CHAPTER - 6

DECOMPOSITION OF $M^{(1,2)^r}$-α-CONTINUITY AND $M^{(1,2)^r}$-αgs-CONTINUITY

6.1 INTRODUCTION

Noiri and Sayed [30] introduced the notion of η-sets and obtained some decompositions of continuity. In this chapter, we introduce new types of sets called $m_{x}^{(1,2)^r}$-αB-sets, $m_{x}^{(1,2)^r}$-η-sets, $m_{x}^{(1,2)^r}$-αgs-open sets, $m_{x}^{(1,2)^r}$-pgs-open sets, $m_{x}^{(1,2)^r}$-η*-sets and $m_{x}^{(1,2)^r}$-η**-sets in biminimal spaces and new classes of mappings called $M^{(1,2)^r}$-αB-continuity, $M^{(1,2)^r}$-η-continuity, $M^{(1,2)^r}$-αgs-continuity, $M^{(1,2)^r}$-pgs-continuity, $M^{(1,2)^r}$-η*-continuity and $M^{(1,2)^r}$-η**-continuity and obtain decompositions of $M^{(1,2)^r}$-α-continuity and $M^{(1,2)^r}$-αgs-continuity.

6.2 SOME RESULTS

Remark 6.2.1

The $m_{x}^{(1,2)^r}$-α-interior $m_{x}^{(1,2)^r}$-α-int(S) of S is the union of all $m_{x}^{(1,2)^r}$-α-open sets contained in S. The $m_{x}^{(1,2)^r}$-α-closure $m_{x}^{(1,2)^r}$-α-cl(S) of S is the intersection of all $m_{x}^{(1,2)^r}$-α-closed sets containing S.

Proposition 6.2.2

Let X have the property [I]. For any subset A of a biminimal space X, the followings hold:

(i) $m_{x}^{(1,2)^r}$-α-cl(A) = A $\cup$ $m_{x}^{(1,2)^r}$-cl($m_{x}^{(1,2)^r}$-int($m_{x}^{(1,2)^r}$-cl(A)));
(ii) \( m_x^{(1,2)^r} - \alpha\)-int(A) = A \( \cap \) \( m_x^{(1,2)^r} - \text{int}(m_x^{(1,2)^r} - \text{int}(A)) \);

(iii) \( m_x^{(1,2)^r} - \text{pint}(A) = A \cap m_x^{(1,2)^r} - \text{int}(m_x^{(1,2)^r} - \text{cl}(A)) \).

**Proposition 6.2.3**

S is a \( m_x^{(1,2)^r} \)-semi-open set if and only if \( m_x^{(1,2)^r} \)-cl(S) = \( m_x^{(1,2)^r} \)-cl(\( m_x^{(1,2)^r} \)-int(S)).

**Remark 6.2.4**

The family of all \( m_x^{(1,2)^r} \)-h-sets of X will be denoted by \( m_x^{(1,2)^r} \)-h(X).

### 6.3 \( m_x^{(1,2)^r} \)-\( \eta^* \)-SETS AND \( m_x^{(1,2)^r} \)-\( \eta^{**} \)-SETS

**Definition 6.3.1**

A subset A of a space X is said to be

(i) an \( m_x^{(1,2)^r} \)-\( \alpha \)-B-set if A = V \( \cap \) T where V is \( m_x^{(1,2)^r} \)-\( \alpha \)-open and T is a \( m_x^{(1,2)^r} \)-t-set;

(ii) an \( m_x^{(1,2)^r} \)-\( \eta \)-set if A = V \( \cap \) T where V is \( m_x^{(1,2)^r} \)-open and T is an \( m_x^{(1,2)^r} \)-\( \alpha \)-closed set;

(iii) an \( m_x^{(1,2)^r} \)-\( \alpha \)-generalized semi-open (written as \( m_x^{(1,2)^r} \)-\( \alpha \)-gs-open) set in X if U \( \subseteq \) \( m_x^{(1,2)^r} \)-\( \alpha \)-int(A) where U \( \subseteq \) A and U is \( m_x^{(1,2)^r} \)-semi-closed in X.

(iv) a \( m_x^{(1,2)^r} \)-pre generalized semi-open [written as \( m_x^{(1,2)^r} \)-pgs-open] set in X if U \( \subseteq \) \( m_x^{(1,2)^r} \)-pint(A) whenever U \( \subseteq \) A and U is \( m_x^{(1,2)^r} \)-semi-closed in X.
The complements of the above mentioned open sets are their respective closed sets.

The collection of all $m^{(1,2)*r}_{x}-\alpha B$-sets (resp. $m^{(1,2)*r}_{x}-\alpha gs$-open sets, $m^{(1,2)*r}_{x}$-pgs-open sets) will be denoted by $m^{(1,2)*r}_{x}-\alpha B(X)$ (resp. $m^{(1,2)*r}_{x}-\alpha gsO(X)$, $m^{(1,2)*r}_{x}$-pgsO(X)).

**Definition 6.3.2**

A subset $A$ of a space $X$ is said to be

(i) an $m^{(1,2)*}_{x}$-$\eta^{*}$-set if $A = V \cap T$ where $V$ is $m^{(1,2)*}_{x}$-semi-open and $T$ is $m^{(1,2)*}_{x}$-$\alpha$-closed in $X$;

(ii) an $m^{(1,2)*}_{x}$-$\eta^{**}$-set if $A = V \cap T$ where $V$ is $m^{(1,2)*}_{x}$-$\alpha gs$-open and $T$ is a $m^{(1,2)*}_{x}$-t-set in $X$.

The collection of all $m^{(1,2)*}_{x}$-$\eta^{*}$-sets (resp. $m^{(1,2)*}_{x}$-$\eta^{**}$-sets) in $X$ will be denoted by $m^{(1,2)*}_{x}$-$\eta^{*}(X)$ (resp. $m^{(1,2)*}_{x}$-$\eta^{**}(X)$).

**Remark 6.3.3**

From the definitions, the followings hold:

(i) Every $m^{(1,2)*}_{x}$-A-set is $m^{(1,2)*}_{x}$-B-set but not conversely.

(ii) Every $m^{(1,2)*}_{x}$-open set is $m^{(1,2)*}_{x}$-$\alpha$-open but not conversely.

(iii) Every $m^{(1,2)*}_{x}$-B-set is $m^{(1,2)*}_{x}$-$\alpha B$-set.

(iv) Every regular $m^{(1,2)*}_{x}$-closed set is $m^{(1,2)*}_{x}$-closed but not conversely.

(v) Every locally $m^{(1,2)*}_{x}$-closed set is $m^{(1,2)*}_{x}$-$\eta$-set.
(vi) Every \( m_x^{(1,2)*} - \alpha \)-open set is \( m_x^{(1,2)*} - \text{semi-open} \) but not conversely.

(vii) Every \( m_x^{(1,2)*} - \eta \)-set is \( m_x^{(1,2)*} - \eta^* \)-set.

(viii) Every \( m_x^{(1,2)*} - \eta \)-set is \( m_x^{(1,2)*} - \alpha \beta \)-set.

(ix) Every \( m_x^{(1,2)*} - \alpha \)-open set is \( m_x^{(1,2)*} - \alpha \text{gs-open} \).

(x) Every \( m_x^{(1,2)*} - \alpha \beta \)-set is \( m_x^{(1,2)*} - \eta^{**} \)-set.

(xi) Every \( m_x^{(1,2)*} - \alpha \text{gs-open} \) set is \( m_x^{(1,2)*} - \eta^{**} \)-set.

(xii) Every \( m_x^{(1,2)*} - \alpha \text{gs-open} \) set is \( m_x^{(1,2)*} - \text{pgs-open} \).

**Theorem 6.3.4**

For a subset \( A \) of a space \( X \), the following are equivalent:

(i) \( A \) is an \( m_x^{(1,2)*} - \eta^* \)-set.

(ii) \( A = U \cap m_x^{(1,2)*} - \alpha \text{-cl}(A) \) for some \( m_x^{(1,2)*} - \text{semi-open} \) set \( U \).

**Proof**

(i) \( \rightarrow \) (ii): Since \( A \) is an \( m_x^{(1,2)*} - \eta^* \)-set, then \( A = U \cap T \) where \( U \) is \( m_x^{(1,2)*} - \text{semi-open} \) and \( T \) is \( m_x^{(1,2)*} - \alpha \)-closed. So, \( A \subseteq U \) and \( A \subseteq T \). Hence \( m_x^{(1,2)*} - \alpha \text{-cl}(A) \subseteq m_x^{(1,2)*} - \alpha \text{-cl}(T) \). Therefore, \( A \subseteq U \cap m_x^{(1,2)*} - \alpha \text{-cl}(A) \subseteq U \cap m_x^{(1,2)*} - \alpha \text{-cl}(T) = U \cap T = A \). Thus \( A = U \cap m_x^{(1,2)*} - \alpha \text{-cl}(A) \).

(ii) \( \rightarrow \) (i): It is obvious because \( m_x^{(1,2)*} - \alpha \text{-cl}(A) \) is \( m_x^{(1,2)*} - \alpha \)-closed.

**Remark 6.3.5**

Observe that since the union of \( m_x^{(1,2)*} - \text{t-sets} \) need not be a \( m_x^{(1,2)*} - \text{t-set} \), then the union of two \( m_x^{(1,2)*} - \eta^{**} \)-sets need not be an \( m_x^{(1,2)*} - \eta^{**} \)-set as seen from the following example.
Example 6.3.6

Let $X = \{a, b, c\}$, $m^1_x = \{\phi, X, \{a, b\}\}$ and $m^2_x = \{\phi, X\}$. Then the sets $\{a\}$ and $\{c\}$ are $m^{(1,2)*}_x$-sets in $X$, but their union $\{a, c\}$ is not an $m^{(1,2)*}_x$-set in $X$.

Remark 6.3.7

We have the following diagram

$$
\begin{align*}
m^{(1,2)*}_x-A(X) & \rightarrow m^{(1,2)*}_x-LC(X) \\
\downarrow & \downarrow \\
m^{(1,2)*}_x-\eta(X) & \rightarrow m^{(1,2)*}_x-\eta^*(X) \\
\downarrow & \downarrow \\
m^{(1,2)*}_x-B(X) & \rightarrow m^{(1,2)*}_x-\alpha B(X) \rightarrow m^{(1,2)*}_x-\eta^{**}(X) \\
& \uparrow \\
m^{(1,2)*}_x-\alpha gsO(X) & \rightarrow m^{(1,2)*}_x-pgsO(X)
\end{align*}
$$

where none of these implications is reversible as shown by the following Examples.

Example 6.3.8

Let $X = \{a, b, c\}$, $m^1_x = \{\phi, X, \{a\}\}$ and $m^2_x = \{\phi, X\}$. Then $\{a, b\}$ is both an $m^{(1,2)*}_x$-set and $m^{(1,2)}_x$-$\alpha$-B-set but not an $m^{(1,2)*}_x$-set in $X$. Also, $\{b\}$ is $m^{(1,2)*}_x$-set but not a locally $m^{(1,2)*}_x$-closed. Moreover, $\{a, b\}$ is $m^{(1,2)*}_x$-$\alpha$-B-set but not a $m^{(1,2)*}_x$-B-set.
Let $X = \{a, b, c\}$, $m^1_x = \{\phi, X, \{a, b\}\}$ and $m^2_x = \{\phi, X\}$. Then $\{c\}$ is an $m^{(1,2)r}_{x} - \eta^{**}$-set but not an $m^{(1,2)r}_{x} - \alpha$gs-open and the set $\{a\}$ is an $m^{(1,2)r}_{x} - \eta^{**}$-set but not an $m^{(1,2)r}_{x} - \alpha B$-set in $X$. Also, $\{a, c\}$ is $m^{(1,2)r}_{x} - \text{pgs-open set}$ but not an $m^{(1,2)r}_{x} - \alpha$gs-open.

**Remark 6.3.9**

(i) The notions of $m^{(1,2)r}_{x} - \eta^{*}$-sets and $m^{(1,2)r}_{x} - \alpha$gs-closed sets are independent.

(ii) The notions of $m^{(1,2)r}_{x} - \eta^{**}$-sets and $m^{(1,2)r}_{x} - \text{pgs-open sets}$ are independent.

**Example 6.3.10**

Let $X = \{a, b, c\}$, $m^1_x = \{\phi, X, \{a, b\}\}$ and $m^2_x = \{\phi, X\}$. Then $\{b, c\}$ is $m^{(1,2)r}_{x} - \alpha$gs-closed but not an $m^{(1,2)r}_{x} - \eta^{*}$-set.

Let $X = \{a, b, c\}$, $m^1_x = \{\phi, X, \{a\}\}$ and $m^2_x = \{\phi, X, \{b\}\}$. Then $\{a, b\}$ is an $m^{(1,2)r}_{x} - \eta^{*}$-set but not $m^{(1,2)r}_{x} - \alpha$gs-closed in $X$.

**Example 6.3.11**

Let $X = \{a, b, c\}$, $m^1_x = \{\phi, X, \{a, b\}\}$ and $m^2_x = \{\phi, X\}$. Then the set $\{c\}$ is an $m^{(1,2)r}_{x} - \eta^{**}$-set but not a $m^{(1,2)r}_{x} - \text{pgs-open set}$ and also the set $\{a, c\}$ is a $m^{(1,2)r}_{x} - \text{pgs-open set}$ but not an $m^{(1,2)r}_{x} - \eta^{**}$-set in $X$.

**Theorem 6.3.12**

Let $X$ have the property [u]. For a subset $A$ of a space $X$, the following are equivalent:

83
(i) A is $m_x^{(1,2)r}\-\alpha$-closed.

(ii) A is an $m_x^{(1,2)r}\-\eta\-set$ and $m_x^{(1,2)r}\-\alpha$gs-closed.

Proof

(i) $\rightarrow$ (ii). Obvious.

(ii) $\rightarrow$ (i). Since A is an $m_x^{(1,2)r}\-\eta\-set$, then $A = U \cap m_x^{(1,2)r}\-\alpha$-cl(A), where U is $m_x^{(1,2)r}\-semi-open$ in X. So, $A \subseteq U$ and since A is $m_x^{(1,2)r}\-\alpha$gs-closed, then $m_x^{(1,2)r}\-\alpha$-cl(A) $\subseteq U$. Therefore $m_x^{(1,2)r}\-\alpha$-cl(A) $\subseteq U \cap m_x^{(1,2)r}\-\alpha$-cl(A) = A. Hence A is $m_x^{(1,2)r}\-\alpha$-closed.

Proposition 6.3.13

Let X have the property [I]. Let A and B be subsets of a space X. If B is an $m_x^{(1,2)r}\-h$-set, then $m_x^{(1,2)r}\-\alpha-int(A \cap B) = m_x^{(1,2)r}\-\alpha-int(A) \cap m_x^{(1,2)r}\-int(B)$.

Theorem 6.3.14

Let X have the property [I]. For a subset S of a space X, the following are equivalent:

(i) S is $m_x^{(1,2)r}\-\alpha$gs-open.

(ii) S is an $m_x^{(1,2)r}\-\eta**\-set$ and $m_x^{(1,2)r}\-pgs$-open.

Proof

Necessity: Trivial.

Sufficiency: Assume that S is $m_x^{(1,2)r}\-pgs$-open and an $m_x^{(1,2)r}\-\eta**\-set$ in X. Then S = A $\cap$ B, where A is $m_x^{(1,2)r}\-\alpha$gs-open and B is a $m_x^{(1,2)r}\-t$-set.
in X. Let $F \subseteq S$, where $F$ is $m_{x}^{(1,2)}$-semi-closed in X. Since $S$ is $m_{x}^{(1,2)}$-pgs-open in X, $F \subseteq m_{x}^{(1,2)}$-pint$(S) = S \cap m_{x}^{(1,2)}$-int$(m_{x}^{(1,2)}$-cl$(S)) = (A \cap B) \cap m_{x}^{(1,2)}$-int$[m_{x}^{(1,2)}$-cl$(A \cap B)] \subseteq A \cap B \cap m_{x}^{(1,2)}$-int$(m_{x}^{(1,2)}$-cl$(A)) \cap m_{x}^{(1,2)}$-int$(m_{x}^{(1,2)}$-cl$(B)) = A \cap B \cap m_{x}^{(1,2)}$-int$(m_{x}^{(1,2)}$-cl$(A)) \cap m_{x}^{(1,2)}$-int$(m_{x}^{(1,2)}$-cl$(B))$, since B is a $m_{x}^{(1,2)}$-t-set. This implies $F \subseteq m_{x}^{(1,2)}$-int$(B)$. Note that A is $m_{x}^{(1,2)}$-αgs-open and that $F \subseteq A$. So, $F \subseteq m_{x}^{(1,2)}$-α-int$(A)$. Therefore, $F \subseteq m_{x}^{(1,2)}$-α-int$(A) \cap m_{x}^{(1,2)}$-int$(B) = m_{x}^{(1,2)}$-α-int$(S)$ by Proposition (6.3.13). Hence S is $m_{x}^{(1,2)}$-αgs-open.

6.4 $M^{(1,2)}$-η*-CONTINUITY AND $M^{(1,2)}$-η**-CONTINUITY

We introduce new classes of mappings as follows:

**Definition 6.4.1**

A function $f: X \rightarrow Y$ is said to be

(i) $M^{(1,2)}$-η-continuous if $f^{-1}(V)$ is an $m_{x}^{(1,2)}$-η-set in X for every $m_{y}^{(1,2)}$-open set V of Y;

(ii) $M^{(1,2)}$-αB-continuous if $f^{-1}(V)$ is an $m_{x}^{(1,2)}$-αB-set in X for every $m_{y}^{(1,2)}$-open set V of Y;

(iii) $M^{(1,2)}$-αgs-continuous (resp. $M^{(1,2)}$-pgs-continuous) if $f^{-1}(V)$ is an $m_{x}^{(1,2)}$-αgs-open set (resp. $m_{x}^{(1,2)}$-pgs-open set) in X for every $m_{y}^{(1,2)}$-open set V of Y.
Definition 6.4.2

A function f: X → Y is said to be $M^{(1,2)}\eta^*-\text{continuous}$ (resp. $M^{(1,2)}\eta^{**}-\text{continuous}$) if $f^{-1}(V)$ is an $m_x^{(1,2)}\eta^*-\text{set}$ (resp. $m_x^{(1,2)}\eta^{**}-\text{set}$) in X for every $m_y^{(1,2)r}\text{-open set}$ V of Y.

Definition 6.4.3

A function f: X → Y is said to be $M^{(1,2)}\eta^*\text{-continuous}$ if $f^{-1}(V)$ is an $m_x^{(1,2)}\eta^*\text{-set}$ in X for every $m_y^{(1,2)r}\text{-closed set}$ V of Y.

Remark 6.4.4

(i) Every $M^{(1,2)r}\alpha\text{-gs-continuous}$ is $M^{(1,2)r}\alpha\text{-pgs-continuous}$ but not conversely.

(ii) $M^{(1,2)r}\alpha\text{-A-continuous}$ is $M^{(1,2)r}\alpha\text{-LC-continuous}$ but not conversely.

(iii) $M^{(1,2)r}\alpha\text{-A-continuous}$ is $M^{(1,2)r}\alpha\text{-B-continuous}$ but not conversely.

(iv) $M^{(1,2)r}\alpha\text{-B-continuous}$ is $M^{(1,2)r}\alpha\text{B-continuous}$.

Remark 6.4.5

It is clear that a function f: X → Y is $M^{(1,2)r}\alpha\text{-continuous}$ if and only if $f^{-1}(V)$ is an $m_x^{(1,2)r}\alpha\text{-closed set}$ in X for every $m_y^{(1,2)r}\text{-closed set}$ V of Y.

Remark 6.4.6

From the definitions stated above we obtain the following diagram.
$M^{(1,2)^r}$-A-continuity $\rightarrow$ $M^{(1,2)^r}$-LC-continuity

\[\downarrow\]

$M^{(1,2)^r}$-\(\eta\)-continuity $\rightarrow$ $M^{(1,2)^r}$-\(\eta^*\)-continuity

\[\downarrow\]

$M^{(1,2)^r}$-B-continuity $\rightarrow$ $M^{(1,2)^r}$-\(\alpha\)B-continuity $\rightarrow$ $M^{(1,2)^r}$-\(\eta^{**}\)-continuity

\[\downarrow\]

$M^{(1,2)^r}$-\(\alpha\)gs-continuity $\rightarrow$ $M^{(1,2)^r}$-pgs-continuity

where none of the implications is reversible as shown by the following Examples.

**Example 6.4.7**

Let $X = Y = \{a, b, c\}$, $m_x^1 = \{\emptyset, X, \{a\}\}$, $m_x^2 = \{\emptyset, X\}$, $m_y^1 = \{\emptyset, Y, \{a\}\}$ and $m_y^2 = \{\emptyset, Y, \{b\}\}$. Then the identity function $f: X \rightarrow Y$ is $M^{(1,2)^r}$-\(\eta^*\)-continuous but not $M^{(1,2)^r}$-\(\eta\)-continuous.

**Example 6.4.8**

Let $X = Y = \{a, b, c\}$, $m_x^1 = \{\emptyset, X, \{a\}\}$, $m_x^2 = \{\emptyset, X\}$, $m_y^1 = \{\emptyset, Y, \{b\}\}$ and $m_y^2 = \{\emptyset, Y, \{b, c\}\}$. Then the identity function $f: X \rightarrow Y$ is $M^{(1,2)^r}$-\(\eta\)-continuous but not $M^{(1,2)^r}$-LC-continuous.

**Example 6.4.9**

Let $X = Y = \{a, b, c\}$, $m_x^1 = \{\emptyset, X, \{a\}\}$, $m_x^2 = \{\emptyset, X\}$, $m_y^1 = \{\emptyset, Y, \{a\}\}$ and $m_y^2 = \{\emptyset, Y, \{b\}\}$. Define a function $f: X \rightarrow Y$ by $f(a) = a$;
f(b) = c; f(c) = b. Then f is $M^{(1,2)*-\alpha B}$-continuous but it is neither $M^{(1,2)*-\eta}$-continuous nor $M^{(1,2)*-B}$-continuous.

**Example 6.4.10**

Let $X = Y = \{a, b, c\}$, $m_x^1 = \{\phi, X, \{b, c\}\}$, $m_x^2 = \{\phi, X, \{c\}\}$, $m_y^1 = \{\phi, Y, \{a\}\}$ and $m_y^2 = \{\phi, Y, \{c\}\}$. Then the identity function $f: X \rightarrow Y$ is $M^{(1,2)*-\eta^{**}}$-continuous but not $M^{(1,2)*-\alpha gs}$-continuous.

**Example 6.4.11**

Let $X = Y = \{a, b, c\}$, $m_x^1 = \{\phi, X, \{a, b\}\}$, $m_x^2 = \{\phi, X\}$, $m_y^1 = \{\phi, Y, \{a\}\}$ and $m_y^2 = \{\phi, Y, \{b\}\}$. Then the identity function $f: X \rightarrow Y$ is $M^{(1,2)*-\eta^{**}}$-continuous but not $M^{(1,2)*-\alpha B}$-continuous.

**Example 6.4.12**

Let $X = Y = \{a, b, c\}$, $m_x^1 = \{\phi, X, \{a, b\}\}$, $m_x^2 = \{\phi, X\}$, $m_y^1 = \{\phi, Y, \{a\}\}$ and $m_y^2 = \{\phi, Y, \{b, c\}\}$. Define a function $f: X \rightarrow Y$ by $f(a) = b; f(b) = a; f(c) = c$. Then f is $M^{(1,2)*-pgs}$-continuous but not $M^{(1,2)*-\alpha gs}$-continuous.

**Remark 6.4.13**

The following Examples show that the concepts of

(i) $M^{(1,2)*-\alpha gs}$-continuity and $M^{(1,2)*-\eta^{**}}$-continuity are independent;

(ii) $M^{(1,2)*-\alpha gs}$-continuity and $M^{(1,2)*-\eta^{**}}$-continuity are independent.
Example 6.4.14

Let \( X = Y = \{a, b, c\} \), \( m^1_x = \{\phi, X, \{a\}\} \), \( m^2_x = \{\phi, X, \{b\}\} \), \( m^1_y = \{\phi, Y, \{c\}\} \) and \( m^2_y = \{\phi, Y, \{a\}\} \). Then the identity function \( f: X \rightarrow Y \) is \( M^{(1,2)_\eta\ast\eta\ast}-\)continuous but not \( M^{(1,2)_\alpha\alpha\ast\eta\ast}-\)continuous.

Example 6.4.15

Let \( X = Y = \{a, b, c\} \), \( m^1_x = \{\phi, X, \{a, b\}\} \), \( m^2_x = \{\phi, X\} \), \( m^1_y = \{\phi, Y, \{a\}\} \) and \( m^2_y = \{\phi, Y\} \). Let \( f: X \rightarrow Y \) be the identity function on \( X \). Then \( f \) is \( M^{(1,2)_\alpha\alpha\ast\eta\ast}-\)continuous but not \( M^{(1,2)_\eta\eta\ast\eta\ast}-\)continuous.

Example 6.4.16

Let \( X = Y = \{a, b, c\} \), \( m^1_x = \{\phi, X, \{a, b\}\} \), \( m^2_x = \{\phi, X\} \), \( m^1_y = \{\phi, Y\} \) and \( m^2_y = \{\phi, Y, \{a\}\} \). Then the identity function \( f: X \rightarrow Y \) is \( M^{(1,2)_\alpha\alpha\ast\eta\ast\eta\ast}-\)continuous but not \( M^{(1,2)_\eta\eta\ast\eta\ast\eta\ast}-\)continuous.

Example 6.4.17

Let \( X = Y = \{a, b, c\} \), \( m^1_x = \{\phi, X, \{c\}\} \), \( m^2_x = \{\phi, X, \{a\}\} \), \( m^1_y = \{\phi, Y, \{b, c\}\} \) and \( m^2_y = \{\phi, Y, \{a\}\} \). The identity function \( f: X \rightarrow Y \) is \( M^{(1,2)_\alpha\alpha\ast\eta\ast\eta\ast\eta\ast}-\)continuous but not \( M^{(1,2)_\eta\eta\ast\eta\ast\eta\ast\eta\ast}-\)continuous.

Remark 6.4.18

The following Examples show that the concepts of \( M^{(1,2)_\alpha\alpha\ast\eta\ast\eta\ast\eta\ast}-\)pgs-continuity and \( M^{(1,2)_\eta\eta\ast\eta\ast\eta\ast\eta\ast}-\)continuity are independent.
Example 6.4.19

Let \( X = Y = \{a, b, c\} \), \( m_1^x = \{\phi, X, \{a, b\}\} \), \( m_2^x = \{\phi, X\} \), \( m_1^y = \{\phi, Y, \{b\}\} \) and \( m_2^y = \{\phi, Y, \{a\}, \{a, c\}\} \). Then the identity function \( f: X \rightarrow Y \) is \( M^{(1,2)^*}\)-pgs-continuous, but it is not \( M^{(1,2)^*\eta^{**}}\)-continuous.

Example 6.4.20

Let \( X = Y = \{a, b, c\} \), \( m_1^x = \{\phi, X, \{c\}\} \), \( m_2^x = \{\phi, X\} \), \( m_1^y = \{\phi, Y, \{a\}\} \) and \( m_2^y = \{\phi, Y, \{b\}\} \). Let \( f: X \rightarrow Y \) be defined by \( f(a) = c \), \( f(b) = b \) and \( f(c) = a \). Then \( f \) is \( M^{(1,2)^*\eta^{**}}\)-continuous, but it is not \( M^{(1,2)^*}\)-pgs-continuous.

6.5 BIMINIMAL DECOMPOSITIONS

Theorem 6.5.1

For a function \( f: X \rightarrow Y \), where \( X \) have the property [u], the following are equivalent:

(i) \( f \) is \( M^{(1,2)^*\alpha}\)-continuous.

(ii) \( f \) is \( M^{(1,2)^*\eta^*}\)-continuous and \( M^{(1,2)^*\alpha_{gs}}\)-continuous.

Proof

The proof follows from Definitions 6.4.3 and 6.4.1(iii), Remark 6.4.5 and Theorem 6.3.12.

Theorem 6.5.2

For a function \( f: X \rightarrow Y \), where \( X \) have the property [I], the following are equivalent:
(i) $f$ is $M^{(1,2)*}$-αgs-continuous.

(ii) $f$ is $M^{(1,2)*}$-η**-continuous and $M^{(1,2)*}$-pgs-continuous.

**Proof**

The proof follows from Theorem 6.3.14.