CHAPTER 4

SOLUTION OF FUZZY TRANSPORTATION PROBLEM
USING ZERO TERMINATION METHOD

In this chapter, solution to a fuzzy transportation problem is obtained using zero termination method. The Transportation costs, supply and demand values are considered to lie in an interval of values. Fuzzy modified distribution method is proposed to find the optimal solution in terms of fuzzy numbers. The solution procedure is illustrated with a numerical example.

4.1 INTRODUCTION

Transportation problem deals with the distribution of a product from various sources to different destinations of demand in such a manner that the total transportation cost is minimized. In order to solve a transportation problem, the decision parameters such as availability, requirement and the unit transportation cost of the model were tried at crisp values. But in real life, supply, demand and unit life transportation cost are uncertain due to several factors. These imprecise data may be represented by fuzzy numbers. To deal with this uncertain situations in transportation problems, many researchers [Chanas et al (1996), Ishibuchi et al (1990), Pandian et al (2010) Herrera et al (1995)] have proposed fuzzy and interval programming techniques for solving the transportation problem. The concept of fuzzy mathematical programming
on a general level was first proposed by Tanaka et al.,(1974) in the framework of fuzzy decision of Bellman and Zadeh (1965).

Des et al., (1999) proposed a method, using fuzzy technique to solve interval transportation problems by considering the right bound and the midpoint interval and T.K.P et al (2009) proposed a new orientated method to solve interval transportation problems by considering the midpoint and width of the interval in the objective function. Stephen Dinagara, Palanivel (2009) proposed method of finding the initial fuzzy feasible solution to a fuzzy transportation problem. But most of the existing techniques provide only crisp solution for fuzzy transportation problem. In general, most of the authors obtained the crisp optimal solution to a given fuzzy transportation problem. In this paper, we propose a new algorithm to find the initial fuzzy feasible solution to a fuzzy transportation problem without converting it to be a classical transportation problem.

In section 2, we recall the basic concepts and results of Trapezoidal fuzzy numbers and the fuzzy transportation problem with Trapezoidal fuzzy number and their related results. In Section 3, we propose a new algorithm of fuzzy interval transportation problem. In Section 4, we propose a new algorithm to find the initial fuzzy feasible solution for the given fuzzy transportation problem and obtained the fuzzy optimal solution, applying the zero termination method. In Section 5, we concise the method of solving a
fuzzy transportation problem using zero termination method on Trapezoidal fuzzy number. Numerical example is illustrated.

4.2 PRELIMINERIES

In this section, some of the necessary definitions have been presented.

4.3 ARITHMETIC OPERATIONS

Let $\tilde{a} = [a_1, a_2, a_3, a_4]$ and $\tilde{b} = [b_1, b_2, b_3, b_4]$ be two trapezoidal fuzzy numbers, then the arithmetic operations on $\tilde{a}$ and $\tilde{b}$ are defined as follows:

**Addition:** $\tilde{a} + \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

**Subtraction:** $\tilde{a} - \tilde{b} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

**Multiplication:** $\tilde{a} \times \tilde{b} = [\min(a_1 b_1, a_1 b_4, a_4 b_1, a_4 b_4),$

$\min(a_2 b_2, b_3 a_3, a_3 b_2, a_3 b_3),$

$\max(a_1 b_1, a_1 b_4, a_4 b_1, a_4 b_4),$

$\max(a_2 b_2, a_2 b_3, a_3 b_2, a_3 b_3)]$

4.4 FUZZY TRANSPORTATION PROBLEM

Let us consider a transportation system based on fuzzy with m origins and n destinations. Let us further assume that the fuzzy transportation cost of one unit of product from $i^{th}$ origin to $j^{th}$ destination be denoted by $C_{ij} = [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}]$, the fuzzy availability of commodity at the origin i be
\[ s_i = [s_i^{(1)}, s_i^{(2)}, s_i^{(3)}, s_i^{(4)}], \] fuzzy commodity needed at the destination \( j \) be
\[ d_j = [d_j^{(1)}, d_j^{(2)}, d_j^{(3)}, d_j^{(4)}]. \] The fuzzy quantity transported from \( i^{th} \) origin to \( j^{th} \) destination be \( X_{ij} = [X_{ij}^{(1)}, X_{ij}^{(2)}, X_{ij}^{(3)}, X_{ij}^{(4)}]. \)

Now, the fuzzy transportation problem based on supply \( s_i \), demand \( d_i \) and the transported quantity \( X_{ij} \) can be related in a table as follows:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2...</th>
<th>...</th>
<th>N</th>
<th>Fuzzy capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( C_{11} )</td>
<td>( C_{12} )</td>
<td>...</td>
<td>( C_{1n} )</td>
<td>( X_{1n} )</td>
</tr>
<tr>
<td>2</td>
<td>( C_{21} )</td>
<td>( C_{22} )</td>
<td>...</td>
<td>( C_{2n} )</td>
<td>( X_{2n} )</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td>...</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>M</td>
<td>( C_{m1} )</td>
<td>( C_{m2} )</td>
<td>...</td>
<td>( C_{mn} )</td>
<td>( X_{mn} )</td>
</tr>
<tr>
<td>Fuzzy demand</td>
<td>( d_1 )</td>
<td>( d_2 )</td>
<td>...</td>
<td>( d_n )</td>
<td>( \sum d_j = \sum s_i )</td>
</tr>
</tbody>
</table>

where \( C_{ij} = [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}], X_{ij} = [X_{ij}^{(1)}, X_{ij}^{(2)}, X_{ij}^{(3)}, X_{ij}^{(4)}], \)

\[ s_i = [s_i^{(1)}, s_i^{(2)}, s_i^{(3)}, s_i^{(4)}] \] and \( d_i = [d_i^{(1)}, d_i^{(2)}, d_i^{(3)}, d_i^{(4)}] \)

The linear programming model representing the fuzzy transportation is given by

\[
\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ c_{ij}^{(1)} X_{ij}^{(1)} + c_{ij}^{(2)} X_{ij}^{(2)} + c_{ij}^{(3)} X_{ij}^{(3)} + c_{ij}^{(4)} X_{ij}^{(4)} \right],
\]

\[
\text{subject to: } \sum_{j=1}^{n} X_{ij} = s_i, \quad \forall i = 1, 2, \ldots, m,
\]

\[
\sum_{i=1}^{m} X_{ij} = d_j, \quad \forall j = 1, 2, \ldots, n,
\]

\[
X_{ij} \geq 0, \quad \forall i, j.
\]
Subject to the constraints

$$\sum_{j=1}^{n} [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}] = [s_i^{(1)}, s_i^{(2)}, s_i^{(3)}, s_i^{(4)}]$$

for $i=1,2,\ldots,m$ (Row sum)

$$\sum_{i=1}^{m} [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}] = [d_j^{(1)}, d_j^{(2)}, d_j^{(3)}, d_j^{(4)}]$$

for $j=1,2\ldots n$ (Column sum)

$$[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}] \geq 0$$

The given fuzzy transportation problem is said to be balanced if

$$\sum_{i=1}^{m} [s_i^{(1)}, s_i^{(2)}, s_i^{(3)}, s_i^{(4)}] = \sum_{j=1}^{n} [d_j^{(1)}, d_j^{(2)}, d_j^{(3)}, d_j^{(4)}]$$

i.e., if the total fuzzy capacity is equal to the total fuzzy demand

4.5 THE COMPUTATIONAL PROCEDURE FOR FUZZY MODIFIED DISTRIBUTION METHOD

4.5.1 Zero Termination Method

The procedure of Zero Termination method is as follows,

**Step 1:** Construct the transportation table

**Step 2:** Select the smallest unit transportation cost value for each row and subtract it from all costs in that row. In a similar way this process is repeated column wise.
Step 3: In the reduced cost matrix obtained from step 2, there will be at least one zero in each row and column. Then we find the termination value of all the zeros in the reduced cost matrix, using the following rule;

The zero termination cost is T= Sum of the costs of all the cells adjacent to zero/ Number of non-zero cells added

Step 4: In the revised cost matrix with zero termination costs has a unique maximum T, allot the supply to that cell. If it has one (or) more Equal max values, then select the cell with the least original cost and allot the maximum possible.

Step 5: After the allotment, the columns and rows corresponding to exhausted demands and supplies are trimmed. The resultant matrix must possess at least one zero in each row and column, else repeat step (2)

Step 6: Repeat step (3) to step (5) until the optimal solution is obtained.

4.5.2 Fuzzy Modified Distribution Method

This proposed method is used for finding the optimal basic feasible solution in fuzzy environment and the following step by step procedure is utilized to find out the same.
1. Find a set of numbers $[u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}]$ and $[v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}]$ for each row and column satisfying $[u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}]+ [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}] = [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}]$ for each occupied cell. To start with, we assign a fuzzy zero to any row or column having maximum number of allocations. If this maximum number of allocation is more than one, choose any one arbitrarily.

2. For each empty (un occupied ) cell, we find the fuzzy sum $[u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}] + [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}]$

3. Find out for each empty cell the net evaluation value $[Z_{ij}^{(1)}, Z_{ij}^{(2)}, Z_{ij}^{(3)}, Z_{ij}^{(4)}] = [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}] - \{[u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}] + [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}]\}$ this step gives the optimality conclusion

i. If all $[Z_{ij}^{(1)}, Z_{ij}^{(2)}, Z_{ij}^{(3)}, Z_{ij}^{(4)}] > [-2,-1,1,2]$ the solution is optimal and a unique solution exists. [-2,-1,1,2] is a trapezoidal fuzzy number equal to crisp zero.

ii. If $[Z_{ij}^{(1)}, Z_{ij}^{(2)}, Z_{ij}^{(3)}, Z_{ij}^{(4)}] \geq [-2,-1,1,2]$ then the solution is fuzzy optimal. But an alternate solution exists.

iii. If at least one $[Z_{ij}^{(1)}, Z_{ij}^{(2)}, Z_{ij}^{(3)}, Z_{ij}^{(4)}] < [-2,-1,1,2]$ the solution is not fuzzy optimal. In this case we go to next step to improve the total fuzzy transportation minimum cost.
iv. Select the empty cell having the most negative value of $[Z_{ij}^{(1)}, Z_{ij}^{(2)}, Z_{ij}^{(3)}, Z_{ij}^{(4)}]$ from this cell we draw a closed path drawing horizontal and vertical lines with corner cells being occupied. Assign sign + and – alternately and find the fuzzy minimum allocation from the cells that have negative sign. This allocation should be added to the allocation having negative sign.

v. The above step yields a better solution by making one (or more) occupied cells as empty and one empty cell as occupied. For this new set of fuzzy basic feasible allocation repeat the steps from [i] till an fuzzy optimal basic feasible solution is obtained.

4.6 NUMERICAL EXAMPLE

To solve the following fuzzy transportation problem starting with the fuzzy initial fuzzy basic feasible solution obtained by Zero Termination Method

<table>
<thead>
<tr>
<th>Table 4.1 The basic fuzzy transportation problem</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>FO1</td>
</tr>
<tr>
<td>FO2</td>
</tr>
<tr>
<td>FO3</td>
</tr>
<tr>
<td>Fuzzy demand</td>
</tr>
</tbody>
</table>

since $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = [4,10,19,27]$
The problem is balanced fuzzy transportation problem.

There exists a fuzzy initial basic feasible solution

**Table 4.2 The initial basic fuzzy feasible solution, by zero termination method**

<table>
<thead>
<tr>
<th></th>
<th>FD1</th>
<th>FD2</th>
<th>FD3</th>
<th>FD4</th>
<th>Fuzzy Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>FO1</td>
<td>[-2,0,2,8]</td>
<td>[-2,0,2,8]</td>
<td>[-2,0,2,8]</td>
<td>[-1,0,1,4]</td>
<td>[0,2,4,6]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[-7,-1,5,11]</td>
<td>[-11,-3,5,13]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FO2</td>
<td>[4,8,12,16]</td>
<td>[4,7,9,12]</td>
<td>[2,4,6,8]</td>
<td>[1,3,5,7]</td>
<td>[2,4,9,13]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[1,3,5,7]</td>
<td>[-12,-2,8,18]</td>
<td></td>
</tr>
<tr>
<td>FO3</td>
<td>[2,4,9,13]</td>
<td>[0,6,8,10]</td>
<td>[0,6,8,10]</td>
<td>[4,7,9,12]</td>
<td>[2,4,6,8]</td>
</tr>
<tr>
<td></td>
<td>[1,3,5,7]</td>
<td>[-5,-1,3,7]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuzzy</td>
<td>[1,3,5,7]</td>
<td>[0,2,4,6]</td>
<td>[1,3,5,7]</td>
<td>[1,3,5,7]</td>
<td></td>
</tr>
<tr>
<td>demand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since the number of occupied cell having \( m+n-l = 6 \) and are also independent, there exist a non-degenerate fuzzy basic feasible solution.

Therefore, the initial fuzzy transportation minimum cost is,

\[
[Z^{(1)},Z^{(2)},Z^{(3)},Z^{(4)}] = [-2,0,2,8][-7,-1,5,11]+[-1,0,1,4][-11,-3,5,13]
+ [2,4,6,8][1,3,5,7] + [1,3,5,7] [-12,-2,8,18]
+ [2,4,9,13] [1,3,5,7] + [0,6,8,10] [-5,-1,3,7]
= [-56,-2,10,88] + [-44,-3,5,52]
+ [2,12,30,56] + [-84,-10,40,126]
+ [2, 12, 45, 91] + [0, -8, 24, 70]
\]

\[
[Z^{(1)},Z^{(2)},Z^{(3)},Z^{(4)}] = [-180, 1, 154483]
= 114.5
\]
To Find the Optimal Solution

Applying the fuzzy modified distribution method, we determine a set of numbers \([u_i(1), u_i(2), u_i(3), u_i(4)]\) and \([v_j(1), v_j(2), v_j(3), v_j(4)]\) each row and column such that \([c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}] = [u_i(1), u_i(2), u_i(3), u_i(4)] + [v_j(1), v_j(2), v_j(3), v_j(4)]\) for each occupied cell.

Since 1st row has maximum number of allocations, we give fuzzy numbers\([u_i(1), u_i(2), u_i(3), u_i(4)] = [-2, -1, 0, 1, 2]\). The remaining numbers can be obtained as given below.

\[
\begin{align*}
[c_{12}^{(1)}, c_{12}^{(2)}, c_{12}^{(3)}, c_{12}^{(4)}] &= [u_1^{(1)}, u_1^{(2)}, u_1^{(3)}, u_1^{(4)}] + [v_2^{(1)}, v_2^{(2)}, v_2^{(3)}, v_2^{(4)}] \\
[v_2^{(1)}, v_2^{(2)}, v_2^{(3)}, v_2^{(4)}] &= [-4, -1, 3, 10] \\
[c_{14}^{(1)}, c_{14}^{(2)}, c_{14}^{(3)}, c_{14}^{(4)}] &= [u_1^{(1)}, u_1^{(2)}, u_1^{(3)}, u_1^{(4)}] + [v_4^{(1)}, v_4^{(2)}, v_4^{(3)}, v_4^{(4)}] \\
[v_4^{(1)}, v_4^{(2)}, v_4^{(3)}, v_4^{(4)}] &= [-3, -1, 2, 6] \\
[c_{23}^{(1)}, c_{23}^{(2)}, c_{23}^{(3)}, c_{23}^{(4)}] &= [u_2^{(1)}, u_2^{(2)}, u_2^{(3)}, u_2^{(4)}] + [v_3^{(1)}, v_3^{(2)}, v_3^{(3)}, v_3^{(4)}] \\
[v_3^{(1)}, v_3^{(2)}, v_3^{(3)}, v_3^{(4)}] &= [-8, -2, 5, 13] \\
[c_{24}^{(1)}, c_{24}^{(2)}, c_{24}^{(3)}, c_{24}^{(4)}] &= [u_2^{(1)}, u_2^{(2)}, u_2^{(3)}, u_2^{(4)}] + [v_4^{(1)}, v_4^{(2)}, v_4^{(3)}, v_4^{(4)}] \\
[u_2^{(1)}, u_2^{(2)}, u_2^{(3)}, u_2^{(4)}] &= [-5, 1, 6, 10] \\
[c_{31}^{(1)}, c_{31}^{(2)}, c_{31}^{(3)}, c_{31}^{(4)}] &= [u_3^{(1)}, u_3^{(2)}, u_3^{(3)}, u_3^{(4)}] + [v_1^{(1)}, v_1^{(2)}, v_1^{(3)}, v_1^{(4)}] \\
[v_1^{(1)}, v_1^{(2)}, v_1^{(3)}, v_1^{(4)}] &= [-12, -5, 6, 23] \\
[c_{32}^{(1)}, c_{32}^{(2)}, c_{32}^{(3)}, c_{32}^{(4)}] &= [u_3^{(1)}, u_3^{(2)}, u_3^{(3)}, u_3^{(4)}] + [v_2^{(1)}, v_2^{(2)}, v_2^{(3)}, v_2^{(4)}] \\
[u_3^{(1)}, u_3^{(2)}, u_3^{(3)}, u_3^{(4)}] &= [-10, 3, 9, 14]
\]
We find, for each empty cell of the sum \([u_i(1),u_i(2),u_i(3),u_i(4)]\) and \([v_j(1),v_j(2),v_j(3),v_j(4)]\).

Next we find net evaluation \([Z_{ij}(1),Z_{ij}(2),Z_{ij}(3),Z_{ij}(4)]\) is given by:

**Table 4.3 Iteration 1**

<table>
<thead>
<tr>
<th>U1=[-2,-1,1,2]</th>
<th>FD1</th>
<th>FD2</th>
<th>FD3</th>
<th>FD4</th>
<th>Fuzzy Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>-ve</td>
<td>[-2,0,2,8]</td>
<td>[-2,0,2,8]</td>
<td>[-2,0,2,8]</td>
<td>[-1,0,1,4]</td>
<td>[0,2,4,6]</td>
</tr>
<tr>
<td>+ve</td>
<td>[-27,-7,8,22]</td>
<td>[-7,-1,5,11]</td>
<td>*[-17,-6,5,18]</td>
<td>[-11,-3,5,13]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>U2=[-5,1,6,10]</th>
<th>FD1</th>
<th>FD2</th>
<th>FD3</th>
<th>FD4</th>
<th>Fuzzy Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ve</td>
<td>[4,8,12,16]</td>
<td>[43,7,9,12]</td>
<td>[2,4,6,45458]</td>
<td>[1,3,5,7]</td>
<td>[2,4,9,13]</td>
</tr>
<tr>
<td>-ve</td>
<td>*[-29,-4,16,33]</td>
<td>*[-16,-2,9,21]</td>
<td>[1,3,5,7]</td>
<td>[-12,-2,8,18]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>U3=[-10,3,9,14]</th>
<th>FD1</th>
<th>FD2</th>
<th>FD3</th>
<th>FD4</th>
<th>Fuzzy Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>-ve</td>
<td>[2,4,9,13]</td>
<td>[0,6,8,10]</td>
<td>[0,6,8,10]</td>
<td>[4,7,9,12]</td>
<td>[2,4,6,8]</td>
</tr>
</tbody>
</table>

Solution in the above table 4.3 is not optimal since \(d_{11}<0\).

In the Table 4.3, the cell (1,1) has the negative value. This non-basic cell is converted into a basic cell without affecting the capacity and demand restrictions. Hence, starting from the cell (1,1), a closed loop is constructed.
Iteration 2

Again applying the fuzzy modified distribution method, we determine a set of numbers

\[ [u_i(1), u_i(2), u_i(3), u_i(4)] \text{ and } [v_j(1), v_j(2), v_j(3), v_j(4)] \]

each row and column such that \([c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}] = [u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}] + [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}] \] for each occupied cell.

Since 1st row has maximum number of allocation, we give fuzzy number

Let \([u_i(1), u_i(2), u_i(3), u_i(4)] = [-2, -1, 0, 1, 2] \). The remaining numbers can be obtained as given below.

\[ [c_{11}^{(1)}, c_{11}^{(2)}, c_{11}^{(3)}, c_{11}^{(4)}] = [u_1^{(1)}, u_1^{(2)}, u_1^{(3)}, u_1^{(4)}] + [v_1^{(1)}, v_1^{(2)}, v_1^{(3)}, v_1^{(4)}] \]

\[ [v_1^{(1)}, v_1^{(2)}, v_1^{(3)}, v_1^{(4)}] = [-4, -1, 3, 10] \]

\[ [c_{14}^{(1)}, c_{14}^{(2)}, c_{14}^{(3)}, c_{14}^{(4)}] = [u_1^{(1)}, u_1^{(2)}, u_1^{(3)}, u_1^{(4)}] + [v_4^{(1)}, v_4^{(2)}, v_4^{(3)}, v_4^{(4)}] \]

\[ [v_4^{(1)}, v_4^{(2)}, v_4^{(3)}, v_4^{(4)}] = [-3, -1, 2, 6] \]

\[ [c_{23}^{(1)}, c_{23}^{(2)}, c_{23}^{(3)}, c_{23}^{(4)}] = [u_2^{(1)}, u_2^{(2)}, u_2^{(3)}, u_2^{(4)}] + [v_3^{(1)}, v_3^{(2)}, v_3^{(3)}, v_3^{(4)}] \]

\[ [v_3^{(1)}, v_3^{(2)}, v_3^{(3)}, v_3^{(4)}] = [-8, -2, 5, 13] \]

\[ [c_{24}^{(1)}, c_{24}^{(2)}, c_{24}^{(3)}, c_{24}^{(4)}] = [u_2^{(1)}, u_2^{(2)}, u_2^{(3)}, u_2^{(4)}] + [v_4^{(1)}, v_4^{(2)}, v_4^{(3)}, v_4^{(4)}] \]

\[ [u_2^{(1)}, u_2^{(2)}, u_2^{(3)}, u_2^{(4)}] = [-5, 1, 6, 10] \]

\[ [c_{31}^{(1)}, c_{31}^{(2)}, c_{31}^{(3)}, c_{31}^{(4)}] = [u_3^{(1)}, u_3^{(2)}, u_3^{(3)}, u_3^{(4)}] + [v_1^{(1)}, v_1^{(2)}, v_1^{(3)}, v_1^{(4)}] \]

\[ [u_3^{(1)}, u_3^{(2)}, u_3^{(3)}, u_3^{(4)}] = [-8, 1, 10, 17] \]

\[ [c_{32}^{(1)}, c_{32}^{(2)}, c_{32}^{(3)}, c_{32}^{(4)}] = [u_3^{(1)}, u_3^{(2)}, u_3^{(3)}, u_3^{(4)}] + [v_2^{(1)}, v_2^{(2)}, v_2^{(3)}, v_2^{(4)}] \]

\[ [v_2^{(1)}, v_2^{(2)}, v_2^{(3)}, v_2^{(4)}] = [-17, -4, 7, 18] \]
We find, for each empty cell of the sum $[u_i(1), u_i(2), u_i(3), u_i(4)]$ and $[v_j(1), v_j(2), v_j(3), v_j(4)]$.

Next we find net evaluation $[Z_{ij}(1), Z_{ij}(2), Z_{ij}(3), Z_{ij}(4)]$ is given by:

**Table 4.4** Iteration 2

<table>
<thead>
<tr>
<th></th>
<th>FD1</th>
<th>FD2</th>
<th>FD3</th>
<th>FD4</th>
<th>Fuzzy Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>U₁=[-2,-1,1,2]</td>
<td>[-2,0,2,8]</td>
<td>[-2,0,2,8]</td>
<td>[-2,0,2,8]</td>
<td>[-1,0,1,4]</td>
<td>[0,2,4,6]</td>
</tr>
<tr>
<td>U₂=[-5,1,6,10]</td>
<td>[4,8,12,16]</td>
<td>[4,7,9,12]</td>
<td>[2,4,6,8]</td>
<td>[1,3,5,7]</td>
<td>[2,4,9,13]</td>
</tr>
<tr>
<td></td>
<td>*[16,-1,12,25]</td>
<td>*[24,-12,34]</td>
<td>[1,3,5,7]</td>
<td>[-12,-2,8,18]</td>
<td></td>
</tr>
<tr>
<td>U₃=[-8,1,10,17]</td>
<td>[2,4,9,13]</td>
<td>[0,6,8,10]</td>
<td>[0,6,8,10]</td>
<td>[4,7,9,12]</td>
<td>[2,4,6,8]</td>
</tr>
<tr>
<td></td>
<td>[-10,-2,6,14]</td>
<td>[-12,-2,8,18]</td>
<td>*[30,-9,9,26]</td>
<td>*[19,-5,9,23]</td>
<td></td>
</tr>
</tbody>
</table>

**Iteration 2** shows the revised allocations based on the calculations.

In table 4.3 are presented in the table 4.4 the solution in table 4.4 is also not optimal because the cell (3, 3) has a negative value. Starting from this cell, a closed loop is constructed.
Iteration 3

Again applying the fuzzy modified distribution method, we determine a set of numbers

\[ [u_i(1), u_i(2), u_i(3), u_i(4)] \text{ and } [v_j(1), v_j(2), v_j(3), v_j(4)] \]

each row and column such that \[ [c_{ij}(1), c_{ij}(2), c_{ij}(3), c_{ij}(4)] = [u_i(1), u_i(2), u_i(3), u_i(4)] + [v_j(1), v_j(2), v_j(3), v_j(4)] \] for each occupied cell.

Since 1\textsuperscript{rd} row has maximum numbers of allocation, we give fuzzy number \([u_3(1), u_3(2), u_3(3), u_3(4)] = [-2, -1, 0, 1, 2] \). The remaining numbers can be obtained as given below.

\[
\begin{align*}
[c_{31}(1), c_{31}(2), c_{31}(3), c_{31}(4)] &= [u_3(1), u_3(2), u_3(3), u_3(4)] + [v_1(1), v_1(2), v_1(3), v_1(4)] \\
[v_1(1), v_1(2), v_1(3), v_1(4)] &= [4, 5, 8, 11] \\
[c_{32}(1), c_{32}(2), c_{32}(3), c_{32}(4)] &= [u_3(1), u_3(2), u_3(3), u_3(4)] + [v_2(1), v_2(2), v_2(3), v_2(4)] \\
[v_2(1), v_2(2), v_2(3), v_2(4)] &= [-2, 5, 9, 12] \\
[c_{33}(1), c_{33}(2), c_{33}(3), c_{33}(4)] &= [u_3(1), u_3(2), u_3(3), u_3(4)] + [v_3(1), v_3(2), v_3(3), v_3(4)] \\
[v_3(1), v_3(2), v_3(3), v_3(4)] &= [-2, 5, 9, 12] \\
[c_{34}(1), c_{34}(2), c_{34}(3), c_{34}(4)] &= [u_3(1), u_3(2), u_3(3), u_3(4)] + [v_4(1), v_4(2), v_4(3), v_4(4)] \\
[v_4(1), v_4(2), v_4(3), v_4(4)] &= [-9, 2, 10, 17] \\
[c_i(1), c_i(2), c_i(3), c_i(4)] &= [u_i(1), u_i(2), u_i(3), u_i(4)] + [v_i(1), v_i(2), v_i(3), v_i(4)] \\
[u_i(1), u_i(2), u_i(3), u_i(4)] &= [-13, -8, -3, 4]
\end{align*}
\]
We find, for each empty cell of the sum $[u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}]$ and $[v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}]$.

Next we determine the net evaluation $[Z_{ij}^{(1)}, Z_{ij}^{(2)}, Z_{ij}^{(3)}, Z_{ij}^{(4)}]$.

### Table 4.5 Iteration 3

<table>
<thead>
<tr>
<th></th>
<th>FD1</th>
<th>FD2</th>
<th>FD3</th>
<th>FD4</th>
<th>Fuzzy Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[-2,0,2,8]</td>
<td>[-2,0,2,8]</td>
<td>[-2,0,2,8]</td>
<td>[-1,0,1,4]</td>
<td>[0,2,4,6]</td>
</tr>
<tr>
<td>$U_1$</td>
<td>[-18,-4,10,24]</td>
<td>*[-18,-6,5,23]</td>
<td>*[-18,-6,5,23]</td>
<td>*[-22,-7,7,26]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ve</td>
<td>+ve</td>
<td>+ve</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[4,8,12,16]</td>
<td>[4,7,9,12]</td>
<td>[2,4,6,8]</td>
<td>[1,3,5,7]</td>
<td>[2,4,9,13]</td>
</tr>
<tr>
<td>$U_2$</td>
<td>*[-17,-1,12,22]</td>
<td>*[-18,-3,9,24]</td>
<td>[-12,-2,8,18]</td>
<td>[-23,-5,13,31]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ve</td>
<td>+ve</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[2,4,9,13]</td>
<td>[0,6,8,10]</td>
<td>[0,6,8,10]</td>
<td>[4,7,9,12]</td>
<td>[2,4,6,8]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+ve</td>
<td></td>
</tr>
</tbody>
</table>

| Fuzzy Demand | [1,3,5,7] | [0,2,4,6] | [1,3,5,7] | [1,3,5,7] |

**Iteration3:** The revised solution based on the calculation in the previous table 4.4 is shown in the table 4.5 in this table, all the penalties of the non-basic cells are positive. Hence, the optimality is reached.
<table>
<thead>
<tr>
<th></th>
<th>FD1</th>
<th>FD2</th>
<th>FD3</th>
<th>FD4</th>
<th>Fuzzy Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>FO1</td>
<td>[-2,0,2,8]</td>
<td>[-2,0,2,8]</td>
<td>[-2,0,2,8]</td>
<td>[-1,0,1,4]</td>
<td>[0,2,4,6]</td>
</tr>
<tr>
<td></td>
<td>[-18,-4,10,24]</td>
<td>*[-18,-6,5,23]</td>
<td>*[-18,-6,5,23]</td>
<td>*[-22,-7,7,26]</td>
<td></td>
</tr>
<tr>
<td>FO2</td>
<td>[4,8,12,16]</td>
<td>[4,7,9,12]</td>
<td>[2,4,6,8]</td>
<td>[1,3,5,7]</td>
<td>[2,4,9,13]</td>
</tr>
<tr>
<td></td>
<td>*[-17,-12,22]</td>
<td>*[-18,-3,9,24]</td>
<td>[-12,-2,8,18]</td>
<td>[-23,-5,13,31]</td>
<td></td>
</tr>
<tr>
<td>FO3</td>
<td>[2,4,9,13]</td>
<td>[0,6,8,10]</td>
<td>[0,6,8,10]</td>
<td>[4,7,9,12]</td>
<td>[2,4,6,8]</td>
</tr>
<tr>
<td>Fuzzy Demand</td>
<td>[1,3,5,7]</td>
<td>[0,2,4,6]</td>
<td>[1,3,5,7]</td>
<td>[1,3,5,7]</td>
<td></td>
</tr>
</tbody>
</table>

Where \(U_i = [u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}]\), \(V_i = [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}]\), and

\[
*\left[ z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}, z_{ij}^{(4)} \right] = \left[ c_{ij}^{(1)}, c_{ij}^{(1)}, c_{ij}^{(1)}, c_{ij}^{(1)} \right] - \left[ [u_i^{(1)}, u_i^{(1)}, u_i^{(1)}, u_i^{(1)}] + [u_i^{(1)}, u_i^{(1)}, u_i^{(1)}, u_i^{(1)}] \right]
\]

Since all \(z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}, z_{ij}^{(4)} > 0\) the solution is fuzzy optimal and unique.

Therefore the fuzzy optimal solution obtained in terms of trapezoidal fuzzy numbers:

\[
\left[ X_{11}^{(1)}, X_{11}^{(2)}, X_{11}^{(3)}, X_{11}^{(4)} \right] = [-18,-4,10,24]
\]
\[
\left[ X_{23}^{(1)}, X_{23}^{(2)}, X_{23}^{(3)}, X_{23}^{(4)} \right] = [-12,-2,8,18]
\]
\[
\left[ X_{24}^{(1)}, X_{24}^{(2)}, X_{24}^{(3)}, X_{24}^{(4)} \right] = [-23,-5,13,31]
\]
\[
\left[ X_{31}^{(1)}, X_{31}^{(2)}, X_{31}^{(3)}, X_{31}^{(4)} \right] = [-23,-7,9,25]
\]
\[
\left[ X_{32}^{(1)}, X_{32}^{(2)}, X_{32}^{(3)}, X_{32}^{(4)} \right] = [-12,-2,8,18]
\]
\[
\left[ X_{33}^{(1)}, X_{33}^{(2)}, X_{33}^{(3)}, X_{33}^{(4)} \right] = [-11,-3,5,13]
\]
Hence, the total fuzzy transportation minimum cost is

\[
[Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}] = [-2,0,2,8] [-18,-4,10,24] + [2,4,6,8] [-12,-2,8,18] + \\
[1,3,5,7][-23,-5,13,31] + [2,4,9,13][-23,-7,9,25] + [0,6,8,10] [-12,-2,8,18] + [0,6,8,10] [-11,-3,5,13]
\]

\[
[Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}] = [-930,-148,318,1188] = 107
\]

4.7 CONCLUSION

In this chapter, we have obtained an optimal solution for the fuzzy transportation problem of minimal cost using the fuzzy trapezoidal membership function, the new algorithm for the fuzzy optimal solution to a fuzzy transportation problem with triangular fuzzy numbers, the new algorithm of zero termination method. The proposed method provides more options and this can serve as an important tool in decision making problem.