as the intersection of all $\tau_1\tau_2$-closed sets containing $A$. Also $\tau_1\tau_2$-$\text{Int}(A)$ denotes the $\tau_1\tau_2$-interior of $A$ and is defined as the union of all $\tau_1\tau_2$-open sets contained in $A$.

Definition 0.4.3 A subset $A$ of $(X, \tau_1, \tau_2)$ is said to be

(i) $(1,2)\alpha$-open[40] if $A \subseteq \tau_1-\text{Int}(\tau_1\tau_2-\text{Cl}(\tau_1-\text{Int}(A)))$.

(ii) $(1,2)$semi-open[40] if $A \subseteq \tau_1\tau_2-\text{Cl}(\tau_1-\text{Int}(A))$

(iii) $(1,2)$pre-open[40] if $A \subseteq \tau_1-\text{Int}(\tau_1\tau_2-\text{Cl}(A))$ and

(iv) $(1,2)$semi-pre-open[40] (briefly $(1,2)$sp-open) if $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1-\text{Int}(\tau_1\tau_2-\text{Cl}(A)))$
The complement of the above mentioned sets are called their respective closed sets. The family of all \((1, 2)\alpha\)-open (resp. \((1, 2)\alpha\)-closed, \((1, 2)\)-semi-open, \((1, 2)\)-semi-closed, \((1, 2)\)-pre-open, \((1, 2)\)-pre-closed, \((1, 2)\)-semi-pre-open and \((1, 2)\)-semi-pre-closed sets is denoted by \((1, 2)\alpha O(X)\) (resp. \((1, 2)\alpha CL(X), (1, 2)SO(X), (1, 2)SCL(X), (1, 2)PO(X), (1, 2)PCL(X), (1, 2)SPO(X)\) and \((1, 2)SPCL(X)\))

**Definition 0.4.4 (40)** If \(A\) is a subset of the bitopological space \(X\), then the \((1, 2)\)-semi-closure (resp. \((1, 2)\alpha\)-closure, \((1, 2)\)-pre-closure and \((1, 2)\)-semi-pre-closure) of \(A\) is denoted by \((1, 2)sCl(A)\) (resp. \((1, 2)\alpha Cl(A), (1, 2)pCl(A)\) and \((1, 2)spcl(A)\)) and is defined as the intersection of all \((1, 2)\)-semi-closed sets (resp. \((1, 2)\alpha\)-closed sets, \((1, 2)\)-pre-closed and \((1, 2)\)-semi-pre-closed sets) containing \(A\). Similarly \((1, 2)\alpha\)-\(Int(A)\) (resp. \((1, 2)\)-\(Int(A)\) and \((1, 2)\)-\(Int(A)\)) is defined to be the union of all \((1, 2)\alpha\)-open sets (resp \((1, 2)\)-semi-open sets and \((1, 2)\)-pre-open sets) contained in \(A\).

**Remark 0.4.5 (40)** For any subset \(A\) of \(X\),

(i) \(\tau_1\)-\(Int(A) \subseteq \tau_1\tau_2\)-\(Int(A)\) and \(\tau_2\)-\(Int(A) \subseteq \tau_1\tau_2\)-\(Int(A)\),

(ii) \(\tau_1\tau_2\)-\(Cl(A) \subseteq \tau_1\)-\(Cl(A)\) and \(\tau_1\tau_2\)-\(Cl(A) \subseteq \tau_2\)-\(Cl(A)\),

(iii) \(\tau_1\tau_2\)-\(Cl(A \cap B) \subseteq \tau_1\tau_2\)-\(Cl(A) \cap \tau_1\tau_2\)-\(Cl(B)\),

(iv) \(\tau_1\tau_2\)-\(Int(A) \cup \tau_1\tau_2\)-\(Int(B) \subseteq \tau_1\tau_2\)-\(Int(A \cup B)\),

(v) \((1, 2)\alpha O(X) = (1, 2)SO(X) \cap (1, 2)PO(X)\) and

(vi) \(\tau_1\tau_2\)-\(Int(X - A) = X - \tau_1\tau_2\)-\(Cl(A)\).

**Definition 0.4.6 (41)** A subset \(A\) of \((X, \tau_1, \tau_2)\) is called a

(i) \((1, 2)\alpha\)-generalized closed set (briefly \((1, 2)\alpha g\)-closed) if \((1, 2)\alpha Cl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U \in (1, 2)\alpha O(X)\).
(ii) (1,2)semi-generalized-closed set (briefly (1,2)sg-closed) if \( (1,2)sCl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \in (1,2)SO(X) \).

(iii) (1,2)-generalized-semi-closed set (briefly (1,2)gs-closed) if \( (1,2)sCl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \in (1,2)oS(X) \).

(iv) (1,2)generalized-semi-pre-closed (briefly (1,2)gsp-closed set) if \( (1,2)spCl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \in (1,2)αO(X) \).

(v) (1,2)pre-generalized-closed (briefly (1,2)pg-closed) if \( (1,2)pCl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \in (1,2)PO(X) \).

(vi) (1,2)generalized-pre-closed (briefly (1,2)gp-closed) if \( (1,2)pCl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \in (1,2)αO(X) \).

The complement of the above mentioned sets are called their respective open sets. The family of all \( (1,2)αg \)-closed sets (resp. \( (1,2)sg \)-closed sets, \( (1,2)gs \)-closed sets, \( (1,2)gsp \)-closed sets, \( (1,2)pg \)-closed sets and \( (1,2)gp \)-closed sets) is denoted by \( (1,2)αGCL(X) \) (resp \( (1,2)SGCL(X) \), \( (1,2)GSCL(X) \), \( (1,2)GSPCL(X) \), \( (1,2)PGCL(X) \) and \( (1,2)GPCL(X) \)). The family of all \( (1,2)αg \)-open sets (resp. \( (1,2)sg \)-open sets, \( (1,2)gs \)-open sets, \( (1,2)gsp \)-open sets, \( (1,2)pg \)-open sets and \( (1,2)gp \)-open sets) is denoted by \( (1,2)αO(X) \)(resp \( (1,2)SGO(X) \), \( (1,2)GSO(X) \), \( (1,2)GSPO(X) \), \( (1,2)PGO(X) \) and \( (1,2)GPO(X) \)).

**Definition 0.4.7** A subset \( A \) of a bitopological space \((X, τ_1, τ_2)\) is called a

(i) \((τ_i, τ_j)\)-g-closed set[29] if \( τ_j-Cl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is open in \( τ_i \)

(ii) \((τ_i, τ_j)\)-gp-closed set[27] if \( τ_j-pCl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is open in \( τ_i \)

(iii) \((τ_i, τ_j)\)-gpr-closed set[9] if \( τ_j-pCl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is regular open in \( τ_i \)
(iv) $(\tau_i, \tau_j)$-$\omega$-closed set if $\tau_j$-$\text{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open in $\tau_i$

(v) $(\tau_i, \tau_j)$-$g^*$-closed set if $\tau_j$-$\text{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $g$-open in $\tau_i$

The family of all $(\tau_i, \tau_j)$-$g$-closed (resp $(\tau_i, \tau_j)$-$gp$-closed, $(\tau_i, \tau_j)$-$gpr$-closed, $(\tau_i, \tau_j)$-$\omega$-closed and $(\tau_i, \tau_j)$-$g^*$-closed) subsets of a botopological space $(X, \tau_1, \tau_2)$ is denoted by $D(\tau_i, \tau_j)$ (resp $GPC(\tau_i, \tau_j), \zeta(\tau_i, \tau_j), C(\tau_i, \tau_j)$ and $D^*(\tau_i, \tau_j)$).

**Definition 0.4.8 (41)** A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be

(i) $(1,2)\alpha$-continuous if the inverse image of every $(1,2)\alpha$-closed set in $Y$ is $(1,2)\alpha$-closed in $X$.

(ii) $(1,2)sg$-continuous if the inverse image of every $(1,2)\alpha$-closed set in $Y$ is $(1,2)sg$-closed in $X$.

(iii) $(1,2)gsp$-continuous if the inverse image of every $(1,2)\alpha$-closed set in $Y$ is $(1,2)gsp$-closed in $X$.

(iv) $(1,2)ag$-irresolute if the inverse image of every $(1,2)ag$-closed set in $Y$ is $(1,2)ag$-closed in $X$.

(v) $(1,2)ag$-continuous if the inverse image of every $(1,2)\alpha$-closed set in $Y$ is $(1,2)ag$-closed in $X$.

**Definition 0.4.9** A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be

(i) $(\tau_i, \tau_j)$-$gp$-$\sigma_k$-continuous if the inverse image of every $\sigma_k$-closed set in $(Y, \sigma_1, \sigma_2)$ is $(\tau_i, \tau_j)$-$gp$-closed in $(X, \tau_1, \tau_2)$.

(ii) $\zeta(\tau_i, \tau_j)$-$\sigma_k$-continuous if the inverse image of every $\sigma_k$-closed set in $(Y, \sigma_1, \sigma_2)$ is $(\tau_i, \tau_j)$-$gpr$-closed in $(X, \tau_1, \tau_2)$.
(iii) $D^*(\tau_i, \tau_j)$-$\sigma_k$-continuous[58] if the inverse image of every $\sigma_k$-closed set in $(Y, \sigma_1, \sigma_2)$ is $(\tau_i, \tau_j)$-$g^*$-closed in $(X, \tau_1, \tau_2)$,

(iv) $D(\tau_i, \tau_j)$-$\sigma_k$-continuous[29] if the inverse image of every $\sigma_k$-closed set in $(Y, \sigma_1, \sigma_2)$ is $(\tau_i, \tau_j)$-$g$-closed in $(X, \tau_1, \tau_2)$,

(v) $\tau_j$-$\sigma_k$-continuous[49] if the inverse image of every $\sigma_k$-closed set in $(Y, \sigma_1, \sigma_2)$ is $\tau_j$-closed in $(X, \tau_1, \tau_2)$,

(vi) bi-continuous[49] if $f$ is $\tau_1$-$\sigma_1$-continuous and $\tau_2$-$\sigma_2$-continuous,

(vii) strongly bi-continuous[49] if $f$ is bi-continuous, $\tau_1$-$\sigma_2$-continuous and $\tau_2$-$\sigma_1$-continuous.

**Definition 0.4.10** A space $(X, \tau_1, \tau_2)$ is called

(i) an Ultra-$T_{1/2}$-space[41] if every $(1, 2)\alpha g$-closed in it is $(1, 2)\alpha$-closed.

(ii) an Ultra-semi-$T_{1/2}$-space[41] if every $(1, 2)sg$-closed set in it is $(1, 2)$semi-closed.