CONCLUSION

In many production systems, setup operations take several days and are very costly. One way to reduce the setup cost per unit time is to delay the production until some number of raw materials accumulate and this is the well known N policy in a queueing context. The N policy results in a longer cycle length which means fewer number of cycles per unit time. But at the same time work in process inventory (WIP) level becomes larger. Thus in real production setting, the N-policy is used to reduce the overall cost per unit time when the setup cost is extremely high compared to the WIP holding cost.

The optimal control of queues under N policy received a great deal of attention following its launch by Yadin and Naor (1963). The researchers working in this area have looked at the problems at different angles, depending on the application. This has given rise to vast and rich literature. Several authors have investigated the N policy in such queueing systems, incorporating diverse characteristics such as batch arrival, setup operations, server breakdowns and server vacations etc.

In Chapters II to V, the author has analysed single server batch arrival queueing systems with double threshold policy, early setup operation and vacations in which the server is typically subject to unpredictable breakdowns while serving the customers. The server provides two stages of heterogeneous service. All the customers undergo the First Essential Service (FES) and as soon as the FES of a customer is completed, the customer either leaves the system with probability \((1 - r)\) or opts the Second Optional Service (SOS) with probability \(r\). The SOS rule combined with \((m, N)\) policy is studied under single and multiple vacations in Chapters II and III respectively. It is assumed in Chapter II that the customer just being served while the server breaks down, waits in the service facility (FES or SOS) and completes the remaining service as soon as the server returns from the repair facility. On the other hand in Chapter III, it is assumed that, if the server fails during the FES of a customer, then the customer may either stay in the service facility
with probability \((1 - q)\) to complete the remaining service or join the head of the queue and receives a new FES with probability \(q\).

The server providing multi-optional \(c\)-types of heterogeneous service in second phase is considered in Chapter IV, i.e., each customer after completing the FES may choose the \(i\)th type of service with probability \(r_i (1 \leq i \leq c)\) or may leave the system with probability \(1 - \sum_{i=1}^{c} r_i\). Also, the server instead of taking vacations at the end of each busy period, offers Bernoulli type single vacation of random length after completing each service of a customer. Chapter V deals with the case that, the server provides SOS consisting of only one type of service but provided with \(M\) different types of vacation following different distributions and the server after completing each service to a customer may choose any one of the vacations or remain in the system according to the requirements.

The models discussed in Chapters II to V are among the most general queueing systems with threshold policies and include many previous works as special cases. The models are successfully described as a Markov process by using supplementary variable technique and the key PGFs of the steady state system size equations are obtained in a closed form. These PGFs give rise to interesting performance measures under the stability condition obtained.

Moreover, system size distributions at departure epochs are obtained. The existence of stochastic decomposition property of the queue size distribution at arbitrary epoch is also demonstrated. Finally, the total expected cost function per unit time is defined and a solution procedure is developed to determine the joint optimal threshold values \(m^*\) and \(N^*\) and the minimum total expected cost per unit time \(T_C(m^*, N^*)\). Numerical examples are presented to illustrate how critically the performance measures are affected by traffic intensity and other parameters. Apart from the usual intuitive conclusions namely, the mean system size increases with arrival rate, mean vacation time, mean setup time, mean repair time, breakdown rates, Bernoulli vacation
probabilities and the probabilities with which the customers choose the SOS, the following interesting results are also discussed.

It is shown that, the optimal thresholds \((m^*, N^*)\) increase as the setup cost \((C_s)\) or holding cost \((C_h)\) decreases, the total cost \(T_c\) \((m^*, N^*)\) per unit time increases as \(C_s\) or \(C_h\) or the mean vacation time increases. It is also noted that, the mean setup time has larger effect on the thresholds than the mean vacation time and the batch size significantly affects the optimal threshold values. Through numerical examples it is verified that \((m, N)\)-policy provides lower average cost than the usual \((N, N)\)-policy.

Chapters VI and VII deal with the vacation queueing models where the server serves the customers at a lower service rate instead of completely stopping the service during vacation period. The survey of the results of working vacation (WV) queues (Tian et al., 2009) shows that till 2009 only single arrival and single service \((M/M/1, M/G/1\) and \(G/M/1)\) WV queueing models are analysed by various authors using matrix analytic approach. Recently, Markovian bulk arrival \((M^X/M/1)\) and batch service \((M/M(a, b)/1)\) working vacation queueing models are studied (Julia Rose Mary and Afthab Begum, 2009, 2010). As an extension of these works, the author considered batch arrival non Markovian working vacation queueing model \(M^X/G/1\) in Chapter VI and obtained the PGF of the system size at arbitrary epochs and departure epochs using supplementary variable technique. And in Chapter VII, a non Markovian bulk service queueing model \(GI/M(a, b)/1\) is investigated using embedded Markov chain technique and the steady state queue size probabilities at prearrival epochs as well as arbitrary epochs are obtained. Many existing queueing systems are proved as special cases of the models of Chapters VI and VII. The expected queue length is calculated numerically by considering deterministic, exponential and Erlang-k type distributions for service times (Chapter VI) and interarrival times (Chapter VII) to illustrate that the queue length is a decreasing function of the vacation parameter \(\eta\) and an increasing function of the arrival rate \(\lambda\). It has
been shown that the effect of $\eta$ on mean queue length becomes less as the vacation service rate ($\mu_v$) approaches to the regular service rate ($\mu_b$).

The study of retrial queues is an important field, since in various scenarios, they are able to capture certain behaviour of real systems more accurate than classical FCFS queues. The literature shows that, retrial queueing systems with general service times and non exponential retrial time distribution have received only little attention. Due to the complexity of retrial queues, publications on performance measures in a closed form are quite rare. Thus in Chapter VIII, the author investigated a more general repairable $M^X/\text{G}/1$ retrial queue with general retrial time, Bernoulli vacation policy, setup time and second multi-optional services allowing reneging of customers in the retrial queue which generalizes previous studies.

The following are some of the possible extensions suggested for future research.

- The waiting time distribution and busy period distribution may be obtained for all the models.
- By suitably defining the control parameters of Bernoulli schedule vacation, the models of Chapters II to IV can be combined into a single model. The results presented in these chapters will help to achieve this.
- The bilevel control policies may be investigated for finite and multi-server queues and for the systems in which interarrival times follow non Markovian distribution.
- The control policy of multi server infinite capacity queueing models may be analysed.
- The retrial queueing models may be investigated under working vacation policy.