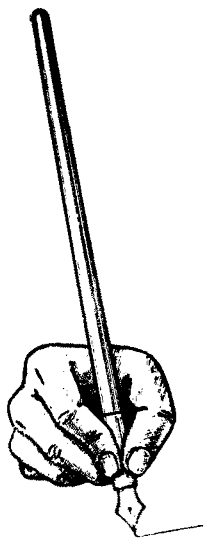

CHAPTER - 6

STUDIES ON THERMAL EFFCTS, NONSTEADY STATE SELF-FOCUSING AND GROWTH OF GAUSSIAN INSTABILITIES FOR TEM_{10} AND HIGHER ORDER MODE LASER BEAMS

In this Chapter we have dealt with thermal focusing/defocusing, non-steady state self-focusing in a relaxing medium and the growth of Gaussian instabilities in laser beams of TEM_{10} and TEM_{20} modes.

6.1 THERMAL FOCUSING/DEFOCUSING OF TEM_{10} AND TEM_{20} MODE LASER BEAMS

In the past a number of attempts have been made both theoretically and experimentally to investigate thermal focusing/defocusing in liquids and dyes.[1-6] Akhmanov et al[1,2] have proposed a theory for thermal self-defocusing of a laser beam. Ghatak and Sharma[7] have modified this theory to include diffraction effect and applied the same to analyze the experimental data.[6] Sodha[8] and Sodha et al[9] also have given a simplified treatment of the steady state thermal effects. In all the theoretical studies mentioned above the propagation of a Gaussian laser beam and the consequent creation of a radial gradient of temperature (and hence of dielectric constant) is seen to be the starting point of investigations. Thermal self-focusing occurs when this gradient is positive, while defocusing takes place for its negative value. We have examined the possibility of such effects for TEM_{10} and TEM_{20} mode laser beams.

6.1.1 Steady State Thermal Effects with TEM_{10} Laser Beam

We have adopted the procedure of Sodha[8] for this study.

Let P = power of a cylindrically symmetric beam at any point z

P_o = power at z = 0

The beam axis is supposed to be coincident with the z-axis. Then

$$P = P_o \exp(-\delta z) \qquad \ldots (6.1)$$

where δ = intensity absorption coefficient of the beam.

The intensity distribution of a cylindrically symmetric TEM_{10} beam is given by

$$A_{10}^2 = \frac{E_o^2}{f^2}\left[1 - \frac{r^2}{r_o^2 f^2}\right]^2 \exp(-r^2/r_o^2 f^2) \qquad \ldots (6.2)$$

where the symbols have their meanings as given in Chapter 3.

The power P(r) of the beam on a circle of radius r is defined as

$$P(r) = \int_o^r A_{10}^2 \, 2\pi r \, dr \qquad \ldots (6.3)$$

Also $$P = \int_0^\infty A_{10}^2 \, 2\pi r \, dr \qquad \ldots (6.4)$$

Using eq.(6.2) in (6.3) and (6.4) the integrals are evaluated as given below :

$$\int_0^r A_{10}^2 \, 2\pi r \, dr = \pi r_o^2 E_0^2 \left[(x^2+4x+3)e^{-x} - 3\right] \qquad \ldots (6.5)$$

and $$\int_0^\infty A_{10}^2 \, 2\pi r \, dr = \pi r_o^2 E_0^2 \qquad \ldots (6.6)$$

where $x = r^2 / r_o^2 f^2$

Hence we have

$$\frac{P(r)}{P} = [(x^2 + 4x + 3)e^{-x} - 3]$$

$$\text{or} \quad P(r) = P_o e^{-\delta z} [(x^2 + 4x + 2) e^{-x} - 3] \qquad \ldots (6.7)$$

(Using eq.(6.1))

Imagine a cylinder of length Δz and radius r with its axis along z-axis. In steady state neglecting the transmission of heat along the axis, the power absorbed from the beam in the cylinder will be conducted normal to its curved surface. Therefore we can write

$$-2\pi r \, \Delta z \, K \frac{\partial T}{\partial r} = - \frac{\partial P(r)}{\partial z} \Delta z$$

$$\text{or} \quad \frac{\partial T}{\partial r} = \frac{1}{2\pi r K} \frac{\partial P(r)}{\partial z} \qquad \ldots (6.8)$$

with K = thermal conducti··ity of the medium

Differentiating eq.(6.7) w.r.t. z we get

$$\frac{\partial P(r)}{\partial z} \simeq \delta \, P_o [3 - (3+ \frac{4t}{f^2} + \frac{t^2}{f^4}) \exp(- \frac{t}{f^2} - \delta z)] \qquad \ldots (6.9)$$

where $t = r^2/r_o^2$

In deriving this expression we have neglected the terms involving df/dz because experimentally the beam gets diverted only by a few minutes after passing through the medium i.e.

$\frac{df}{dz}$ has a negligibly small value during this passage.

Putting eq.(6.9) in (6.8) we obtain

$$\frac{\partial T}{\partial r} \simeq \frac{\delta P_o}{2\pi r K}\left[\left(3+\frac{4r^2}{r_o^2 f^2}+\frac{r^4}{r_o^4 f^4}\right)\exp\left(-\frac{r^2}{r^2 f^2}\right)-3\right]\exp(-\delta z) \quad \ldots (6.10)$$

It is obvious that $\left(\frac{\partial T}{\partial r}\right)_{r=0} \simeq 0$. In order to consider the temperature close to the axis we adopt the condition that $r \ll r_o$ so that eq.(6.10) can be approximated to

$$\frac{\partial T}{\partial r} \simeq \frac{\delta P_o}{2\pi K}\left[-\frac{r^5}{r_o^6 f^6}-\frac{3r^3}{r_o^4 f^4}+\frac{r}{r_o^2 f^2}\right]\exp(-\delta z) \quad \ldots (6.11)$$

Differentiating again

$$\frac{\partial^2 T}{\partial r^2} = -\frac{\delta P_o}{2\pi K}\left[-\frac{5r^4}{r_o^6 f^6}-\frac{9r^2}{r_o^4 f^4}+\frac{1}{r_o^2 f^2}\right]\exp(-\delta z)$$

Hence

$$\left(\frac{\partial^2 T}{\partial r^2}\right)_{r=0} \simeq -\frac{\delta P_o}{2\pi K r_o^2 f^2}\exp(-\delta z) \quad \ldots (6.12)$$

This is the same result as the eq.(6.5) of Ref.(9). Obviously the eqs.(6.6) - (6.13) of the said reference are also valid for the TEM_{10} case.

6.1.2. Thermal Effects With Donought Mode

For 'donought' mode we have

$$A_{01}^{2*} = \frac{E_o^2}{r_o^2 f^4} r^2 \exp(-r^2/r_o^2 f^2) \quad \ldots (6.13)$$

Hence the following integrals are evaluated

$$\int_0^r A_{01}^{2*} \, 2\pi r \, dr = \pi r_o^2 E_o^2 \left[1 - \exp(-x) - x \exp(-x)\right] \quad \ldots (6.14)$$

and

$$\int_0^\infty A_{01}^{2*} \, 2\pi r \, dr = \pi r_o^2 E_o^2 \quad \ldots (6.15)$$

where $x = r^2/r_o^2 f^2$

Then

$$\frac{P(r)}{P} = 1 - \exp(-x) - x \exp(-x)$$

i.e. $P(r) = P_o e^{-\delta z} \left[1 - \exp(-x) - x \exp(-x)\right]$ ... (6.16)

Differentiating w.r.t. z we get

$$\frac{\partial P(r)}{\partial z} \simeq - \delta P_o e^{-\delta z} \left[1 - \exp(-r^2/r_o^2 f^2 - (r^2/r_o^2 f^2)\exp(-r^2/r_o^2 f^2)\right] \quad \ldots (6.17)$$

In getting this equation we have neglected the terms involving $\frac{df}{dz}$ on the basis of argument made in Sec.6.1.1. With the help of eq.(6.8) in the steady state we obtain

$$\frac{\partial T}{\partial r} = \frac{\delta P_o}{2\pi r K} \left[1 - \exp(-r^2/r_o^2 f^2) - (r^2/r_o^2 f^2) \exp(-r^2/r_o^2 f^2)\right] e^{-\delta z} \quad \ldots (6.18)$$

Obviously $\left(\frac{\partial T}{\partial r} \right)_{r=0} = 0$

Applying the condition $r << r_o f$, eq.(6.18) can be approximated to

$$\frac{\partial T}{\partial r} = - \frac{\delta P_o}{2\pi r K} \left(\frac{r^4}{r_o^4 f^4} \right) \exp(-\delta z)$$

so that

$$\frac{\partial^2 T}{\partial r^2} = - \frac{2 \delta P_o}{\delta K r_o^4 f^4} r^2 \exp(-\delta z) \qquad \ldots (6.19)$$

Since $\left(\frac{\partial^2 T}{\partial r^2} \right)_{r=0} = 0$ it is obvious from eq.(6.6) of Ref.(9) that

$$T(r) - T(0) \simeq 0$$

which means $\Phi = 0$.

Thus there is no nonlinearity induced on account of variation of dielectric constant with temperature. The thermal focusing/defocusing is not possible for 'donought' mode.

6.1.3 Time Dependent Thermal Effects With TEM_{10} Laser Beam

As mentioned earlier Sodha et al[9] have reviewed the theory of Akhmanov and co-workers[1,2] for thermal self-focusing in which the diffraction effect was included by Ghatak and Sharma.[7] The dielectric constant due to thermal self-action is given by

$$\epsilon = \epsilon_{or} - i\,\epsilon_{oi} + \frac{d\,\epsilon_{or}}{d\,T} T(r, z, t) \qquad \ldots (6.20)$$

where ϵ_{or} and ϵ_{oi} are the real and imaginary parts ($\epsilon_{oi} << \epsilon_{or}$), while $\frac{d\,\epsilon_{or}}{d\,T}$ represents the rate of change of ϵ_{or} with temperature. The temperature rise is given by T(r, z, t) which can be obtained from the heat conduction equation :

$$\rho\, C_p \frac{\partial T}{\partial t} = K \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + Q \qquad \ldots (6.21)$$

where ρ = density (gm/cc)

C_p = sp. heat (cal/gm^{o} K)

Q = source term (Cal/cm^3 sec)

K = thermal conductivity (cal/cm sec ^{o}K)

In eq. (6.21) the term with $\frac{\partial^2 T}{\partial z^2}$ is neglected which is justified when the nonlinear part of the dielectric constant

$$\phi = \frac{d\,\epsilon_{or}}{dT} T << (\epsilon_{or} - i\, \epsilon_{oi})$$

Now for an inhomogeneous and absorbing medium (characterized by eq.(6.20) we can obtain the following equation for beam-width parameter[10]

$$\frac{1}{f} \frac{d^2 f}{dz^2} = \frac{1}{2} \frac{1}{\epsilon_{or}} \frac{d\,\epsilon_{or}}{dT} \left(\frac{1}{r} \frac{\partial T}{\partial r} \right) + \frac{1}{k_o^2 r_o^2 f^4} \qquad \ldots (6.22)$$

with k_o as the real part of the wave vector k. This equation can be solved if we know $\frac{\partial T}{\partial r}$ from the solution of eq.(6.21)

By definition, the power of TEM_{10} beam would be

$$P = \int_0^{\infty} \frac{c}{n} \frac{\epsilon_{or}}{8\pi} \left\{ \frac{E_o^2}{f^2}\left[1 - \frac{r^2}{r_o^2 f^2}\right]\exp\left(-\frac{r^2}{r_o^2 f^2}\right)\right\} e^{-\delta z}\, 2\pi r\, dr$$

$$\simeq P_o\, e^{-\delta z} \qquad \ldots (6.23)$$

which has been obtained by neglecting r^3 and higher terms under $r << r_o f$ and using the relations given below

$$\epsilon_{or} = n^2 \text{ and } P_o \simeq \frac{nc}{8} E_o^2\, r_o^2$$

Similarly the expression for the source term Q arising from the heating of the medium by the TEM_{10} laser beam is found to be

$$Q \simeq \left[\frac{1}{J} \frac{P_o\, \delta\, e^{-\delta z}}{\pi\, f^2 r_o^2} - \frac{2P_o\, \delta\, e^{-\delta z}}{J\, \pi\, f^4 r_o^4} r^2 \right] \exp(-r^2/r_o^2 f^2) \qquad \ldots (6.24)$$

where J = 4.2 joules/cal.

Let $T = T_1 + T_2$ be the solution of eq.(6.21) such that

$$K \left[\frac{d^2 T_2}{dr^2} + \frac{1}{r}\frac{d\,T_2}{dr} \right] = -Q \qquad \ldots (6.25)$$

Equating eqs.(6.24) and (6.25) and integrating w.r.t. r we have got

$$r\frac{dT_2}{dr} = -\frac{P_o\, \delta\, e^{-\delta z}}{2KJ\, \pi}\left[1 + \exp(-r^2/r_o^2 f^2) \right]$$

$$- \frac{P_o\, \delta\, e^{-\delta z}}{K\, J\, \pi} \frac{r^2}{r_o^2 f^2} \exp(-r^2/r_o^2 f^2) \qquad \ldots (6.26)$$

Now T_1 (r, z, t) satisfies

$$\rho c_p \frac{\partial T_1}{\partial t} = K \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T_1}{\partial r} \right)$$

which after differentiation gives

$$\rho c_p \frac{\partial \psi}{\partial t} = r \frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial \psi}{\partial r} \right] \qquad \ldots (6.27)$$

with $\psi = r \frac{\partial T_1}{\partial r}$

This equation can be solved by the Laplace transform technique under the boundary conditions that at $t = 0$, $r \frac{\partial T}{\partial r} = 0$ and that $\frac{\partial T}{\partial r}$ is finite everywhere

$$r \frac{\partial T_1}{\partial r} = \psi = \frac{P_o \delta e^{-\delta z}}{2\pi K J} \left[1 - \exp \left\{ - \frac{r^2}{r_o^2 (f^2 + \frac{t}{\mathcal{J}})} \right\} \right] \qquad \ldots (6.28)$$

with $\mathcal{J} = r_o^2 \rho c_p / 4K$

Adding eqs.(6.26) and (6.28) we get the complete solution of eq.(6.21) for $r \ll r_o f$

$$r \frac{\partial T}{\partial r} \simeq - \frac{P_o \delta e^{-\delta z}}{2\pi KJ r_o^2 f^2} r^2 \frac{1}{(1 + \frac{f^2 \mathcal{J}}{t})}$$

$$\therefore \frac{1}{r} \frac{\partial T}{\partial r} \simeq \frac{P_o \delta e^{-\delta z}}{2\pi K J r^2 f^2} \left[1 + \frac{f^2 \mathcal{J}}{t} \right]^{-1} \qquad \ldots (6.29)$$

which is found to be the same as eq.(6.30) of Ref.(9). Putting this in eq.(6.22) we obtain the differential equation

$$\frac{d^2f}{dz^2} = \frac{P\,\exp(-\delta z)}{f(1 + \frac{f^2 \mathcal{J}}{t})} + \frac{\beta}{f^3} \qquad \ldots (6.30)$$

with $P = -\frac{dn}{dT}\,\frac{P_o\,\delta}{2\pi\, n_o r_o^2\, KJ}$, $\beta = \frac{1}{k_o^2\, r_o^4}$

For most materials $\frac{dn}{dT} < 0$ so that $p > 0$

Hence the numerical solution of eq.(6.30) and the calculations therefrom would follow the results for the Gaussian laser beam case.

6.2 NONSTEADY-STATE SELF-FOCUSING OF TEM_{10} AND HIGHER ORDER MODES

In order the self-focusing to be stronger than diffraction, a large change in refractive index ($\Delta n \sim 10^{-7}$ esu) is required to be induced in the medium. Such a large Δn can be produced only by a laser intensity higher than several MW/cm^2 normally available with pulsed lasers. The amplitude variation of the input laser pulse leads to the involvement of time dependence in the self-focusing phenomena. The focusing of a nonstationary e.m. wave in a nonlinear medium has been investigated in the past.[11,12] The characteristic features of this wave propagation are listed below :

i) The displacement vector $\bar{D}$ depends both on the instantaneous and the past values of $\bar{E}$ vector. This gives rise to temporal dispersion.

ii) The high amplitude portion of the pulse is focused at a smaller distance in the medium as compared to the low amplitude portion. Consequently the amplitude envelope of the pulse is distorted thereby leading to the formation of a filament as the locus of moving foci.

iii) High and low amplitude portions have different phase velocities. The phase of the disturbance is therefore, modulated according to its amplitude.

iv) The relaxation time of the nonlinearity (J) is finite. As a result, either initial or lagging part of the pulse may not be focused even when the power of the beam exceeds its critical value.

Considering the propagation of a laser pulse along z-axis in a relaxing nonlinear dielectric the following coupled equations for intensity and eikonal can be derived[11]

$$\frac{\partial A_{oo}^2}{\partial z} + \frac{\partial S}{\partial r}\frac{\partial A_{oo}^2}{\partial r} + A_{oo}^2\left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right) = 0 \qquad \ldots (6.31)$$

$$2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{\phi}{\epsilon_o} + \frac{1}{k^2 A_{oo}}\left(\frac{\partial^2 A_{oo}}{\partial r^2} + \frac{1}{r}\frac{\partial A_{oo}}{\partial r}\right) \quad \ldots (6.32)$$

where ϕ = nonlinear dielectric constant at time t

$$= \frac{e_2\, e^{-t/\mathcal{T}}}{\mathcal{T}} \int_{-\infty}^{t} e^{t/\mathcal{T}}\, EE^* \, dt \quad \ldots (6.33)$$

$$S = S_o(\xi) + \frac{\phi(t)}{2\epsilon_o} z,$$

$\xi = t - \frac{z}{v_g}$, v_g = group velocity

$A_{oo}^2 = E_{oo}^2 F\left(t - \frac{z}{v_g}\right)$, E_{oo} = constant (normalizing field)

We assume that the propagating laser pulse has an intensity distribution corresponding to TEM_{10} mode. Therefore we take the following relations as the solutions of eqs.(6.31) and (6.32)

$$A_{10}^2 = \frac{E_{10}^2(\xi)}{f_1^2(\xi, z)}\left[1 - 2\rho + \rho^2\right]\exp(-\rho) \quad \ldots (6.34)$$

and

$$S = \frac{r^2}{z}\beta(\xi, z) + \phi_1(\xi, z) \quad \ldots (6.35)$$

with

$$\beta(\xi, z) = \frac{1}{f_1(\xi, z)}\frac{\partial}{\partial z} f_1(\xi, z)$$

$$\rho = r^2/r_o^2 f_1^2(\xi, z)$$

Under paraxial ray approximation eq.(6.34) is approximated to

$$A_{10}^2 \simeq A_{10}^2 \Big|_{r=0} \left(1 - \frac{3r^2}{r_o^2\, f_1^2}\right) \qquad \ldots (6.36)$$

where $A_{10}^2 \Big|_{r=0} = E_{10}^2 / f_1^2$

By the usual method the LHS of eq.(6.32) can be evaluated from eq.(6.35)

$$2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{1}{f_1}\frac{d^2 f_1}{dz^2} r^2 + \frac{2\,\partial \phi_1}{\partial z} \qquad \ldots (6.37)$$

Similarly the RHS of eq.(6.32) is obtained as

$$\text{RHS} = \frac{\phi_1}{\epsilon_o} + \frac{r^2}{k^2 r_o^4\, f_1^4} \qquad \ldots (6.38)$$

Now using eqs.(6.33) and (6.36) with the introduction of variable ξ we can get

$$\phi_1 \simeq \frac{e_2\, \bar{e}^{\xi/J}}{J}\left[\int_{-\infty}^{\xi} \frac{E_{10}^2\, e^{\xi/J}}{f_1^2}\, d\xi - \frac{3r^2}{r_o^2}\int_{-\infty}^{r} \frac{E_{10}^2\, e^{\xi/J}}{f_1^4}\, d\xi\right] \qquad \ldots (6.39)$$

From eqs.(6.37) - (6.39) after equating the coefficients of r^2 from both the sides of the resulting equation we obtain

$$\frac{1}{f_1}\frac{\partial^2 f_1}{\partial z^2} = \frac{1}{R_d^2\, f_1^4} - \frac{3\, e_2}{J}\frac{\bar{e}^{\xi/}}{\epsilon\, r_o^2}\int_{-\infty}^{r} \frac{E_{10}^2}{f_1^4}\, e^{\xi/J}\, d\xi \qquad \ldots (6.40)$$

We can take E_{10}^2 and f_1^4 to be constants provided the characteristic time of their variation is negligibly small as compared to . Then evaluating the remaining integral we have

$$\frac{1}{f_1}\frac{\partial^2 f_1}{\partial z^2} = \frac{1}{R_d^2 f_1^4} - \frac{3\epsilon_2}{\epsilon_o r_o^2}\frac{E_{10}^2(\xi)}{f_1^4} \quad \ldots (6.41)$$

This can be solved under the usual initial conditions (viz. $f_1(\xi,0) = 1$ and $\left.\frac{\partial f_1}{\partial z}\right|_{z=0} = 0$) to obtain

$$f_1^2 = 1 - \left[\frac{3E_{10}^2}{E_{oo}^2}\frac{1}{R_{nl}^2} - \frac{1}{R_d^2} \right] z^2 \quad \ldots (6.42)$$

with $R_{nl} = r_o \sqrt{\epsilon_o/\epsilon_2 E_{oo}^2}$, $R_d = kr_o^2$

Eq.(6.42) differs from eq.(10.31) of Ref.(11) by a factor 3 multiplied to second RHS term. From this equation we can easily deduce the length z_R at which $f = 0$

$$(z_R)^{-1} = \left[\frac{3E_{10}^2(\xi)}{E_{oo}^2}\frac{1}{R_{nl}^2} - \frac{1}{R_d^2} \right]^{1/2} \quad \ldots (6.43)$$

Now for a TEM_{00} beam which is Gaussian in space and time we have

$$\frac{E_o^2(t,z)}{E_{00}^2} = \exp\left[- \left(\frac{t}{\mathcal{T}} - \frac{z}{\mathcal{T} v_g} \right)^2 \right]$$

By analogy we write the following expression for the TEM_{10} beam

$$\frac{E_{10}^2(\xi)}{E_{00}^2} = [1 - (\frac{\xi}{\mathcal{T}})^2]^2 \exp[-(\frac{\xi}{\mathcal{T}})^2] \qquad \ldots (6.44)$$

Putting this in eq.(6.43) and using the notation $\gamma = (R_d/R_{nl})^2$ we get

$$(\frac{Z_R}{R_d}) = \{\gamma[1-(\xi/\mathcal{T})^2]^2 \exp[-(\xi/\mathcal{T})^2]-1\}^{-1/2} \qquad \ldots (6.45)$$

With the help of this equation we can rewrite eq.(6.42) as

$$f_1^2 = 1 - \{\gamma[1-(\xi/\mathcal{T})^2]^2 \exp[-(\xi/\mathcal{T})^2]-1\} (Z/R_d)^2 \qquad \ldots (6.46)$$

Now

$$A_{10}^2 \Big|_{r=0} = \frac{E_{10}^2(\xi)}{f_1^2(\xi, Z)}$$

$$= \frac{E_{00}^2 [1-(\xi/\mathcal{T})^2]^2 \exp[-(\xi/\mathcal{T})^2]}{f_1^2(\xi, \mathcal{T})} \quad \text{using eq.(6.44)}$$

$$\therefore \quad \frac{A_{10}^2(r=0, \xi)}{E_{00}^2} = \frac{[1-(\xi/\mathcal{T})^2]^2 \exp[-(\xi/\mathcal{T})^2]}{f_1^2(\xi, \mathcal{T})} \qquad \ldots (6.47)$$

Using eqs.(6.45) - (6.47) we can calculate the location of focus point and the axial intensity of a TEM_{10} mode laser beam as a function of time.

Proceeding as above we have obtained the relevant expressions for a TEM_{20} mode laser beam as listed below :

$$\left(\frac{Z_R}{R_d}\right) = \left\{ \gamma\left[1-2\left(\frac{\xi}{\tau}\right)^2 + \frac{1}{2}\left(\xi/\tau\right)^4\right]^2 \exp\left[-\left(\xi/\tau\right)^2\right]-1\right\}^{-\frac{1}{2}} \quad \ldots (6.48)$$

$$f_2^2 = 1 - \left\{ \gamma\left[1-2\left(\xi/\tau\right)^2 + \frac{1}{2}\left(\xi/\tau\right)^4\right]^2 \exp\left[-\left(\xi/\tau\right)^2\right] - 1 \right\} \left(\frac{z}{R_d}\right)^2 \quad \ldots (6.49)$$

$$\frac{A_{20}^2(r=0,\xi)}{E_{00}^2} = \frac{\left[1-2\left(\xi/\tau\right)^2 + \frac{1}{2}\left(\xi/\tau\right)^4\right]^2 \exp\left[-\left(\xi/\tau\right)^2\right]}{f_2^2(\xi,\tau)} \quad \ldots (6.50)$$

The calculations of moving foci and the axial intensities as functions of time are presented and discussed in Sec.(6.4).

6.3 GROWTH OF GAUSSIAN INSTABILITIES IN TEM_{10} AND HIGHER ORDER LASER BEAMS

So far we have not considered any fluctuation in the intensity of the incident beam. A small increment in the intensity in any part of the beam causes an increase in the dielectric constant in that region[13]. This part of the region attracts more and more power from the surrounding portion of the beam. This perturbation grows as the beam propagates inside the medium provided its life time is greater than the nonlinearity relaxation time. Such fluctuations in very intense laser beams finally lead to the formation of multiple filaments in the medium. The analysis of Gaussian type of perturbation superposed on a laser beam with a smooth Gaussian intensity profile has been carried out by different workers.[14-16] In the present work we have considered the growth of Gaussian instabilities which are present in the otherwise smooth intensity profiles of isolated TEM_{10} and higher order mode laser beams. We have adopted the treatment proposed by Abbi and Kothari.[16]

For a linearly polarized quasimonochromatic e.m.wave propagating along the z direction in a medium, Abbi and Kothari[16] have given the coupled equations for the intensity and eikonal in slightly different forms

$$2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{\epsilon_2}{\epsilon_o}\left|A_o\right|^2 + \frac{1}{k^2 A_o}\nabla_\perp^2 A_o$$

$$\frac{\partial A_o}{\partial z} + \left(\frac{\partial S}{\partial r}\right)\left(\frac{\partial A_o}{\partial r}\right) + \frac{A_o}{2}\nabla_\perp^2 S = 0 \qquad \ldots (6.51)$$

where $\nabla_\perp^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$

$$A = A_o \exp(-ikS)$$

Eqs.(6.51) are equivalent to those given in Sec.(2.2)

We consider a single spike of Gaussian profile propagating co-axially with the main TEM_{10} laser beam

$$A = (A_{10} + e_1 + ie_2)\exp(-ikS) \qquad \ldots (6.52)$$

with $e_1 + ie_2$ = complex electric field of perturbation with

$e_1, e_2 << A_{10}$

A_{10} = unperturbed amplitude of TEM_{10} beam

Putting eq.(6.52) in (6.51) the following two coupled equations can be obtained[16]:

$$\frac{e_1}{A_{10}} \nabla_\perp^2 A_{10} - 2k \frac{\partial e_2}{\partial z} = \nabla_\perp^2 e_1 + k e_2 \nabla_\perp^2 S + 2k \frac{\partial S}{\partial r} \frac{\partial e_2}{\partial r} + \frac{2\epsilon_2}{e_o} k^2 \left| A_{10} \right|^2 e_1 \qquad \ldots (6.53)$$

and

$$\frac{e_2}{A_{10}} \nabla_\perp^2 A_{10} + 2k \frac{\partial e_1}{\partial z} = \nabla_\perp^2 e_2 - k e_1 \nabla_\perp^2 S - 2k \frac{\partial S}{\partial r} \frac{\partial e_1}{\partial r} \qquad \ldots (6.54)$$

For TEM_{10} laser beam we take

$$A_{10}^2 = A_{10}^2 \Big|_{r=0} \left(1 - \frac{r^2}{r_o^2 f^2}\right)^2 \exp\left(- \frac{r^2}{r_o^2 f^2}\right)$$

$$\simeq A_{10}^2 \Big|_{r=0} \left(1 - \frac{3r^2}{r_o^2 f^2}\right) \qquad \ldots (6.55)$$

(for $r << r_o f$)

Also $$S(r,Z) = \phi(z) + \frac{1}{2} r^2 \beta(z) \qquad \ldots (6.56)$$

$$e_{1,2} = e_{10,20} \exp\left[\alpha(z)\right] \exp\left[- \frac{r^2}{2b^2(z)} \right] \qquad \ldots (6.57)$$

where $\alpha(z)$ = perturbation growth parameter

$b(z)$ = size of Gaussian perturbation

From eqs.(6.55) - (6.57) we obtained the expressions given below :

$$\frac{\partial e_{1,2}}{\partial z} = e_{10,20} \left[\frac{d\alpha}{dz} + \frac{r^2}{b^3} \frac{db}{dz} \right] \exp\left[\alpha(z)\right] \exp\left[-\frac{r^2}{2b^2}\right] \quad \ldots (6.58)$$

$$\nabla_\perp^2 e_{1,2} = -\frac{2 e_{10,20}}{b^2} \exp(\alpha) \exp(-r^2/2b^2)$$

$$+ \frac{e_{10,20}}{b^4} \exp(\alpha)\, r^2 \exp(-r^2/2b^2) \quad \ldots (6.59)$$

$$\nabla_\perp^2 S = 2\beta(z) \text{ and } \frac{\partial S}{\partial r} = r\,\beta(z) \quad \ldots (6.60)$$

$$\frac{\partial e_{1,2}}{\partial r} = -e_{10,20} \exp(\alpha) \frac{r}{b^2} \exp(-r^2/2b^2) \quad \ldots (6.61)$$

By making use of these results in eq.(6.53) and equating the coefficients of terms without r on both sides of the resulting equation we got

$$\frac{d\alpha}{dz} = \frac{1}{k} \left(\frac{e_{10}}{e_{20}}\right) \left[-\frac{3}{r_o^2 f^2} + \frac{1}{b^2} - \frac{\epsilon_2}{\epsilon_o} k^2 \frac{E_o^2}{f^2}\right] - \beta(z) \quad \ldots (6.62)$$

Similarly equating the coefficients of r^2 terms we obtain

$$\frac{db}{dz} = \frac{b^3}{2k} \left\{ \left[\frac{1}{r_o^4 f^4} - \frac{1}{b^4} + \frac{6\epsilon_2 E_o^2 k^2}{\epsilon_o r_o^2 f^4 b^3} \right] \left(\frac{e_{10}}{e_{20}}\right) + \frac{2k}{b^2} \beta(z) \right\} \quad \ldots (6.63)$$

By the above described procedure eq.(6.54) leads to the following relation :

$$\frac{d\alpha}{dz} = \frac{3}{r_o^2 f^2 k} \left(\frac{e_{20}}{e_{10}} \right) - \frac{1}{kb^2} \left(\frac{e_{20}}{e_{10}} \right) \qquad \ldots (6.64)$$

which can be rewritten as

$$\frac{e_{10}}{e_{20}} = [(3/r_o^2 f^2 k) - 1/kb^2] / \left(\frac{d\alpha}{dz} + \beta \right) \qquad \ldots (6.65)$$

Putting this value in eq.(6.62) and solving the resulting quadratic equation we obtain

$$\frac{d\alpha}{dz} = \frac{1}{k} \left\{ \left(\frac{1}{b^2} - \frac{3}{r_o^2 f^2} \right) \left[\frac{\epsilon_2}{\epsilon_o} \frac{k^2 E_o^2}{f^2} - \left(\frac{1}{b^2} - \frac{3}{r_o^2 f^2} \right) \right] \right\}^{\frac{1}{2}} - \beta(z) \qquad \ldots (6.66)$$

This is the equation for the perturbation growth parameter related to TEM_{10} mode beam. It is analogous to eq.(10) of Ref.(16) except for the multiplying factor 3 for $1/r_o^2 f^2$ terms.

Now from eq.(13) of Ref.(16) we write

$$\alpha(z) = \int_0^z G_1[b(z'), z'] \, dz' + \text{const} \qquad \ldots (6.67)$$

where G_1 represents the RHS of eq.(6.66)

$$\therefore \quad \frac{\partial G_1}{\partial b} = \frac{1}{k} \left\{ - \frac{\epsilon_2}{\epsilon_o} \frac{k^2 E_o^2}{f^2} \frac{2}{b^3} + 4 \left(\frac{1}{b^2} - \frac{3}{r_o^2 f^2} \right) \frac{1}{b^3} \right\} \Big/ \left\{ \left(\frac{1}{b^2} - \frac{3}{r_o^2 f^2} \right) \left[\frac{\epsilon_2}{\epsilon_o} \frac{k^2 E_o^2}{f^2} - \left(\frac{1}{b^2} - \frac{3}{r_o^2 f^2} \right) \right] \right\}^{\frac{1}{2}} \qquad \ldots (6.68)$$

For extremum α values we must have $\frac{\partial G_1}{\partial b} = 0$ i.e. its numerator should vanish. This condition gives rise to the expression for the size of the perturbation

$$\frac{1}{b^2} = \left(\frac{1}{2} \frac{\epsilon_2}{\epsilon_o} k^2 E_o^2 + \frac{3}{r_o^2} \right) \frac{1}{f^2} \qquad \ldots (6.69)$$

It again differs in $1/r_o^2 f^2$ term from the eq.(14) of Ref.(16). We also sought for the expression of optimum value of α with the help of eqs.(6.67) and (6.68). However, it has been found that the evaluation of the integrals involved in these equations ultimately leads to the same result as reported in Ref.(16) viz.

$$\alpha(z)\Big|_{opt} = \frac{k}{4} \frac{\epsilon_2 |E_o|^2}{\epsilon_o} Z_{ff} \ln \left(\frac{Z_{ff} + z}{Z_{ff} - z} \right) - \ln f(z) \qquad \ldots (6.70)$$

$$\text{with} \quad 1/Z_{ff}^2 = \frac{\epsilon_2 |E_o|^2}{\epsilon_o r_o^2} - \frac{1}{k^2 r_o^4} \qquad \ldots (6.71)$$

where $k = \frac{2\pi}{\lambda_o}$, λ_o = wavelength in vacuum

r_o = radius of the Gaussian beam

Z_{ff} = self-focusing length

Hence the behaviour of growth parameter α as a function of distance will not differ from that for a Gaussian beam.

The entire procedure has been repeated for the growth of Gaussian instabilities in a TEM_{20} beam. The expressions for $\frac{d\alpha}{dz}$ and $\frac{1}{b^2}$ differ in this case by a factor 5 multiplied to $1/r_o^2\, f^2$ terms, whereas there is no change observed in the expression for the optimum α.

6.4 RESULTS AND DISCUSSION

From the results reported in Sec.(6.1) it is seen that within the paraxial ray approximation the characteristic features of both the steady state and time dependent thermal effects of TEM_{10} and higher order mode laser beams will not differ from those already available for the Gaussian beam. Also there is no possibility of observing such effects in the case of a 'donought' mode laser beam.

The expressions for the nonsteady state self-focusing of TEM_{10} and TEM_{20} laser beams derived in Sec.(6.2) have been utilized in getting the plots for moving foci and the axial intensity as a function of time. The locations of focus points for TEM_{10} and TEM_{20} modes are calculated using eqs.(6.45) and (6.48). The same are reported in Figs. 6.1 and 6.2 by choosing two different sets of parameters as mentioned therein. For comparison the plots of TEM_{00} modes are also given. It is seen that each curve is a parabola. As the order of mode increases the branches of the parabolic curve come closer together towards (Z/R_d) axis. In Figs.6.3-6.6 we have presented the plots of the axial intensity as a function of time calculated from eqs.(6.47) and (6.50). In each figure the plot for TEM_{00} is also given. The figures are drawn for different sets of parameters as indicated therein. It is seen that within the paraxial ray approximation the width of intensity profile decreases for a higher order mode.

In Sec.(6.3) we have discussed the growth of Gaussian instabilities for TEM_{10} and higher order mode laser beams. It has been already stated that only the expressions for $(d\alpha/dz)$ and $(1/b^2)$ differ by certain multiplying factors from the relevant expressions for a Gaussian laser beam and the expression for optimum value of (α) remains unaltered. We therefore conclude that the behaviour of growth parameter is not affected by the shapes of intensity profiles for TEM_{mo} laser beams. Only the size of perturbation is slightly altered.

6.5 SUMMARY

Our studies reported in this chapter show that the phenomena of thermal focusing/defocusing and the growth of Gaussian instabilities do not depend upon the shapes of intensity profiles for TEM_{mo} laser modes. Only the location of focus point and axial intensity as a function of time are considerably affected from mode to mode.

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oOo

Table 6.1

```
10 REM Plot of Axial Intensity of Gaussian Beam
15 REM As Function of Time for Plasma
20 REM Programming by Shri.S.R.Bote
30 RD=10
40 T=E-8
50 VG=1.5E+10
60 G=10
70 Z=.2*RD
80 LPRINT "Gamma=";G,"Tau( )=";T,"z=";Z
90 LPRINT "-----------------------------------
100 LPRINT "T1","I","f^2"
110 LPRINT "-----------------------------------
120 FOR T1=-1.6 TO 1.6 STEP .2
130 F=1-(EXP(-(T1+(Z/(VG*T)))^2)*G-1)*(Z/RD)^2
140 I=EXP(-(T1+(Z/(VG*T)))^2)/F
150 LPRINT T1,I,F
160 NEXT T1
170 LPRINT "-----------------------------------
180 END
```

```
Gamma= 10       Tau( )=-8       z= 2
-----------------------------------------
T1              I               f^2
-----------------------------------------
1.6             7.660928E-02                    1.009078
-1.4            .1431988        .9836566
-1.2            .2506566        .9452289
-.9999999       .4120292        .8928482
-.8             .6359948        .829083
-.6             .9168739        .7609295
-.4             1.218841        .6991425
-.2             1.465324        .6556843
2.980232E-08                    1.5625          .64
.2              1.4>5;24        .6556843
.4              1.218841        .6991425
.6              .9168738        .7609295
.8              .6359948        .829083
1               .4120291        .8928482
1.2             .2506565        .9452289
1.4             .1431987        .9836566
-----------------------------------------
```

Table 6.2

```
10 REM Plot of Axial Intensity of TEM10 Mode
15 REM As Function of Time for Plasma
20 REM Programming by Shri.S.R.Bote
30 RD=10
40 T=E-8
50 VG=1.5E+10
60 G=50
70 Z=.3*RD
80 LPRINT "G=";G,"T=";T,"Z=";Z
90 LPRINT "-----------------------------------------------------------------"
100 LPRINT "T1","I","f^2"
110 LPRINT "-----------------------------------------------------------------"
120 FOR T1=-1.6 TO 1.6 STEP .2
130 R=T1+Z/(VG*T)
140 F=1-((1-R^2)^2*G*EXP(-R^2)-1)*(Z/RD)^2
150 I=(1-R^2)^2*EXP(-R^2)/F
160 LPRINT T1,I,F
170 NEXT T1
180 LPRINT "-----------------------------------------------------------------"
190 END
```

```
G= 50               T= 8              Z= 3
------------------------------------------------------------------
T1                  I                 f^2
------------------------------------------------------------------
-1.6                .7728559          .2434203
-1.4                .2566368          .505832
-1.2                5.191237E-02                          .8835887
-.9999999           4.796222E-15                          1.09
 .8                 .0873337          .782483
-.6                -1.458319         -.1959573
 .4                 .3721375         -1.615727
-.2                -.3059034         -2.894586
 2.980232E-08                        -.2932551            -3.41
 .2                -.3059034         -2.894586
 .4                 .3721375         -1.615727
 .6                -1.458322         -.195957
 .8                 8.733365E-02                          .7824831
 1                  0                 1.09
 1.2                5.191243E-02                          .8835885
 1.4                .2566371          .5058317
------------------------------------------------------------------
```

Table 6.3

```
10 REM Plot of Axial Intensity of TEM2o Mode
20 REM As Function of Time for Plasma
30 REM Programming by Shri.S.R.Bote
40 RD=10
50 T=E-8
60 VG=1.5E+10
70 G=10
80 Z=.1*RD
90 LPRINT "G=";G,"T=";T,"Z=";Z
100 LPRINT "---------------------------------------------"
110 LPRINT "T1","I","f^2"
120 LPRINT "---------------------------------------------"
130 FOR T1=-1.6 TO 1.6 STEP .2
140 R=T1+Z/(VG*T)
150 F=1-((1-2*R^2+.5*R^4)^2*EXP(-R^2)*G-1)*(Z/RD)^2
160 I=(1-2*R^2+.5*R^4)^2*EXP(-R^2)/F
170 LPRINT T1,I,F
180 NEXT T1
190 LPRINT "---------------------------------------------"
200 END
```

G: 10 T=-8 Z= 1

T1	I	f^2	
-1.6	5.471618E-02		1.004504
-1.4	.1412069	.9959367	
-1.2	.1696134	.9931548	
-.9999999	9.189604E-02		1.000803
-.8	2.953201E-03		1.009702
-.6	8.280348E-02		1.001706
-.4	.4220465	.9690995	
-.2	.877323	.9285373	
2.980232E-08		1.098901	.91
.2	.8773229	.9285373	
.4	.4220464	.9690995	
.6	8.280343E-02		1.001706
.8	2.953206E-03		1.009702
1	9.189608E-02		1.000803
1.2	.1696135	.9931548	
1.4	.1412069	.9959367	

Table 6.4

```
10 REM The Location of Focus for TEMØØ,TEM1Ø &TEM2Ø Modes
20 REM As a Function of Time
30 REM Programming by Shri.S.R.Bote
40 REM G =(Rd/Rnl)^2
50 REM T1=(t/T),T=Relaxation Time for Nonlinearity
60 REM Zo=Zr/Rd for TEMØØ
70 REM Z1=Zr/Rd for TEM1Ø
80 REM Z2=Zr/Rd for TEM2Ø
90 INPUT Z
100 G=1Ø
110 VG=1.5E+1Ø
120 T=1E-Ø8
130 ZT=Z/(VG*T)
140 LPRINT "Gamma=";G,"Tau( )=";T,"z=";Z
150 LPRINT "-----------------------------------------------------"
160 LPRINT "T1","Zo","Z1","Z2"
170 LPRINT "-----------------------------------------------------"
180 FOR T1=-.5 TO .5 STEP .1
190 ZO=1/(G*EXP(-(T1-ZT)^2)-1)^.5
200 Z1=1/(G*(1-(T1-ZT)^2)^2*EXP(-(T1-ZT)^2)-1)^.5
210 Z2=1/(G*(1-2*(T1-ZT)^2+.5*(T1-ZT)^4)^2*EXP(-(T1-ZT)^2)-1)^.5
220 LPRINT T1,ZO,Z1,Z2
230 NEXT T1
240 LPRINT "-----------------------------------------------------"
250 END
```

Gamma= 1Ø   Tau( )= 1E-Ø8   z= 1Ø

T1	Zo	Z1	Z2	
-.5	.3998904	.6532972	2.15494	
-.4	.3768078	.5Ø4992	.749Ø421	
-.3	.3593959	.4244994	.5137298	
-.2	.3468214	.3769941	.4119298	
-.1	.3385254	.3493598	.36Ø7857	
-1.49Ø116E-Ø8		.3341576	.3358173	.33749Ø5
9.999999E-Ø2		.3335392	.3339516	.3343648
.2	.3366452	.3434499	.35Ø4864	
.3	.3436Ø18	.3659546	.39Ø88	
.4	.3546994	.4Ø58763	.4714312	
.5	.37Ø4215	.4731Ø94	.6432491	

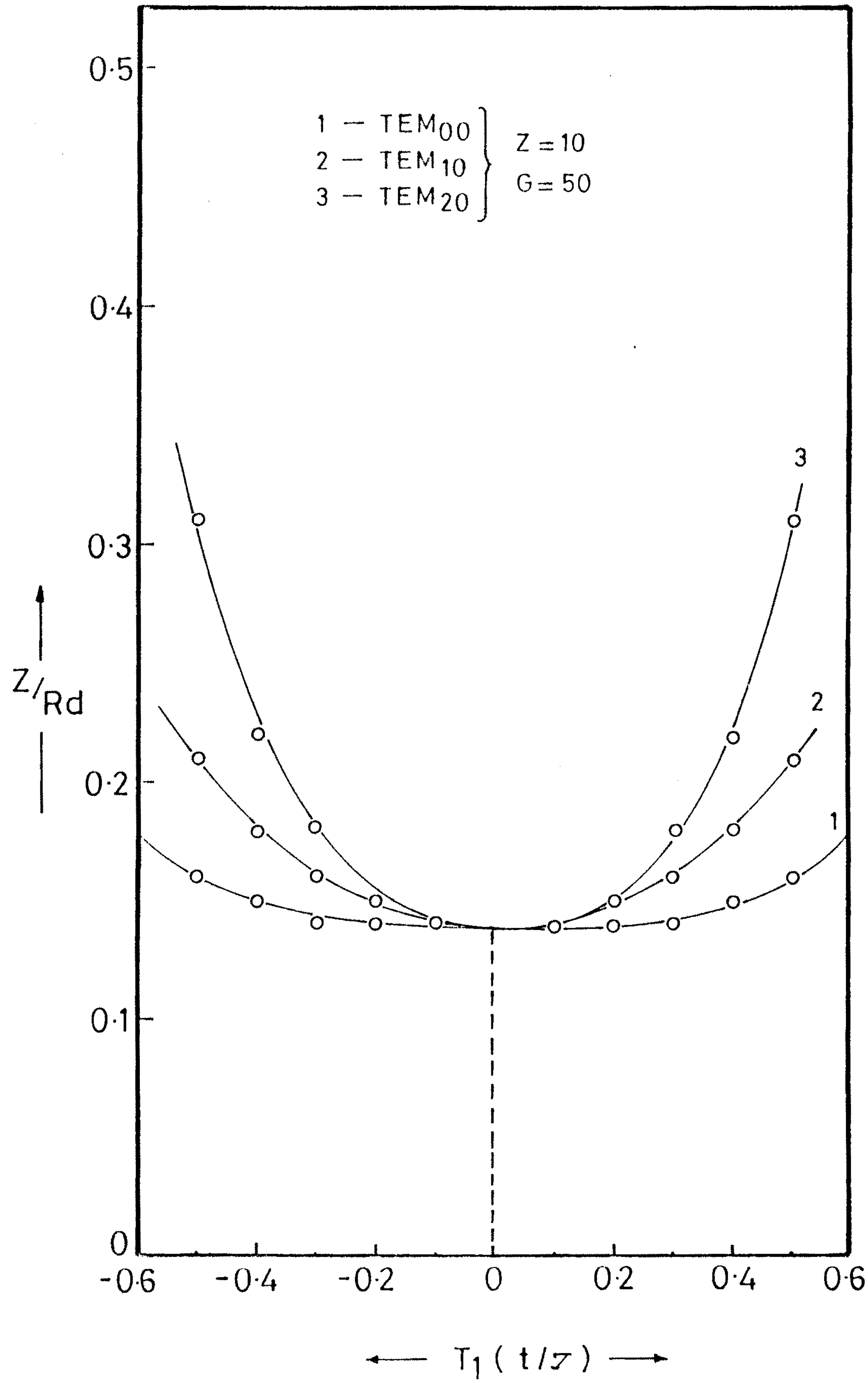

Fig. 6·1 The location of focus point for TEM_{00} mode TEM_{10} mode and TEM_{20} mode as a function of time.

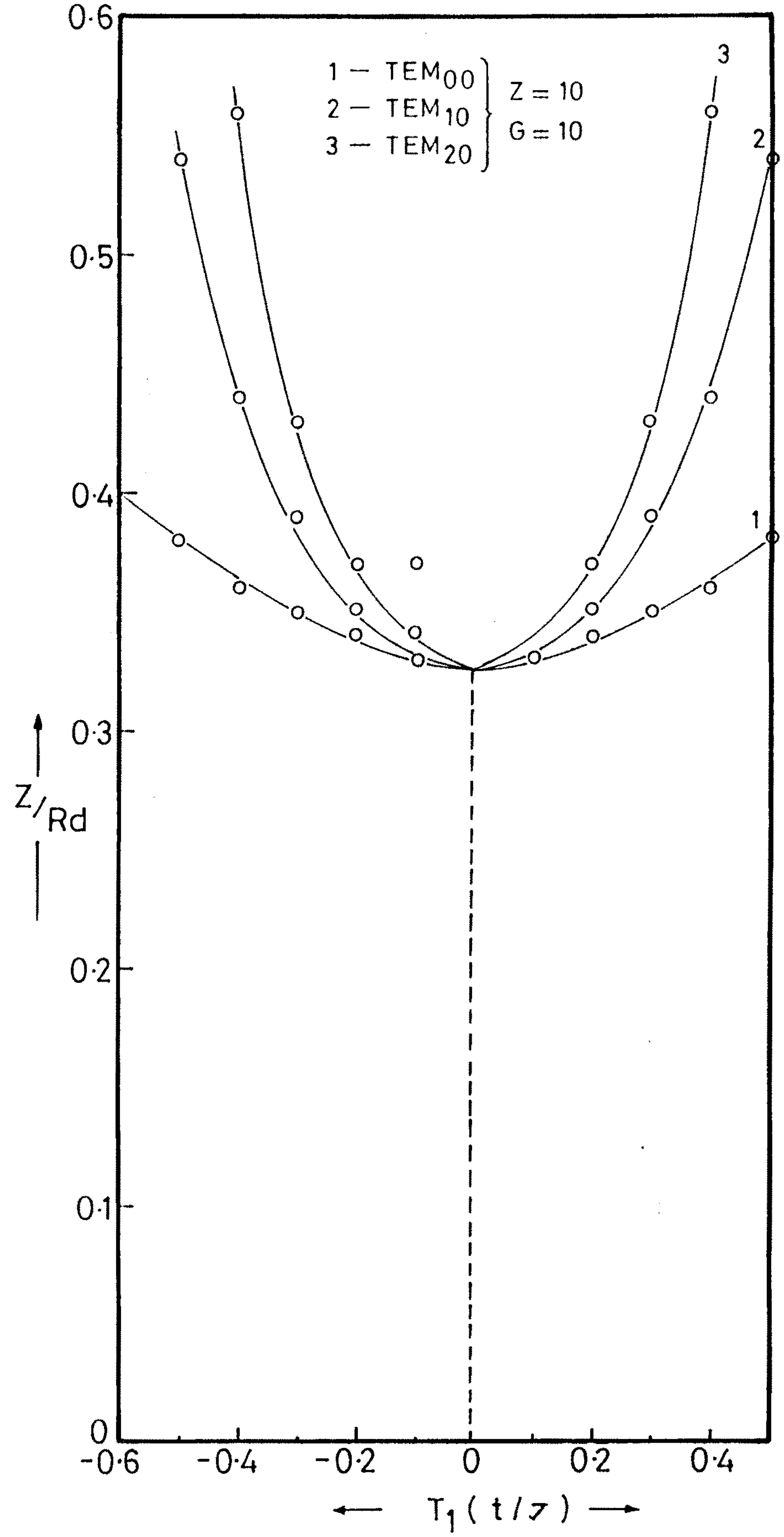

Fig. 6·2 The location of focus point for TEM_{00} mode TEM_{10} mode and TEM_{20} mode as a function of time.

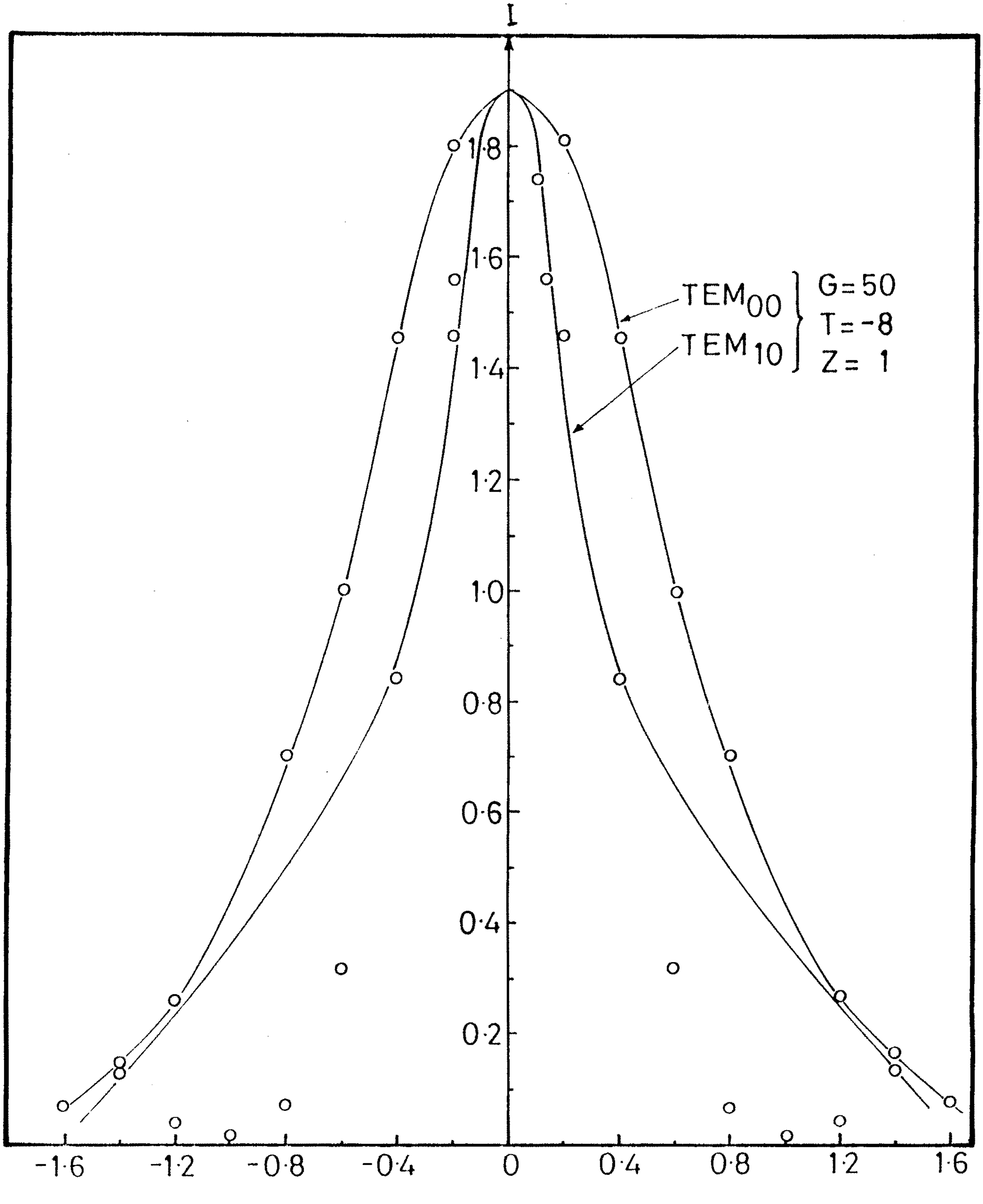

Fig.6·3 Plot of axial intensity of TEM_{00} and TEM_{10} modes as a function of time.

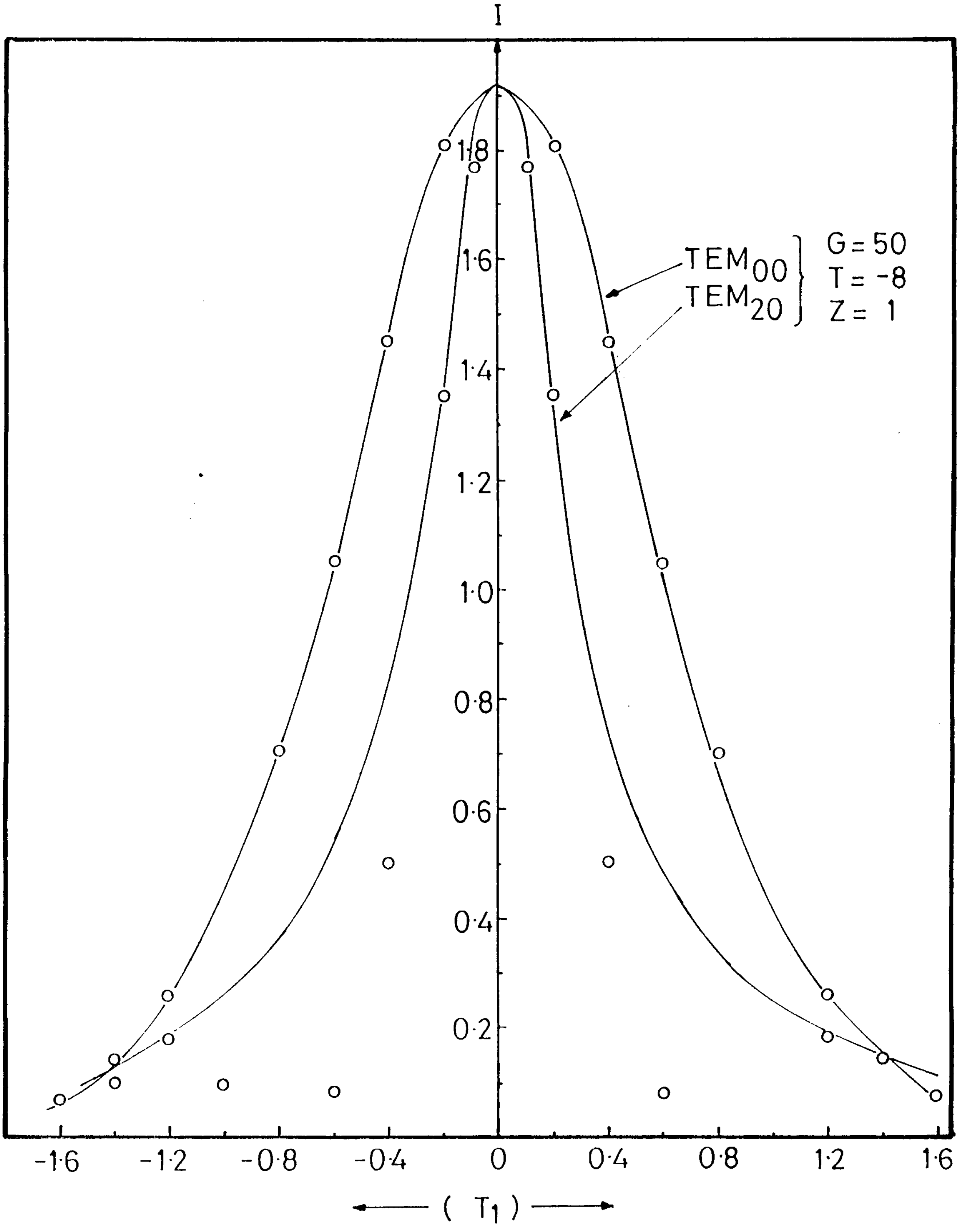

Fig. 6·4 Plot of axial intensity of TEM_{00} and TEM_{20} modes as a function of time.

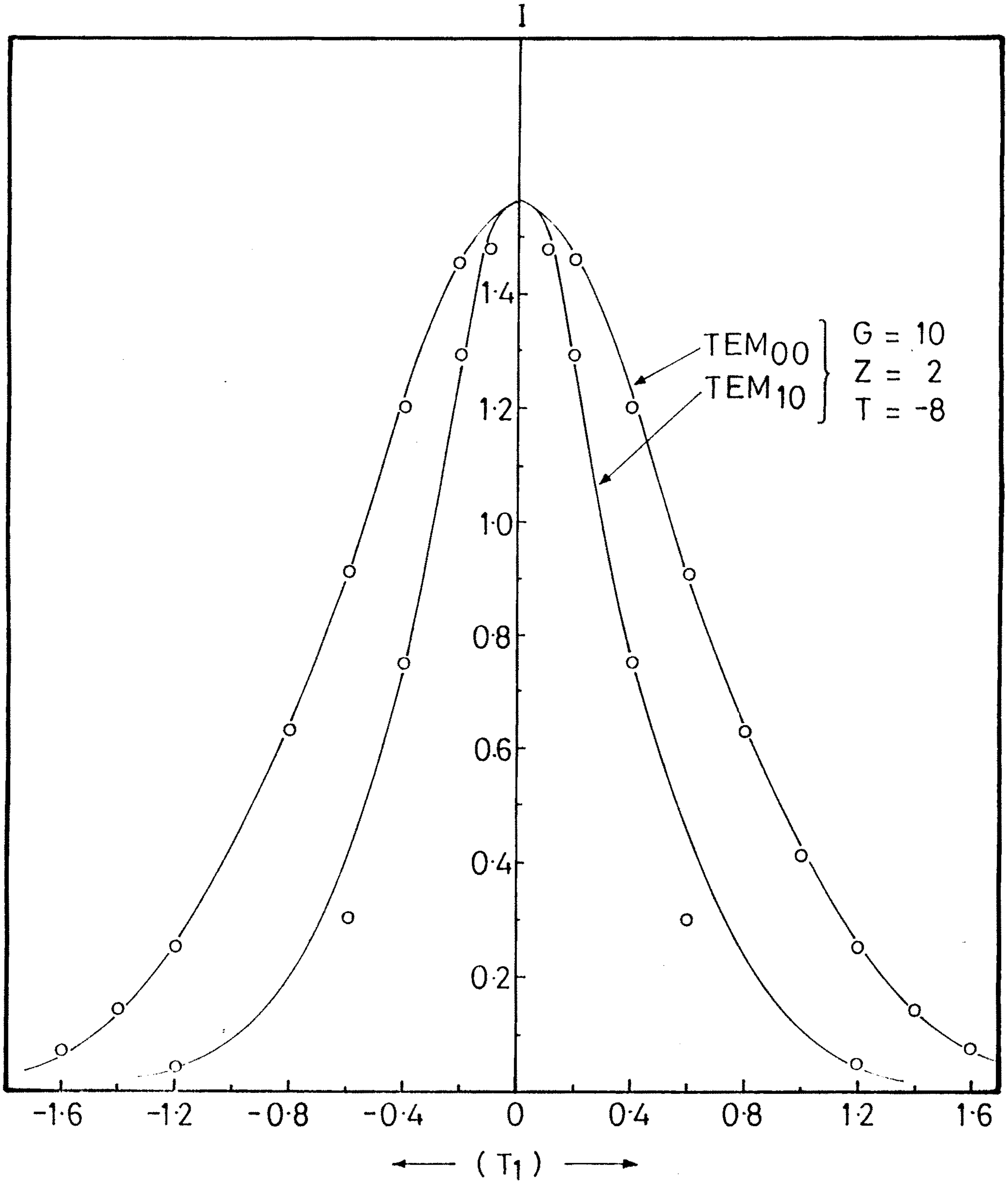

Fig. 6·5 Plot of axial intensity of TEM_{00} and TEM_{10} modes as a function of time.

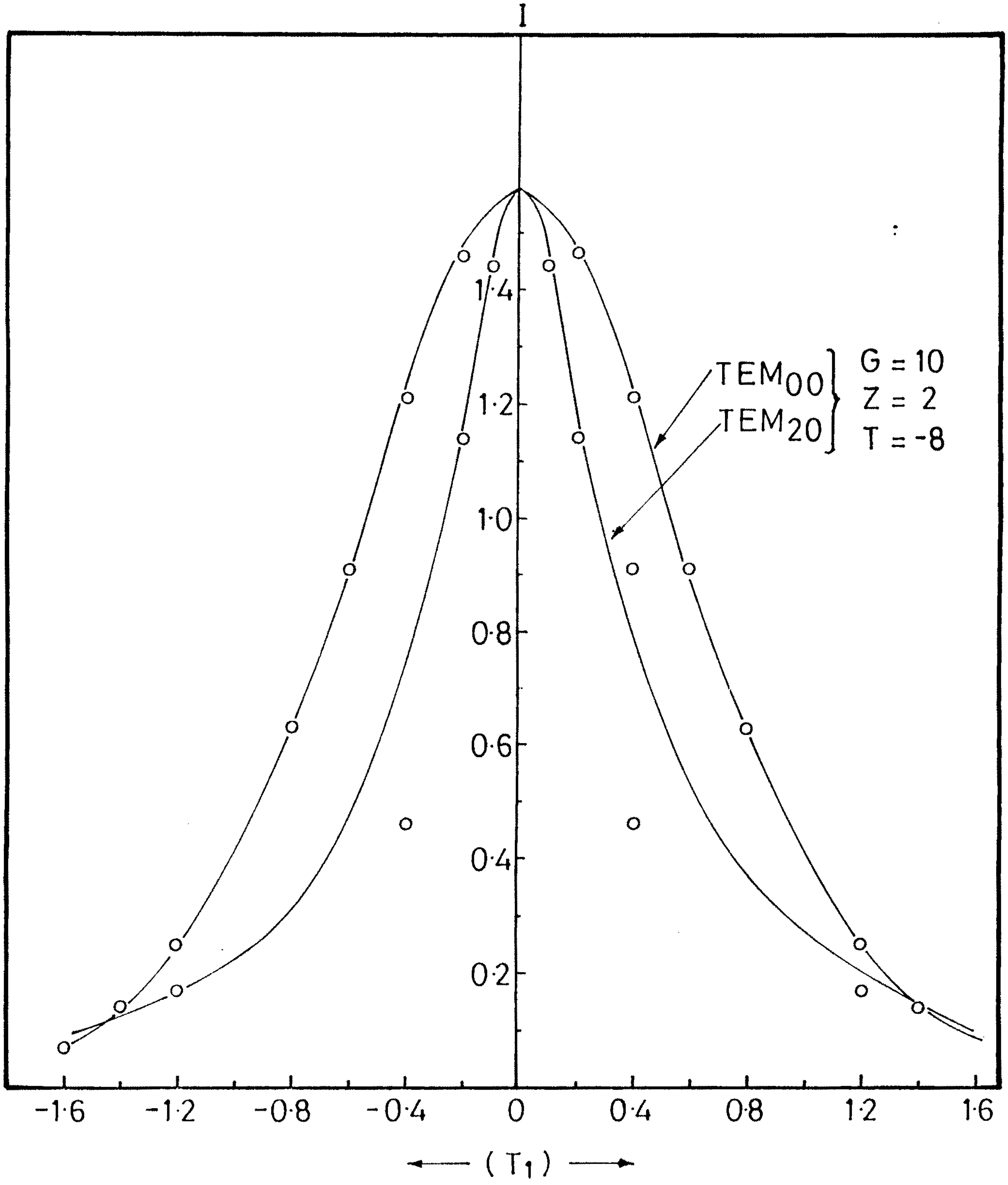

Fig. 6·6 Plot of axial intensity of TEM_{00} and TEM_{20} modes as a function of time.