CHAPTER - 0
INTRODUCTION

Topology plays an important part in many branches of modern mathematics. Topology is a collection of subsets of any set which satisfies certain conditions. In 1996 the mathematics Professor of Japan Professor Maki took an effort leaving off few conditions from the conditions which are required for topology and minimal spaces were brought to the knowledge of the modern topologists. The objectives of this Ph.D. work is four fold.

1. Working on minimal spaces, biminimal spaces and Ideal minimal spaces,

2. Obtaining new results,

3. Sharpening of many known results and,

4. Exploring generalization of certain known results.

In the First chapter …

After advent of the concepts of closure as well as interior of a set many generalized open and closed sets were introduced in topological spaces in the recent past. Particularly $\alpha$-open sets [55], semi-open sets [37] and preopen sets [47] were introduced and furthermore the related functions for the above mentioned open sets were introduced by many mathematicians over the years. Especially $\alpha$-continuous, $\alpha$-irresolute functions and $\alpha$-quotient functions [36] were introduced and studied.
Looking upon the above topological notions this research study is carried out applying the concepts of minimal spaces and its generalization. In this work sets and functions defined in topological spaces are studied and investigated on the minimal spaces [45, 66].

**In the Second chapter …**

In 1970 Professor Levine [38] introduced the most useful notion of generalized closed sets for modern mathematicians. Consequently many modifications of the so called g-closed sets were defined and investigated and they are applied to introduce several low separation axioms. Quite recently Noiri [58, 56] investigated g-closed sets and introduced mg-closed sets and locally m-closed sets under minimal conditions. Since the work has been attractive and impressive the same attempt is taken up in this research study in order to unify certain kind of modification of \( \hat{g} \)-closed sets which was due to Veerakumar. The notion of \( *(m, n) \)-normal spaces is introduced and the characterizations of such notion are obtained as we expected.

**In the Third chapter …**

The idea of decompositions [39, 83, 84] in topological spaces is considered as an important tool in the field of general topology. The contribution of Professor Tong is still remembered in the minds of every researchers doing research. Similar attempt of Professor Tong has been taken up by various Professors at different point of time. Taking care of the contributions of modern topologists a unique work is done in this research study in line with the decomposition of continuity in topological spaces under minimal conditions.
In the Fourth chapter …

In 1961 Levine [39] obtained a decomposition of continuity. Later Professor Rose improved Levine’s decomposition. In 1986 Tong [83] obtained a decomposition of continuity and proved that his decomposition is independent of Levine’s. In 1989, Tong [84] improved upon his earlier decomposition and obtained yet another decomposition of continuity. In 1990, Ganster and Reilly [20] obtained a decomposition of continuity improving the first result of Tong. A year later, Ganster and Reilly [21] improved Tong’s second decomposition. When the development of decomposition in topological spaces have been going on, the idea is tried to apply to the minimal spaces and new decompositions of continuity in minimal spaces are being obtained. This research study is one among such type of works.

A new class of sets and a new type of maps are introduced in this chapter under minimal conditions and some decompositions of generalized continuity in minimal spaces are obtained. Examples are given at vantage point of this chapter.

In the Fifth chapter …

Locally closed sets [19] and its corresponding functions were introduced by Ganster and Reilly. This new idea was an instrumental and this temptation leads to so many contributions of modern mathematicians. This idea was helpful to this research study in order to introduce a new class of sets in accordance with the notion of locally closed sets. So in this chapter stronger forms of locally m-closed sets [56, 58] are
introduced and studied. This study investigates some properties and characterizations of these sets with some theorems, examples and counter examples.

**In the Sixth chapter …**

In this chapter a combined study is taken up. The minimal spaces introduced by Maki were developed by Popa and Noiri in the recent past. In general topology the concepts of ideal are indeed very important tools. The works on ideals done by Kuratowski [35], Vaidyanathaswamy [85], Jankovic and Hamlett [33] are considered as a motivated one in applying topological ideals to generalize the most basic properties in the General Topology. Combining minimal spaces and ideal, ideal minimal spaces have been introduced by two mathematics Professors belonging to Turkey. This idea is helpful to this research study to obtain an amount of works which is required for this chapter. Hence the so called g-closed sets is studied under the condition of minimal spaces and ideal topological spaces. Many nice results are obtained at every section of this chapter.

**In the Seventh chapter …**

Submaximality is very useful concept in classical general topological spaces which was introduced by Professor Hewitt [31] in 1943. Lot of works in topological spaces has been done using this concept of submaximality since then 1943. Recently Professor E. Ekici [16] has applied this idea in ideal topological spaces and the systematic study of submaximal spaces was taken up by Ekici after two Professors [63]. So
using the concept of submaximality in ideal minimal spaces, a new space called m-I-submaximal ideal minimal spaces is introduced and studied in this chapter. Several characterizations and properties of m-I-submaximal ideal minimal spaces are obtained. Properties I and U are also introduced to get over some difficulties on the path of research. At every places the notions have been substantiated with suitable examples.

**In the Eigth chapter…**

Study of ideal topological spaces, minimal spaces and ideal minimal spaces were introduced by Kuratowski, Maki and Ozbakir, respectively on some point of time in the Scenerio of mathematical world. Such notions were developed by Jankovic and Hamlett, Popa and Noiri and Ozbakir and Yildirim, respectively. In this circumstances, few results proved in ideal topological spaces are taken up in this research study and those results were applied to the ideal minimal spaces. This attempt brought many nice and useful results in every sections of this chapter. A comparative study for the subsets defined in this chapter is made and a diagram of implications is obtained at important sections of this chapter.