CHAPTER 2

CHAPTER - 2

FUZZY NEW MODELS – SFCMs, SFRMs AND FLCMs

From the study and analysis of the pilot survey, discussions and interviews with students and teachers, we formulated an elaborate questionnaire. From the study of this elaborate questionnaire and mainly discussions with students, we discovered that the students were not a position to write even a line or two of their own in any language (English or in their mother tongue). This was literally a big shock to us. So to overcome this problem we made a very short questionnaire with very simple answers to the questions as yes or no or to some extent or to a large extent. Based on the study of the answers from this short questionnaire we desired to construct a special model which would cater to all these issues and this resulted in two new fuzzy models. In section one Super Fuzzy Cognitive Maps (SFCMs) models, Domain Super Fuzzy Relational Maps (DSFRMs) and Range Super FRMs (RSFRMs) models are developed and described. These models are appropriate when they use multi experts with varying attributes. These models make use of the concept of super matrices and super special vectors described in chapter 0 of this thesis. In section two, the notion of Fuzzy Linguistic Cognitive Maps (FLCMs) model is introduced. This model exploits the concept of fuzzy linguistic attributes instead of using the interval [0, 1].

2.1 NEW SUPER FUZZY MODELS

This section has three subsections. In subsection 2.1.1, the new concept of Super Fuzzy Cognitive Maps (SFCMs) model is introduced and described by an illustration. In the subsection 2.1.2, the concept of four types of new Super Fuzzy Relational Maps (SFRMs) models are defined and developed. In the final subsection 2.1.3, the new Super Fuzzy Cognitive Relational Maps (SFCRMs) model is introduced and defined.

2.1.1 Description of the New Super Fuzzy Cognitive Maps (SFCMs) Models with Illustration

In this section we for the first time define a Super Fuzzy Cognitive Maps (SFCMs) model and describe how it functions. Here we give the description of the multi expert super FCMs model using the special super fuzzy diagonal matrix. (chapter 00; pp. 13-15) .

DEFINITION 2.1.1.1: Suppose n experts want to work with a problem P using a FCM model, then how to form an integrated dynamical system which can function simultaneously using the n experts opinion.

Suppose the first expert spells out the attributes of a problem as $x_1^1, x_2^1, \ldots, x_{t_1}^1$, *the second expert gives the attributes as* $x_1^2, x_2^2, \ldots, x_{t_2}^2$ *and so on. Thus the* i^{th} *expert gives the attributes with which he/she wishes to work as* $x_1^i, x_2^i, \ldots, x_{t_i}^i$; *i = 1, 2, 3, …, n. Now we model the problem using the special super diagonal fuzzy matrix; this supermatrix will be called as the super connection matrix of the Super FCMs (SFCMs). We see the special feature of this special super fuzzy diagonal matrix will be that all the diagonal matrices are square matrices and the main diagonal of each of these submatrices of the special super fuzzy diagonal matrix is zero. The special super fuzzy diagonal matrix for the problem P takes the following form and is denoted by* M_D.

$$M_D = \begin{array}{cc} & \begin{array}{cccc} x_1^1\, x_2^1 \ldots x_{t_1}^1 & x_1^2\, x_2^2 \ldots x_{t_2}^2 & \ldots & x_1^n\, x_2^n \ldots x_{t_n}^n \end{array} \\ \begin{array}{c} x_1^1 \\ x_2^1 \\ \vdots \\ x_{t_1}^1 \\ x_1^2 \\ x_2^2 \\ \vdots \\ x_{t_2}^2 \\ \vdots \\ x_1^n \\ x_2^n \\ \vdots \\ x_{t_n}^n \end{array} & \left[\begin{array}{c|c|c|c} M_1^1 & (0) & \cdots & (0) \\ \hline (0) & M_2^2 & \cdots & (0) \\ \hline \vdots & \vdots & \vdots & \vdots \\ \hline (0) & (0) & \cdots & M_n^n \end{array}\right] \end{array}.$$

We see M_i^i is a fuzzy matrix with main diagonal elements to be zero i.e.;

$$M_i^i = \begin{bmatrix} 0 & m_{12}^i & \ldots & m_{1t_i}^i \\ m_{21}^i & 0 & \ldots & m_{2t_i}^i \\ \vdots & \vdots & & \vdots \\ m_{t_i 2}^i & m_{t_i 2}^i & \ldots & 0 \end{bmatrix}$$

where $m_{jk}^i \in [0, 1]$, $1 \leq j, k \leq t_i$; $i = 1, 2, \ldots, n$.

This model will be known as the multi expert Super Fuzzy Cognitive Maps (SFCMs) model and the associated fuzzy supermatrix would be known as the special diagonal fuzzy supermatrix. We also define the matrix M_D as the super dynamical multi expert system.

Example 2.1.1.1: Suppose we have 3 experts who wish to work with a problem using FCMs. The problem they wish to investigate is to analyze the Indian political situation to predict the possible electoral winner or how people tend to prefer a particular politician and so on or so forth. All of them choose to use the FCMs model.

The first expert wishes to work with the following six nodes:

x_1^1 - Language

x_2^1 - Community

x_3^1 - Service to people

x_4^1 - Finance they have

x_5^1 - Media they can accesses to

x_6^1 - Party's strength and opponents strength.

The second expert wants to work with the following five nodes:

x_1^2 - Working members of the party

x_2^2 - Party's popularity in media

x_3^2 - The local communities strength and weakness, in relation to the politicians community

x_4^2 - Media's accessibility

x_5^2 - Popularity of the politician in the context of public opinion.

The third expert wishes to work with the following attributes or nodes:

x_1^3 - Language and caste of the public where he stakes

x_2^3 - The finance the politician can spend in propaganda

x_3^3 - Opponents strength

x_4^3 - Parties weaknesses

x_5^3 - Service done by the party in that village

x_6^3 - The party's popularity in media

x_7^3 - Public figure configuration.

Now using these 3 experts we obtain the following special diagonal super fuzzy matrix M_D which is as follows:

$$
M_D = \begin{array}{cc}
 & \begin{array}{ccc} x_1^1\ x_2^1\ x_3^1\ x_4^1\ x_5^1\ x_6^1 & x_1^2\ x_2^2\ x_3^2\ x_4^2\ x_5^2 & x_1^3\ x_2^3\ x_3^3\ x_4^3\ x_5^3\ x_6^3\ x_7^3 \end{array} \\
\begin{array}{c} x_1^1 \\ x_2^1 \\ x_3^1 \\ x_4^1 \\ x_5^1 \\ x_6^1 \\ \hline x_1^2 \\ x_2^2 \\ x_3^2 \\ x_4^2 \\ x_5^2 \\ \hline x_1^3 \\ x_2^3 \\ x_3^3 \\ x_4^3 \\ x_5^3 \\ x_6^3 \\ x_7^3 \end{array} & \left[\begin{array}{c|c|c} (x_{ij}^1) & (0) & (0) \\ \hline (0) & (x_{ij}^2) & (0) \\ \hline (0) & (0) & (x_{ij}^3) \end{array}\right]
\end{array}.
$$

We see (x_{ij}^1) is a 6×6 matrix in which $x_{ii}^1 = 0$ for i = 1, 2, 3, …, 6; i.e., the main diagonal terms of (x_{ij}^1) are zero and the $x_{ij}^1 \in \{0, 1, -1\}$; $1 \leq i, j \leq 6$. Similarly (x_{ij}^2) is a 5×5 matrix in which $x_{kk}^2 = 0$, k = 1, 2, ..., 5 and $x_{ij}^2 \in \{0, 1, -1\}$; $1 \leq i, j \leq 5$. The fuzzy matrix (x_{ij}^3) is such that $x_{jj}^3 = 0$, j = 1, 2, ..., 7 with $x_{ij}^3 \in \{0, 1, -1\}$. The diagonal elements of this special fuzzy supermatrix are all square matrices; $1 \leq i, j \leq 7$.

However this problem is for mere illustration. We just give the special diagonal fuzzy super matrix, M_D with some entries from $\{0, 1, -1\}$.

$$M_D = \begin{array}{c@{}c} & \begin{array}{cccccccccccccccccc} x_1^1 & x_2^1 & x_3^1 & x_4^1 & x_5^1 & x_6^1 & x_1^2 & x_2^2 & x_3^2 & x_4^2 & x_5^2 & x_1^3 & x_2^3 & x_3^3 & x_4^3 & x_5^3 & x_6^3 & x_7^3 \end{array} \\ \begin{array}{c} x_1^1 \\ x_2^1 \\ x_3^1 \\ x_4^1 \\ x_5^1 \\ x_6^1 \\ x_1^2 \\ x_2^2 \\ x_3^2 \\ x_4^2 \\ x_5^2 \\ x_1^3 \\ x_2^3 \\ x_3^3 \\ x_4^3 \\ x_5^3 \\ x_6^3 \\ x_7^3 \end{array} & \left[\begin{array}{cccccc|ccccc|ccccccc} 0 & 1 & -1 & 0 & 0 & 1 & & & & & & & & & & & & \\ 1 & 0 & 0 & 1 & 1 & 0 & & & & & & & & & & & & \\ -1 & 0 & 0 & 1 & 0 & 0 & & & (0) & & & & & & (0) & & & \\ 0 & 0 & 1 & 0 & -1 & 0 & & & & & & & & & & & & \\ 1 & 0 & 0 & 0 & 0 & 1 & & & & & & & & & & & & \\ 0 & 1 & -1 & 0 & 0 & 0 & & & & & & & & & & & & \\ \hline & & & & & & 0 & 1 & 0 & 1 & 0 & & & & & & & \\ & & & & & & 1 & 0 & 1 & 0 & 1 & & & & & & & \\ & & (0) & & & & 0 & 1 & 0 & -1 & 0 & & & & (0) & & & \\ & & & & & & 1 & 0 & -1 & 0 & 1 & & & & & & & \\ & & & & & & 0 & 0 & 1 & -1 & 0 & & & & & & & \\ \hline & & & & & & & & & & & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ & & & & & & & & & & & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ & & & & & & & & & & & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ & & (0) & & & & & & (0) & & & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ & & & & & & & & & & & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ & & & & & & & & & & & 1 & -1 & 0 & 0 & 0 & 0 & -1 \\ & & & & & & & & & & & -1 & 0 & 1 & 0 & 1 & 0 & 0 \end{array}\right] \end{array}.$$

Now let us see how the super dynamical multi expert system M_D functions.

Suppose the expert wishes to work with

$$X = [1\ 0\ 0\ 0\ 0\ 1 \mid 0\ 1\ 0\ 0\ 0 \mid 0\ 0\ 1\ 0\ 0\ 1\ 0\ ];$$

we want to obtain the hidden pattern of this state vector X.

$$X M_D = [0\ 2\ {-2}\ 0\ 0\ 1 \mid 1\ 0\ 1\ 0\ 1 \mid 2\ {-1}\ 0\ 0\ 1\ 1\ {-1}];$$

after updating and thresholding we get

$$X_1 = [1\ 1\ 0\ 0\ 0\ 1 \mid 1\ 1\ 1\ 0\ 1 \mid 1\ 0\ 1\ 0\ 1\ 1\ 0\ ].$$

We find the effect of X_1 on the super dynamical system M_D.

$$X_1 M_D = [1\ 2\ {-2}\ 1\ 1\ 1 \mid 1\ 2\ 2\ {-1}\ 1 \mid 3\ 0\ 1\ 0\ 2\ 3\ 1],$$

after updating and thresholding we get the resultant X_2 to be

$$X_2 = [\ 1\ 1\ 0\ 1\ 1\ 1 \mid 1\ 1\ 1\ 0\ 1 \mid 1\ 0\ 1\ 0\ 1\ 1\ 1\ ]$$

and so on until we arrive at fixed point or a limit cycle.

Thus we see we can use the same C-program used for FCMs only what we need is to partition the hidden pattern properly. We see because of the programs we can use any number of experts opinion. Further the demerit of combined FCMs which at times tends to zero if we have 1 and –1 as a entry is over come by the SFCMs. Further as every experts' feeling is given by a single super fuzzy row vector this helps in easy comparison and computation.

2.1.2 New Super Fuzzy Relational Maps model

In this section we introduce four types of new Super Fuzzy Relational Maps (SFRMs) models.

DEFINITION 2.1.2.1: *Suppose we have some n experts working on a real world problem and give their opinions. They all agree upon to work with the same domain space elements / attributes / concepts; using FRMs model but do not concur on the attributes from the range space; then we can use the special super fuzzy row vector (matrix) to model the problem using Domain Super Fuzzy Relational Maps (DSFRMs) model.*

The DSFRMs matrix associated with this model is given by S_M;

$$S_M = \begin{array}{c} \\ D_1 \\ D_2 \\ \vdots \\ D_m \end{array}\begin{array}{c} \begin{array}{cccc} t_1^1 \ldots t_{r_1}^1 & t_1^2 \ldots t_{r_2}^2 & & t_1^n \; t_2^n \ldots t_{r_n}^n \end{array} \\ \left[\begin{array}{c|c|c|c} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{array}\right] \end{array}$$

$$= \left[S_M^1 \mid S_M^2 \mid \ldots \mid S_M^n \right]$$

where each S_M^i is a m × $t_{r_i}^i$ matrix associated with a FRMs given by the i^{th} expert having $D_1, \ldots, D_m$ to be the domain attributes and ($t_1^i, t_2^i, \ldots, t_{r_i}^i$) to be the range attributes of the i^{th} expert, i = 1, 2, …, n and S_M, the DSFRMs matrix will be a special super row vector / matrix (1 ≤ i ≤ n) which serves as the super special dynamical system of DSFRMs model.

However if n is even a very large value using the mode of programming one can easily obtain the resultant pair of super row vectors or the super hidden pattern for any input supervector which is under investigation.

These DSFRMs will also be known as Domain constant FRMs model for all the experts choose to work with the same domain space attributes only the range space attributes are varying.

Next we proceed on to define the notion of super FRMs with constant range space attributes and varying domain space attributes.

DEFINITION 2.1.2.2: *Let some m experts give opinion on a real world problem who agree upon to make use of the same space of attributes / concepts from the range space using FRMs but want to use different concepts for the domain space then we make use of the newly constructed special super column vector as the matrix to construct this new model. Thus the associated special super column matrix S^M is*

$$S^M = \begin{array}{cc} & \begin{array}{cccc} R_1 & R_2 & \cdots & R_s \end{array} \\ \begin{array}{c} D_1^1 \\ D_2^1 \\ \vdots \\ D_{t_1}^1 \\ D_1^2 \\ \vdots \\ D_{t_2}^2 \\ \vdots \\ D_1^m \\ \vdots \\ D_{t_m}^m \end{array} & \left[\begin{array}{cccc} & & & \\ & & & \\ & & & \\ & & & \\ \hline & & & \\ & & & \\ & & & \\ \hline & & & \\ \hline & & & \\ & & & \\ & & & \end{array} \right] \end{array}.$$

The column attributes i.e., the range space remain constant as $R_1, \ldots, R_s$ for all m experts; only the row attributes for any i^{th} expert is $D_1^i, D_2^i, \ldots, D_{t_i}^i$;

i = 1, 2, …, m; vary from expert to expert. This system will be known as the Range constant Fuzzy Super FRMs or shortly denoted as RSFRMs model.

However it is pertinent to keep on record that by transposing the super matrix of the super DSFRMs dynamical system we get the super matrix of the super DSFRMs dynamical system we get the super matrix of the super RSFRMs. Thus both the models are equivalent. We will illustrate these two definitions by examples i.e., by real world problems in the 4th section of this chapter.

One may be interested in finding a model which has both the range and the column vectors varying, for some experts may not agree upon the constant row or constant column FRMs model.

We construct a new model in that case.

DEFINITION 2.1.2.3: *Suppose we have m experts who wish to work with different sets of both row and column attributes i.e., domain and range space using FRMs, then to accommodate or form a integrated matrix model to cater to this need. We make use of the super diagonal fuzzy matrix, to model such a problem. Suppose the first expert works with the domain attributes* $D_1^1, \ldots, D_{t_1}^1$ *and range attributes* $R_1^1, \ldots, R_{n_1}^1$*, The second expert works with domain attributes* $D_1^2, \ldots, D_{t_2}^2$ *and with range attributes* $R_1^2, \ldots, R_{n_2}^2$ *and so on. Thus the mth expert works with* $D_1^m, \ldots, D_{t_m}^m$ *domain attributes and* $R_1^m, \ldots, R_{n_m}^m$ *range attributes. We have the following diagonal fuzzy supermatrix to model the situation. We are under the assumption that all the attributes both from the domain space as well as from the range space of the m experts are different. The fuzzy super matrix associated with this new mutliexperts model is given by the diagonal super matrix* M_F *where*

$$
M_F = \begin{array}{c} \\ D_1^1 \\ \vdots \\ D_{t_1}^1 \\ D_1^2 \\ \vdots \\ D_{t_2}^2 \\ \vdots \\ D_1^m \\ \vdots \\ D_{t_m}^m \end{array}
\begin{array}{c}
\begin{array}{cccc} R_1^1\, R_2^1 \ldots R_{n_1}^1 & R_1^2\, R_2^2 \ldots R_{n_2}^2 & \cdots & R_1^m\, R_2^m \ldots R_{t_m}^m \end{array} \\
\left[\begin{array}{c|c|c|c}
M_1 & (0) & \cdots & (0) \\ \hline
(0) & M_2 & \cdots & (0) \\ \hline
\vdots & \vdots & \vdots & \vdots \\ \hline
(0) & (0) & \cdots & M_n
\end{array}\right]
\end{array}
$$

where each M_i is a $t_i \times n_i$ matrix associated with the FRMs, we see except, the diagonal strip all other entries are zero. We call this super matrix as a special diagonal fuzzy super matrix and this model will be known as the Special Diagonal Super FRMs Model which will be denoted briefly as (SDSFRMs). M_F is defined as the special super dynamical system associated with the SDSFRMs mutiexpert model.

Now we proceed onto describe yet another new super fuzzy model.

2.1.3 Super Fuzzy Cognitive Relational Maps (SFCRMs) model

Finally in this subsection we define yet a new super fuzzy model which is mainly constructed to study the combined effect of the two models described in chapter one and chapter three of this thesis.

DEFINITION 2.1.3.1: *Suppose there are two experts who work on the same problem and one of them want to work with the FRMs model and another one with FCMs model then we define the new model as the Super Fuzzy Cognitive Relational Maps (SFCRMs) model.*

The associated matrix of this model is a super diagonal matrix.

Suppose the first expert uses m attributes say $t_1, \ldots, t_m$ and uses the FCMs model and the second experts uses the FRMs model with $s_1, \ldots, s_d$ as domain attributes and $n_1, n_2, \ldots, n_r$ as the range attributes. We see if N_S denotes the super diagonal matrix then

$$N_S = \begin{array}{c} \\ t_1 \\ t_2 \\ \vdots \\ t_m \\ s_1 \\ s_2 \\ \vdots \\ s_d \end{array} \begin{array}{c} \begin{array}{cccccccc} t_1 & t_2 & \ldots & t_m & n_1 & n_2 & \ldots & n_r \end{array} \\ \left[\begin{array}{c|c} A & (0) \\ \hline (0) & B \end{array}\right] \end{array}$$

where A is a $m \times m$ connection matrix of the FCMs and B is a $d \times r$ connection matrix associated with the FRMs. Thus N_S denotes the super fuzzy dynamical system of the SFCRMs model.

We can extend this to any number of experts then the multi expert Super Fuzzy Cognitive Relational Maps (SFCRMs) model is as follows.

$$\text{Let } N_S = \left[\begin{array}{c|c|c|c} A_1 & (0) & \ldots & (0) \\ \hline (0) & A_2 & \ldots & (0) \\ \hline \vdots & \vdots & \vdots & \vdots \\ \hline (0) & (0) & \ldots & A_m \end{array}\right]$$

where some of the A_i's are connection matrices associated with FCMs and some of the A_j's are FRMs so that the combined structure is a Super Fuzzy Cognitive Relational Maps (SFCRMs) model whose associated multiexpert super dynamical fuzzy system is a super diagonal matrix.

2.2 NEW SUPER FUZZY LINGUISTIC MAPS MODELS

In this section the definition of two new types of fuzzy linguistic models are carried out. This section has two subsections. In subsection 2.2.1 the new Fuzzy Linguistic Cognitive Maps (FLCMs) model is introduced and illustrated. The concept of Fuzzy Linguistic Relational Maps model (FLRMs model) is defined and described in subsection 2.2.2.

2.2.1 New Fuzzy Linguistic Cognitive Maps (FLCMs) Model

In this section we first introduce the new notion of fuzzy linguistic set L. L contains terms like best, good, bad, very good, better, poor, very bad, fair, worst; etc; where all of them are only adjectives. We further assume L also contains zero and each pair of terms in L are comparable; that is L is a totally ordered set. Now instead of using values from the set [0, 1] we use only elements from L which are also fuzzy in their outlook. If we have a matrix M whose entries are from the set L then we call M to be a fuzzy linguistic matrix.

Just we will illustrate this situation by an example.

$$M = \begin{bmatrix} \text{good} & \text{bad} & 0 & \text{fair} \\ 0 & \text{v.good} & \text{bad} & 0 \\ \text{best} & 0 & \text{v.fair} & \text{good} \\ \text{worst} & \text{v.bad} & 0 & \text{best} \\ \text{fair} & 0 & \text{good} & \text{better} \end{bmatrix}.$$

M is a 5 × 4 fuzzy linguistic matrix.

Let V = (good, bad, 0, best, worst); we see V is a fuzzy linguistic row matrix.

Let $S = \begin{bmatrix} \text{good} \\ 0 \\ \text{bad} \\ \text{worst} \\ \text{best} \\ \text{v.bad} \\ \text{fair} \\ 0 \end{bmatrix}$; S is defined as the fuzzy linguistic column matrix.

We can also have the fuzzy linguistic square matrix given by

$$T = \begin{bmatrix} 0 & \text{good} & \text{bad} & \text{worst} \\ \text{fair} & 0 & \text{good} & \text{bad} \\ \text{v.bad} & \text{best} & 0 & \text{fair} \\ 0 & \text{good} & \text{better} & \text{best} \end{bmatrix}.$$

We will be using these sort of fuzzy linguistic matrices in the new fuzzy linguistic models like fuzzy linguistic cognitive maps model and fuzzy linguistic relational maps model.

Finally we define the new notion of fuzzy linguistic directed graphs.

Suppose we have a graph for which the vertices are some attributes; need not necessarily be fuzzy linguistic terms but the edges of these graphs i.e., the edges connecting these vertices are necessarily fuzzy linguistic terms. Then we call the directed graph to be a fuzzy linguistic graph. We need this notion in the construction of both fuzzy linguistic cognitive maps as well as fuzzy linguistic relational maps. We just illustrate the concept of fuzzy linguistic graphs by an example or two.

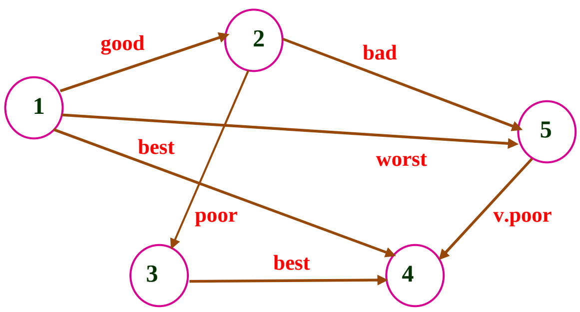

FIGURE 2.2.1.1

This is a fuzzy linguistic graph with 5 vertices and edge values take its entries from L. The fuzzy linguistic matrix M associated with this fuzzy linguistic graph is as follows:

$$M = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \text{good} & 0 & \text{poor} & \text{worst} \\ 0 & 0 & \text{poor} & 0 & \text{bad} \\ 0 & 0 & 0 & \text{best} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \text{v.poor} & 0 \end{bmatrix} \end{array}.$$

We give yet another example of a fuzzy linguistic graph.

Let us have the vertices to be $V_1, V_2, \ldots, V_6$. The directed fuzzy linguistic graph is as follows:

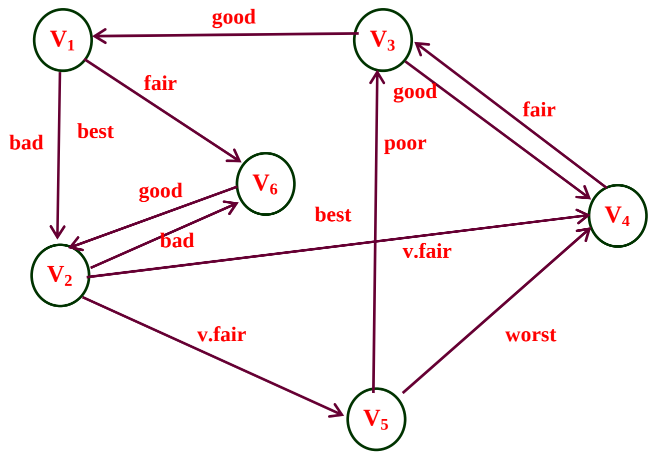

FIGURE 2.2.1.2

For this fuzzy linguistic graph we have the following fuzzy linguistic matrix N.

$$N = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{array} \begin{array}{c} \begin{array}{cccccc} V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \end{array} \\ \begin{bmatrix} 0 & \text{bad} & 0 & 0 & 0 & \text{fair} \\ 0 & 0 & 0 & \text{poor} & \text{v.fair} & \text{bad} \\ \text{good} & 0 & 0 & \text{good} & 0 & 0 \\ 0 & 0 & \text{fair} & 0 & 0 & 0 \\ \text{best} & 0 & \text{poor} & \text{worst} & 0 & 0 \\ 0 & \text{good} & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

We now proceed into define the new notion of Fuzzy Linguistic Cognitive Maps (FLCMs) model.

Let us consider $C_1, C_2, \ldots, C_n$ to be some n attributes / concepts associated with the problem in hand. Now instead of having the 'ON' or 'OFF' state of the concepts $C_1, \ldots, C_n$ as in FCMs we with these concepts associate linguistic states like; always, never, often, very often, not that often, much, very much, not that much, at times, etc., and denote it by L and L is assumed to have only finite number of elements in it, further L is assumed to be ordered and contains 0. So at one time the state C_i may be only one of these states; and '0' when it is none of these states. Clearly these states are not values between [0, 1] but they are linguistic states and not numbers; that is why we say the dynamical system which we are defining as the Fuzzy Linguistic Cognitive Maps (FLCMs) model.

Suppose at an instant the attribute / concept C_i is in the state 'often' and its impact on the another attribute C_j $(i \neq j)$ increases the effect 'often' in C_j (that is increase in C_i increases C_j then we map in the linguistic graph with $C_1, \ldots, C_n$ as nodes, the vertex C_i to C_j as 'positive often'. If decrease in C_i decreases C_j we mark as 'positive often'. If increase (decrease) in C_i decrease (increase) C_j we map as 'negative often'. However C_i cannot have impact on C_i so C_i to C_i is 0 also if C_i has no impact on C_j then also we put it as 0.

Now our state vectors will be $X = (a_1, a_2, \ldots, a_n)$ where $a_i \in$ {linguistic variables associated with the problem} $\cup \{0\} = L$; $o(L) < \infty$, $1 \leq i \leq n$.

Now using the fuzzy linguistic graph we get the fuzzy linguistic connection matrix.

Let G be some fuzzy linguistic graph with n vertices and M the corresponding fuzzy linguistic $n \times n$ matrix and if $x = (a_1, \ldots, a_n)$ be the state vector then we define M to be the fuzzy linguistic dynamical system; now if $M = (m_{ij})$; $m_{ij} \in L$, $1 \leq i, j \leq n$ then $x M = (b_1, \ldots, b_n)$ where $b_i \in L$; $1 \leq i \leq n$.

Let $xM = x_1$ after updating, that is the ON state of a vector which we started with must remain in the ON state till the end. This process is termed as upating. Now we find $x_1M = x_2$ (say) then we find x_2M we continue finding such vectors and finally we arrive at a fixed point or a limit cycle since L is a finite set.

This linguistic fixed point or the linguistic limit cycle is defined as the fuzzy linguistic hidden pattern. If $x_1 = (t_1, \ldots, t_n)$ is the fixed point (or limit cycle) we see the direct impact of $x = (a_1, \ldots, a_n)$ on the problem and we then interpret the corresponding result.

We will illustrate this first by an example.

Example 2.2.1.1: Suppose we are interested in studying the child labour problem. Let $(C_1, C_2, \ldots, C_6)$ be six attributes / concepts associated with it.

C_1 - Child labour
C_2 - Good teacher
C_3 - School drop out
C_4 - Poverty
C_5 - Public encouraging child labour
C_6 - Broken family.

These concepts need not be explained as they are self explanatory we take the following fuzzy linguistic variables as the state vectors.

Let L = {0, often, + often, –often, very much, much, not that much, little, very little, more etc} (Any other L can be got by the expert as per his / her need).

So the state vectors as well as the related fuzzy linguistic matrices take their values from the set L. Also the edges of the fuzzy linguistic graphs take their values from L.

Now using the experts opinion we have the following fuzzy linguistic graph.

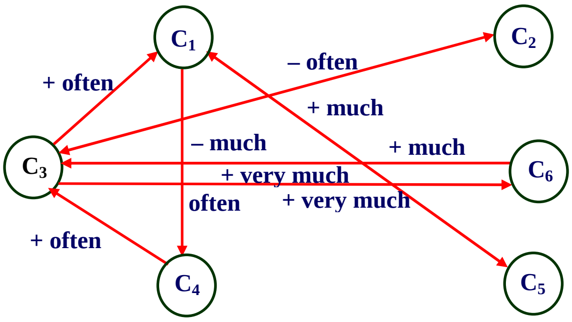

FIGURE 2.2.1.3

Now associated with this linguistic graph we have the following fuzzy linguistic matrix M; where M is the dynamical system associated with the Fuzzy Linguistic Cognitive Maps (FLCMs) that is with the problem of child labour.

$$
M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array}
\begin{array}{c}
\begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\
\left[\begin{array}{cccccc}
0 & 0 & 0 & +\text{often} & +\text{very much} & 0 \\
0 & 0 & -\text{often} & 0 & 0 & 0 \\
+\text{often} & -\text{much} & 0 & 0 & 0 & +\text{much} \\
0 & 0 & +\text{often} & 0 & 0 & 0 \\
+\text{very much} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & +\text{much} & 0 & 0 & 0
\end{array}\right]
\end{array}.
$$

Now our state vector takes values from L. The min and max operations on L are defined as follows.

Let

min $\{0, a_i\} = 0$ and max $\{0, a_i\} = a_i$ for all $a_i \in L$.

For $a_i \in L$, min $\{a_i, a_i\} = a_i$ and max $\{a_i, a_i\} = a_i$;

min $\{a_i, -a_i\} = -a_i$ and max $\{a_i, -a_i\} \in a_i$ for all $a_i \in L$.

For instance

min {often, very often} = often,

max {often, very often} = very often,

min {much, often} = often,

max {much, often} = much,

min {–much, often} = – much

and max {–much, often} = often.

Like this operations on L are performed.

Now suppose we want to study the effect of state vector "Child labour often occurs" under the condition 'often' $\in$ L and all other states are in the off state or zero state. To find the effect of x = (+ often, 0, 0, 0, 0, 0) on the dynamical system M. max min {x, M} = xM = (0, 0, 0, + often, + often, 0) after updating the state vector as the first concept was in the on state with '+ often' in the final result as well as in every step it should continue to remain in the '+ often' state. So let x_1 = (+ often, 0, 0, + often, + often, 0).

Now we find max (min $\{x_1, M\}$) = $x_1M = x_1 \times M$ using as before max $\{\min (a_i, m_{ij})\}$ where $x_1 = (a_1, a_2, \ldots, a_6)$ and $M = (m_{ij})$; $m_{ij}, a_i \in L$, $1 \leq i, j \leq 6$.

Thus max $\{\min (x_1, M)\}$ = x_1M = (+ often, 0, 0, + often, + often, 0) $\times$

$$\begin{bmatrix} 0 & 0 & 0 & +\text{ often} & +\text{ very much} & 0 \\ 0 & 0 & -\text{ often} & 0 & 0 & 0 \\ +\text{ often} & -\text{ much} & 0 & 0 & 0 & +\text{ much} \\ 0 & 0 & +\text{ often} & 0 & 0 & 0 \\ +\text{ very much} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +\text{ much} & 0 & 0 & 0 \end{bmatrix}$$

= (+ often, 0, + often, + often, + often, 0) leading to a fixed point.

Thus the hidden pattern clearly presents if 'often' child labour is in vogue it is 'often' due to poverty C_4 and C_5, public encouraging often child labour.

It is pertinent to mention here that + can be replaced by positive and – by negative. However an expert can choose without +ve or –ve with 0 as the least which is described in the following.

Let L be a fuzzy linguistic set associated with the n fuzzy linguistic attributes $C_1, C_2, \ldots, C_n$ of a problem P where each C_i takes every fuzzy linguistic term in L at one time or the other; for i = 1, 2, …, n.

Now we see C_i may at one time take a fuzzy linguistic term from L and we study its influence on other attributes. Further if C_i is influencing the attribute C_j then ($i \neq j$), we see the influence or association is again a fuzzy linguistic term from L. The state of C_i at any time can be 0 or any other fuzzy linguistic term from L.

Thus if $(a_1, a_2, \ldots, a_n) = X$ denotes the state fuzzy linguistic vector where a_i's are fuzzy linguistic terms and the attributes C_i enjoys at that time is a_i with $a_i \in L$; $1 \leq i \leq n$. Now the influence of each of the fuzzy concepts on each other $C_1, C_2, \ldots, C_n$ is denoted by the fuzzy linguistic graph similar to the directed graph of the fuzzy cognitive maps. These graphs of FLCMs take edge values from L and the edge values are not +1 or –1 or values from [0, 1], but they are basically linguistic terms from L.

Now associated with this fuzzy linguistic graph we obtain a $n \times n$ fuzzy linguistic matrix M whose entries are from L and the diagonal elements are zero.

Thus if

$$M = \begin{bmatrix} 0 & x_{12} & x_{13} & \cdots & x_{1n} \\ x_{21} & 0 & x_{23} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ x_{n1} & x_{n2} & x_{n3} & \cdots & 0 \end{bmatrix} \text{ where } x_{ij} \in L$$

(some of the x_{ij}'s can be zero).

M is defined as the fuzzy linguistic dynamical system associated with the fuzzy linguistic cognitive maps model.

If $X = (a_1, \ldots, a_n)$ where $a_i \in L$ then we can find min (max (X, M)) or max (min (X, M)) or min (min (X, M)) or max (max (X, M)) according to the needs of the expert and the nature of the problem at hand.

We find max (min (X_1, M)) (say) if X, is the value after updating that is whatever the attribute enjoyed the 'ON' state in the beginning or as the initial condition should continue to be in the 'ON' state till the end. Only the effect of other attributes which were in the 0 state are found, then we once again find max (min (X_1, M)) if X_2 is the value after updating all the fuzzy linguistic states except 0's; we again find max (min (X_2, M)) and so on until we arrive at a fixed fuzzy linguistic vector or a fuzzy linguistic vector as a limit cycle; this is always achieved as L is finite. We call this fuzzy linguistic fixed point or fuzzy linguistic limit cycle as a fuzzy linguistic hidden pattern of the fuzzy linguistic dynamical system M.

We describe the working of the Fuzzy Linguistic Cognitive Maps (FLCMs) model by the following example.

Example 2.2.1.2: Consider the problem of finding the eight transit system which includes the level of service and the convenience factors. We have the following eight attributes:

C_1 - Frequency of the service along a route
C_2 - In-vehicle travel time along the route
C_3 - Travel fare along the route
C_4 - Speed of the vehicles along the route
C_5 - Number of intermediate points in the route
C_6 - Waiting time
C_7 - Number of transfers in the route
C_8 - Congestion in the vehicle.

The fuzzy linguistic terms associated with C_1, C_2, …, C_8 as well as the problem are L = {0, often, always, a little, much, very much, usually, some times}. (It is pertinent to mention here more number of fuzzy linguistic terms or lesser number of fuzzy linguistic terms can be associated which is solely the wish of the expert).

We now give the fuzzy linguistic graph whose vertices are $C_1, C_2, \ldots, C_8$ and the edges take values from L are as follows.

Frequency of the vehicle along the route often depends on the in-vehicle travel time along the route; for if we have 'often' that is the frequency is more we see the in-vehicle travel time will also be less as the crowd will be less. Likewise the other attributes are all related in a similar way. The fuzzy linguistic graph given by an expert who is a person travelling in that route is given in the following:

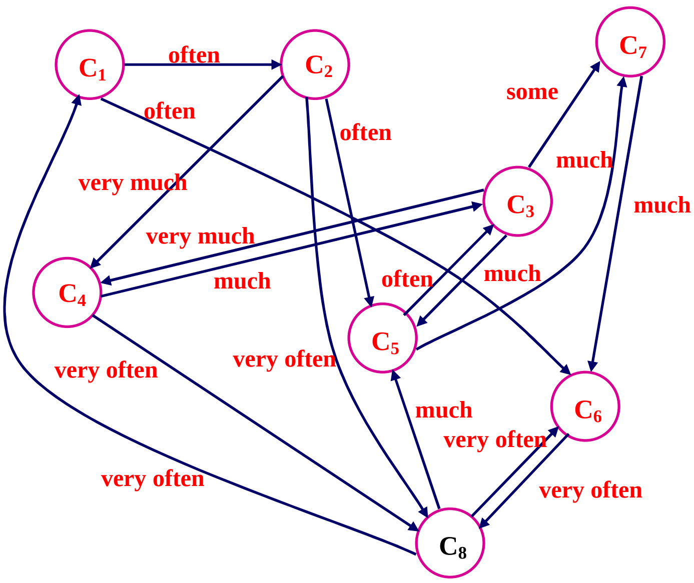

FIGURE 2.2.1.4

We now give the corresponding fuzzy linguistic matrix M of the fuzzy linguistic graph.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \end{array} \\ \left[\begin{array}{cccccccc} 0 & \text{often} & 0 & 0 & 0 & \text{often} & 0 & 0 \\ 0 & 0 & 0 & \text{very much} & \text{often} & 0 & 0 & \text{very often} \\ 0 & 0 & 0 & \text{very much} & \text{much} & 0 & \text{some} & 0 \\ 0 & 0 & \text{much} & 0 & 0 & 0 & 0 & \text{very often} \\ 0 & 0 & \text{often} & 0 & 0 & 0 & \text{much} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \text{very often} \\ 0 & 0 & 0 & 0 & 0 & \text{much} & 0 & 0 \\ \text{very often} & 0 & 0 & 0 & \text{much} & \text{very often} & 0 & 0 \end{array}\right] \end{array}.$$

M is defined as the fuzzy linguistic dynamical system associated with the problem.

Now we find the hidden pattern of any fuzzy linguistic state vector X on M, where X = (often, 0, some, 0, often, some, 0, often) is the fuzzy linguistic state vector supplied by an expert.

To study the effect of X on M using max {max {X, M}} after updating is (often, often, some, very much, often, some, much, often) = X_1.

Now max {max {X_1, M}} after updating is say X_2;

X_2 = (often, very much, some, very much, often, some, very much, often).

Now we find max {max (X_2, M)} after updating we get say X_3;

X_3 = (often, very much, some, very much, often, some, very much, often).

We see the fuzzy linguistic hidden pattern of the fuzzy linguistic state vector X on the fuzzy linguistic dynamical system M is a fixed point given by

X_3 = (often, very much, some, very much, often, some, very much, often) as X_2=X_3.

Thus if the expert takes the fuzzy linguistic states that frequency of the vehicles along the route is 'often', and the travel fare is 'some' i.e.; not much or very much and the number of intermediate points in the route is often (not very often or much or very much) and 'some' time is the waiting time with congestion in the vehicle is often we see; the fuzzy linguistic resultant is that the in-vehicle travel time along the route is very much, speed along the route is also very much and the number of transfers in that route is very much. Thus we have illustrated the model with max {max} operation.

Now suppose another expert wants to use for the same fuzzy linguistic state vector X and the same dynamical system, to study the effect using the 'max min' operation.

Now max {min (X, M)} after updating we get say X_1,

X_1 = (often, often, some, some, often, some, often, often).

Now we find max {min (X, M)} after updating we get say X_2;

X_2 = (often, often, some, some, often, some often, often) which is a fuzzy linguistic hidden pattern of the fuzzy linguistic state vector X under 'max min' operation.

Now when we use 'max min' operation on the fuzzy linguistic state vector X the effect of speed of the vehicle is 'some' that is the fuzzy linguistic position of C_4 remains at 'some' further the number of transfer in that route is only 'often'. Thus an expert may prefer to work with 'max min' operation on the fuzzy dynamical system instead of the 'max max' operation. This is only an illustration of how the fuzzy linguistic cognitive model functions.

We can use 'min min' or 'min max' operation also. It is at the liberty of the expert to use any of the operations to study the problem. However it is important to mention here that max max operation always yield very high results and min min the least or low results. So the experts if he/she wants to work with medium prediction can use max min or min max operations.

2.2.2 Fuzzy Linguistic Relational Maps (FLRMs) Model

Here we introduce yet another new fuzzy linguistic model known as the fuzzy linguistic relational maps model. This model is more a generalization of the fuzzy linguistic cognitive maps model that is a model built analogous to the fuzzy relational maps model. For with any problem P we have an association of the attributes or concepts which are related with fuzzy linguistic terms. If the attributes / concepts can be divided into two disjoint spaces say D and R where D is the fuzzy linguistic attributes known as the fuzzy linguistic domain space of the problem and R the fuzzy linguistic range space of the problem and let L be the fuzzy linguistic space which takes all the fuzzy linguistic terms associated both with R and D.

We see if $(D_1, D_2, \ldots, D_m)$ are the concepts in the fuzzy linguistic domain space D and $(R_1, R_2, \ldots, R_n)$ are the fuzzy linguistic concepts of the fuzzy linguistic range space R, then the fuzzy linguistic graph with $\{D_1, D_2, \ldots, D_m$ and $R_1, R_2, \ldots, R_n\}$ as the vertices and edges will be between D_i's and R_j's only $1 \leq i \leq m$, $1 \leq j \leq n$ and the fuzzy linguistic edges will take its fuzzy linguistic values from the fuzzy linguistic space L.

Associated with this fuzzy linguistic graph we will get the fuzzy linguistic matrix. The related fuzzy linguistic matrix will be known as the fuzzy linguistic dynamical system associated with the fuzzy linguistic relational maps model.

We will have a pair of fuzzy linguistic state vectors one associated with the fuzzy linguistic domain space D and the other associated with the fuzzy linguistic range space R. As in case of fuzzy linguistic cognitive maps models we can get the fuzzy linguistic hidden pattern which will be a pair of fuzzy linguistic state vectors associated with one fuzzy linguistic state vector from D or R. As in case of fuzzy linguistic cognitive models the fuzzy linguistic hidden pattern can be a linguistic fixed point or a linguistic limit cycle which will be a pair of vectors.

We can use any of the four types of operations {max, max} or {min, max} or {max, min} or {min, min} to find the fuzzy linguistic hidden pattern.

We will illustrate this situation by an example.

Example 2.2.2.1: Let us study the employee - employer relation using the fuzzy linguistic relational maps model. Suppose we have the following fuzzy linguistic concepts / attributes associated with the employee taken as the domain space.

D_1 - Pay with allowances and bonus
D_2 - Only pay to employee
D_3 - Pay with allowance to employee
D_4 - Best performance by the employee
D_5 - Average performance by the employee
D_6 - Employee works for more number for hours
D_7 - Employee works for less number of hours.

Suppose the following nodes / concepts are taken as the range space of the employer.

R_1 - Maximum profit to the employer
R_2 - Only profit to the employer
R_3 - Neither profit nor less to the employer
R_4 - Loss to the employer
R_5 - Heavy loss to the employer.

The fuzzy linguistic terms associated with the fuzzy linguistic domain and range spaces be taken as L.

L = {0, gain, loss, no loss no gain, just gain, just loss, gain, heavy loss, good gain}.

Any expert can suggest / take any other fuzzy linguistic terms as L. It is solely left for the expert to choose whatever he / she wishes to work with.

We give the associated fuzzy linguistic graph of the problem using both the fuzzy linguistic domain space and range space.

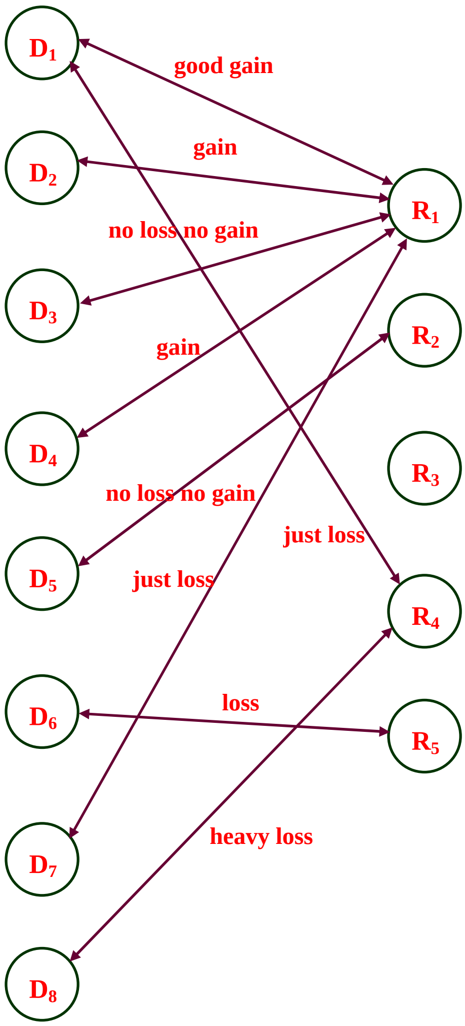

FIGURE 2.2.2.1

We give the associated fuzzy linguistic matrix N of the fuzzy linguistic graph.

$$N = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \end{array} \begin{array}{c} \begin{array}{ccccc} R_1 & R_2 & R_3 & R_4 & R_5 \end{array} \\ \begin{bmatrix} \text{good gain} & 0 & 0 & \text{just loss} & 0 \\ \text{gain} & 0 & 0 & 0 & 0 \\ \text{no loss no gain} & 0 & 0 & 0 & 0 \\ \text{gain} & 0 & 0 & 0 & 0 \\ 0 & \text{no loss no gain} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \text{loss} \\ \text{just loss} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \text{heavy loss} & 0 \end{bmatrix} \end{array}.$$

Now we find the resultant of any fuzzy linguistic vector on N, the dynamical system associated with the problem.

Let X = (gain, 0, loss, 0, gain, loss, 0, 0) be the given fuzzy linguistic state vector. The effect of X on the fuzzy linguistic dynamical system N is as follows:

Max {max (X, N)} = ((good gain, gain, gain, gain, gain)} = Y.

We find max (max $\{Y, N^T\}$) = (gain, gain, loss, gain, gain, loss, gain, gain) = X_1 (say) after updating.

We find max (max $\{X_1, N\}$) = (good gain, gain, gain, gain, gain) = Y_1 = Y.

Thus the fuzzy linguistic hidden pattern is a fixed pair given by {(gain, gain, loss, gain, gain, loss, gain, gain), (good gain, gain, gain, gain, gain)} and the employer always gets gain when the attribute D_1 enjoys the fuzzy linguistic state 'gain' with 'loss' for the attribute D_3; 'gain' by average performance by the employee - D_4 and the fuzzy linguistic term loss if the performance of the employee is poor. For we have used max max operation that is why the resultant is also high.

Under these conditions if the employer gives pay with allowance and bonus to the worker the employer gets 'gain'.

This is only an illustrative example and interested researchers can apply these models to real world problems. For more about these concepts refer [101].