P R E F A C E

Thin Plates and Shells of different shapes frequently occur in many structures and the study of their bending properties is imperative to a design engineer. With the increased use of strong and light-weight structures, specially in aerospace engineering and in the vibrations of machine parts, numerous problems naturally arise which have to be dealt with appropriate bending conditions.

The formulation of elasticity problems including the effect of temperature variation was studied by Duhamel as early as 1837, shortly after the basic formulation of the theory of elasticity. The linear theory of thermoelasticity is based on the assumption that the deflections are small in comparison with the thickness of the plates and shells. This assumption follows from the hypothesis that the component of strains ϵ_{11}, ϵ_{22} and ϵ_{12} in the deflected middle surface have negligible magnitude. Based on the linear theory numerous problems of thermal stresses and deflections including thermal buckling and vibrations of elastic plates and shells have been studied in view of the fact these problems have enormous applications in aeronautics, highspeed aircrafts, missiles and in nuclear and in chemical engineering.

Among the noteworthy works in this field are the books and monographs by Hoff, N.J. (1958), Gatewood, B.E. (1957), Nowacki, W (1962), Jones, D.J (1965), Farkas, M (1976) and Nowinski, J.L (1978). Extensive bibliography of works have been included in these works. Also may be mentioned some other research works in the field of linear thermoelasticity and these are by Vinson, J.R (1958), Bijlaard, P.P (1959), Sunakawa, M and Uemura. M (1960), Das, Y.C and Navaratna, D.R (1962), Sekiya, T., Sumi, S., Matsumoto, E., Katayama, T and Sugimoto, I (1967), Sarkar, S (1963), Panonneau, G and Marangoni, R.D (1970), Akoz, A.Y and Tauchert, T.R (1971, 1975, 1978), Roychoudhury, S.K (1972), Rao, K.S., Bapu Rao, M.N and Ariman, T (1973), Prabhu, M.S.S and Durvasula, S (1973, 1974), Katayama, T and Sektya, T (1975), Ganesan, N (1978), Sarkar, S.K. (1968), Biswas, P (1976, 1977), Irie, T and Yamada, G (1978), Buckens, F (1979), Misra, J.C and Samanta, S (1980), Tomar, J.S and Tiwari, V.S (1981), Das, S (1981), Rao, K.C.V.S (1984) and Tomar, J.S and Gupta, A.K (1984).

The main bulk of the classical approach in applied mechanics rests on the assumption that the phenomena involved can be adequately described by linear mathematical model. With the advent of modern technology and systems exposed to oppressive operational conditions this hypothesis could no longer be retained. Moreover, forces, deformations, velocities, temperatures and other factors become excessive, and nonlinear effects come into play and their influence can no longer be ignored. This situation occurs also in a particular field of applied mechanics involving plates and shallow shells.

Thus when the deflections are no longer small in comparison with the thickness, the supplementary stresses in the middle plane must be taken into account in deriving the differential equations governing the deflections of plates and shells. In this way one gets the nonlinear differential equations in the classical nonlinear theory.

The coupled nonlinear partial differential equations for analysing the large deflections of plates were initially established by von Karman (1910). These equations which are of the fourth order with respect to the unknown deflection W and stress function ψ enables one to determine W and ψ. von Karman's equations are generally difficult to deal with because of its coupled nonlinearity and as yet no general solution of these equations is known. Approximate and numerical methods have to be adopted for the solution of different plate and shell problems.

The outstanding research workers who employed von Karman's equations to analyse nonlinear behaviour of thin plates and shallow shells are Schmidt, R (1963), Saner, H.F (1963), Nowinski, J.L (1963), Nowinski, J.L and Ismail, I.A (1965), Chu, H.N and Harmann, G (1966), Kennedy, J.B and Simon, N.G (1967), Cooley, I.D (1969), Satyamoorthy, M and Pandalai, K.A.V (1970, 1972, 1978, 1981), Prathap, G and Varadhan T.K (1978, 1979), Nath, Y and Alwar, R.S (1978, 1980), Karmakar, B.M (1979), Banerjee, B and Datta, S (1979), Reddy, J.N (1981), Choudhury, S.K (1981). Recently (1980) Chia, C.Y has published an excellent book entitled "Nonlinear

Analysis of Plates" in which problems on orthotropic and laminated Plates have been analysed in addition to other problems with an extensive bibliography of other related works.

von Karman's equations have been extended to thermal loading in the static and dynamic cases and these generalised equations have been quoted by Nowacki, W and Chia, C.Y in their respective books. Very recently these equations have further been extended to skew plates at large amplitudes including a thermal gradient by Dalamangas, A (1984). Mansfield, E.H (1982) has successfully employed von Karman's equations extended to thermal loading in the dynamic case for investigating the large deflection vibrations of heated elliptic plates. Also, with the help of complex variable theory and conformal mapping, nonlinear analysis of regular polygonal plates at elevated temperature has been investigated by Biswas, P (1981). Bailey, C.D and Greetham, J.C (1973) investigated the free vibrations of thermally -stressed plates with various boundary conditions.

In 1955, Berger, H.M proposed a pair of quasilinear partial differential equations for analysing the large deflections of isotropic plates. These equations are in the decoupled form and have obvious advantages for the solution of problems. Berger's method is based on the neglect of the second strain invariant of the middle surface strains in the expression corresponding to the total potential energy of the system. An application of the variational technique to this simplified energy expression yields the equations of equilibrium of the plate in the decoupled form. Although no complete explanation of this method has been set forth, yet the results obtained by him for simply-supported and clamped plates are in good agreement with those obtained from more precise analysis. Following this approximate method, different nonlinear plate problems have been solved with remarkable ease by many investigators, among which mention may be made of the works of Nash, W and Modeer, J (1959), Basuli, S (1961, 1968), Sinha, S.N (1963), Nowinski, J.L and Ismail, I.A (1964), Bera, R (1965), Banerjee, B (1967, 1968), Pal, M.C (1969, 1970, 1973), Wu, C and Vinson, J.R (1969), Ramachandran, J (1973), Sathyamoorthy, M and Pandalai, K.A.V (1974), Sircar, R (1974), Biswas, P (1975, 1977, 1978), Kamiya, W (1976, 1977, 1978, 1980), Banerjee, M.M (1976, 1981), Banerjee, B and Datta, S (1979), Sathyamoorthy, M (1981) and Choudhury, S.K (1984).

Nash and Modeer extended Berger's method to investigate the nonlinear dynamic behaviour of elastic plates and obtained solutions which are in excellent agreement with those obtained from classical equations.

Basuli employed Berger's equations to obtain the solution of a clamped circular plate under concentrated load at large deflections. The method of solution in the paper is very interesting. He also extended the method for plates under normal pressure and heating.

Sinha, S.N. obtained an interesting solution of circular and rectangular plates on elastic foundation of the Winker type.

Banerjee, B discussed the large amplitude free vibrations of elliptic plates based on Berger's approximations. Mathieu functions have been used in this investigation.

Prabhu, M.S.S. extended Berger's analyses for the large deformations of skew plates.

Kamiya further extended Berger's idea of the neglect of second invariant from the total potential energy for the large deflection analysis of sandwich plates and shallow shells.

Chia and Satyamoorthy, investigated large deflections of laminated and orthotropic plates with attention to shear and rotatory inertia with the help of Berger's method. During the later part of the sixtees and in the seventees Berger's equations extended to thermal loading in the static and dynamic cases were extensively used for different plate and shell problems.

Recently Nowinski, J.L. and Ohnabe, H (1972) and Prathap, G (1979) pointed out certain inaccuracies in Berger's equations and concluded that these equations lead to quite meaningless and absurd results for plates with movable edge conditions. This is due to the fact that the neglect of e_2, the second strain invariant of the middle surface strains for movable edges of the plates, fails to imply the freedom of rotation in the meridian plane where the meridian stress

$$\sigma_{rr} = \frac{E}{1-\nu^2}\left[\frac{\partial u}{\partial r} + \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2 + \nu \, {}^{u}/_{r}\right]$$

exists. For movable edges the in-plane displacement u is never zero and thus Berger's equations leads to absurd results. For immovable clamped edges $u = o$ and $\frac{\partial u}{\partial r} = o$ on the boundary and therefore Berger's equations give rise to satisfactory results. For simply-supported immovable edges $u = o$ but $\frac{\partial u}{\partial r} \neq o$ on the boundary and Berger's equations give fairly accurate results.

To overcome the difficulties involved in the von-Karman and Berger's methods, Dutta and Banerjee [1981] suggested a modified energy expression by bringing directly the expression for σ_{rr} in the potential energy of the system and derived a new set of differential equations, also in the decoupled form. They observed accuracy of these equations for circular plates with movable and immovable edge conditions.

Following this method of approach very recently a number of papers have appeared in different Journals in support of this approach. With the help of this modified approach the elastic plate and shell problems for both movable and immovable edges have been investigated by Banerjee, B (1983), Sinharay, G.C. and Banerjee, B (1985, 1986) and Ghosh, S.K (1986).

The object of the present thesis is to study non-linear vibrations of elastic plates and shells at elevated temperature by employing the three methods described above. The thesis is divided into three chapters. In the 1st chapter von-Karman equations extended to thermal loading in the dynamic case have been employed for dealing with the free vibrations of some elastic plates and shallow shells. Four problems have been considered of which the first problem is the non linear free vibrations and thermal buckling of isotropic circular plates at elevated temperature. von-Karman equations in terms of stress function and transverse displacement have been employed in the analysis. Besides determining stress function for the clamped plate with immovable edges, a relation connecting relative amplitudes, thermal loading parameter and relative time-periods for linear and non-linear vibrations has been obtained. The variations have been presented in the form of graph. It is observed that the ratios of time-periods for non-linear and linear vibrations (i.e. T^*/T) are less for plates with thermal effect than for those without thermal effect, that is, the effect of temperature is to deminish the relative time-periods. Moreover it is observed that the effect of temperature parameter on the relative time-periods is similar as that of plates subjected to the in-plane compresive forces as investigated by Biswas [1981]. Criterion for critical buckling temperature has been deduced as a limiting case.

The second problem consists of non linear free vibration analysis of orthotropic circular plates at elevated temperature. Here von-Karman equations in terms of displacement components have been employed in the analysis. Results for both movable and immovable edges have been presented.

The next problem analyses the non linear vibrations and thermal buckling of polygonal plates at elevated temperature by conformal transformation. If the boundary of a plate is a curve natural to any of the common co-ordinate system, the governing equations can be solved in terms of known functions. For more " exotic" boundaries, the natural co-ordinates must first be determined and after this is done, the solution could inevitably involve some unfamilar functions. Therefore, a common co-ordinate system and its associated function are advantageous for plates having complicated boundaries. If the given domain can be conformally mapped onto a unit circle, the problem then reduces to the solution of the transformed differential system. This is known as the conformal mapping technique which is based on the complex variable theory.

The conformal mapping technique is more useful because

(i) If the mapping function is known, the static, thermal and dynamic behaviour of plates of any shape can be obtained from the solution of the same differential equation and thus minimises the computational labour.

(ii) The solutions of irregular-shaped plates are appreciated now-a-days in modern design. These solutions can only be obtained with ease and accuracy with the help of this method.

From a critical survey of literature on arbitrary-shaped plates by the method of conformal mapping technique, only a few works could be located. These are mainly due to Laura, P.A. and Shahady, P (1969), Banerjee, B and Dutta, S (1979), Banerjee, M.M (1976), Biswas, P (1976, 1981) and Das, S (1985).

In this third problem von-Karman equations in the dynamic case extended to thermal loading have been transformed into the complex domain and with the help of conformal mapping and Galerkin's procedure the vibrational characteristic of different regular polygonal plates have been investigated for clamped immovable edges. Critical buckling temperature for such plates have been deduced as limiting cases and compared with available results.

The fourth problem deals with the thermal stresses and vibrations of a shallow spherical shell under elevated temperature. The ratio of non linear and linear frequencies have been presented graphically for different shell geometries and thermal loading parameters. Softening type of non linearity, that is, the increase of frequencies with increasing amplitudes have been observed. Membrane stresses can conveniently be determined from the analysis.

The second chapter is devoted to the investigation of some elastic plates by Berger's approximation. This method is applied to the cases of a right-angled isoceles triangular plates and an equilateral triangular plate with simply-supported boundary conditions. For the case of the equilateral triangular plates tri-linear co-ordinates have been used. A parabolic plate with clamped immovable edges has also been considered. So far such plate-shape has eascaped the attention of researchers perhaps due to the mathematical complexities involved in dealing with such a problem. It has been exhibited that by using Berger method of approximation the non-linear free vibrations of triangular and parabolic plates can conveniently be investigated.

In the third chapter Berger's equations for heated elastic plates and shells have been derived from the modified strain energy expression in the light of the proposition made by Banerjee and Dutta [1981]. This chapter consists of two problems of which the first one is the non linear vibrations and thermal buckling of elastic circular plates at elevated temperature. Results for both movable and immovable edges have been obtained with ease and accuracy. The second problem is the non-linear vibrations of a shallow spherical shells due to thermal gradient. At high temperature modulus of elasticity of materials becomes function of space variable [Hoff, (1958)]. The vibrational characteristic of continuous elastic system must then be based on non-homogeneous elastic theory. So far non-linear analysis of such situations for shallow shells does not appear to be reported in the literature. In this problem basic governing equations for the non-linear vibration analysis of a shallow spherical shell subjected to thermal gradient have been derived considering the modified energy expression and based on non-homogeneous theory, the flexural rigidity being the function of the radial distance (r). Some numerical results showing the dependence of the ratio of non-linear and linear frequencies on the temperature co-efficient have been presented.

The thesis concludes with the observation on the three methods employed to different plate and shell problems.