SYNOPSIS

The thesis entitled **"On Study of Bianchi Type Models in Bimetric Theory of Relativity"** consists of six chapters and is mainly devoted to the study of Bianchi type models in bimetric theory of relativity proposed by Rosen (1940, 73, 74). Furthermore, some plane symmetric cosmological models and plane wave solutions in N-dimensional space-times are studied in bimetric and general relativity theory.

The first chapter is introductory and gives brief information of the subject which is relevant and necessary for understanding the work contained in the subsequent chapters. Though the general theory of relativity has its own importance amongst the existing theories of gravitation, it has some unsatisfactory features. In an attempt to get rid of the difficulties, Rosen (1973, 74, 77) has proposed a theory of gravitation which is usually known as bimetric theory of relativity. In this theory in addition to the physical metric tensor g_{ij} there is a second metric tensor f_{ij} called background metric which describes the geometry of the space-time which one would have if there were no matter present in the universe. Thus at a point of the space-time there are two line elements.

$$ds^2 = g_{ij}\, dx^i\, dx^j$$

$$d\sigma^2 = f_{ij}\, dx^i dx^j .$$

We then have two kinds of covariant differentiation :

g - differentiation based on g_{ij} (denoted by semicolon;)

and f - differentiation based on f_{ij} (denoted by vertical bar |).

The field equations of bimetric relativity derived from variational principle are

$$K_{ij} = N_{ij} - \frac{1}{2} N \, g_{ij} = -8\pi\kappa \, T_{ij} \quad ,$$

where

$$N^{i}_{j} = \frac{1}{2} f^{ab} (g^{hi} \, g_{hj|a})_{|b} \quad ,$$

$$\kappa = \left(\frac{g}{f} \right)^{1/2}, \; g = \det (g_{ij}) \;\; , \;\; f = \det(f_{ij}) .$$

This theory has simpler mathematical structure than that of Einstein general relativity. It does not admit black hole.

In chapter II, we have extended the work of Mohanty and Sahoo (2002) in higher dimensional bimetric gravitation theory. We have investigated that the five dimensional Bianchi type-I cosmological model given by

$$ds^2 = -dt^2 + A^2 dx_1^2 + B^2 dx_2^2 + C^2 dx_3^2 + D^2 dx_4^2,$$

where A, B, C and D are functions of time only, does not exist in a case of scalar meson field and mesonic perfect fluid.

Chapter III deals with the study of Bianchi type-III cosmological model given by

$$ds^2 = -dt^2 + e^{2\alpha} dx^2 + e^{2(\beta + x)} dy^2 + e^{2\gamma} dz^2$$

in four dimensional space-time and

$$ds^2 = -dt^2 + e^{2\alpha}dx^2 + e^{2(\beta+x)}dy^2 + e^{2\gamma}dz_1^2 + e^{2\delta}dz_2^2$$

in five dimensional space-time, where $\alpha, \beta, \gamma, \delta$ are functions of time t only.

We have investigated that the above both the models does not exist in case of both scalar meson field and mesonic perfect fluid.

In chapter IV, we have considered the general cylindrically-symmetric metric given by Marder (1958)

$$ds^2 = A^2(dt^2 - dx^2) - B^2dy^2 - C^2dz^2,$$

where A, B and C are functions of time t only, and shown that the models does not admit perfect fluid source in Rosen's bimetric theory of gravitation and hence a vacuum cosmological model is investigated as

$$ds^2 = e^{2T}(dT^2 - dX^2 - dY^2 - dZ^2),$$

a conformally flat and free from singularity. Furthermore, the special case of the model is studied in bimetric theory of relativity and obtained the analogous result for the above model.

In chapter V, we have studied a non-static plane symmetric space-time

$$ds^2 = -dx^2 - dy^2 - dz^2 + dt^2 - d\psi^2,$$

where ψ is a function of x and t only, and a search is made for material distribution and electromagnetic distribution in general and bimetric theory of relativity. Some special cases are also discussed.

Chapter VI deals with the study of plane gravitational wave solutions for N-dimensional space-time in bimetric theory of relativity. In this chapter we have reformulated the generalized definition of Takeno (1961) [Thengane (2000)] in bimetric relativity as follows :

A plane wave g_{ij} is a non-flat solution of field equations

$$N_i^j = 0, \quad (i, j = 1, 2, \cdots, n)$$

in an empty region of the space-time such that

$$g_{ij} = g_{ij}(Z), \; Z = Z(x^i),$$

where

$$x^i = x^1, x^2, \cdots, x^{n-1}, t$$

in some suitable coordinate system such that

$$g^{ij} Z_{,i} Z_{,j} = 0, \quad Z_{,i} = \frac{\partial Z}{\partial x^i} \quad ,$$

$$Z = Z(x^{n-1}, t) \quad , \qquad Z_{,(n-1)} \neq 0 \quad , \qquad Z_{,n} \neq 0 \; .$$

The signature convention adopted is

$$g_{ll} < 0, \; \begin{vmatrix} g_{ll} & g_{lk} \\ g_{kl} & g_{kk} \end{vmatrix} > 0, \; \begin{vmatrix} g_{ll} & g_{lk} & g_{lm} \\ g_{kl} & g_{kk} & g_{km} \\ g_{ml} & g_{mk} & g_{mm} \end{vmatrix} < 0 \quad ,$$

[not summed for $l, k, m = 1, 2, \cdots, (n-1)$].

$$\begin{vmatrix} g_{11} & g_{12} & \cdots & \cdots & g_{1(n-1)} \\ g_{21} & g_{22} & \cdots & \cdots & g_{2(n-1)} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ g_{(n-1)1} & g_{(n-1)2} & \cdots & \cdots & g_{(n-1)(n-1)} \end{vmatrix} < 0\,, \text{ when } n \text{ is even}$$

and

$$\begin{vmatrix} g_{11} & g_{12} & \cdots & \cdots & g_{1(n-1)} \\ g_{21} & g_{22} & \cdots & \cdots & g_{2(n-1)} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ g_{(n-1)1} & g_{(n-1)2} & \cdots & \cdots & g_{(n-1)(n-1)} \end{vmatrix} > 0\,, \text{ when } n \text{ is odd.}$$

$g_{nn} > 0$.

We have investigated that the plane gravitational wave solutions for N-dimensional space-time are given by g_{ij} satisfying

$$N\, \rho_i' + M\, \sigma_i' = 0$$

which is equivalent to

$$\overline{\overline{w}}\, \rho_i' + \overline{w}\, \sigma_i' = 0 = \overline{\overline{\phi}}\, \rho_i' + \overline{\phi}\, \sigma_i' \quad .$$

The investigation shows that the Takeno's (1961) formats are again retained in bimetric relativity.