The ultimate aims of this paper are to investigate the response of a prestressed concrete pavement slab to an applied load, and to determine the manner by which the slab fails.

Two of the steps leading to the achievement of these aims are to study the response to load of solid materials in general, and to establish criteria for failure.

PART 2

THE RESPONSE TO LOAD, AND THE FAILURE CRITERIA, OF SOLID MATERIALS
2. THE RESPONSE TO LOAD, AND THE FAILURE CRITERIA, OF SOLID MATERIALS.

2.1 Fundamentals Of The Structure Of Materials

i) The fundamental building blocks of matter are possibly the electron, the proton and the neutron. ('Possibly', because the number of fundamental particles that are being discovered is still increasing; and it is as yet not certain as to the part that all of them play in the scheme of things.) The number of electrons, protons and neutrons that are to be found in an atom of a given element, and the manner in which the electrons position themselves in the various shells around the proton-neutron nucleus, constitute the structure of the atom, and are defined by the four quantum numbers. The properties of the atom depend on the structure. The particles are held in equilibrium by the electrostatic forces which develop between themselves.

ii) A real material consists of an aggregation of atoms, molecules, groups of atoms and molecules, crystals, etc. At each stage the fundamental atoms or the aggregations cohere to each other by means of one or more of the three primary bonds, or by a weaker secondary
bond such as the van der Waals bond. The patterns which the particles assume within the material constitute its structure.

iii) The nature of the basic atom or molecule of a substance is an invariant. The state of aggregation of the molecules, however, varies according to given conditions, and depends on the nature and strength of the intermolecular bond, and on the energy levels of the molecules at any given time.

iv) Real materials can be characterised at three levels:

a) the atomic or molecular level, wherein the particles are discrete, and form a discontinuous, nonhomogeneous body. The behaviour is described by the velocity, position and interaction forces between particles.

b) the structural level, wherein the elements of the material have finite dimensions and are macroscopically or microscopically observable, and wherein the body is considered continuous, but non-homogeneous.

c) the engineering level, wherein the material is considered continuous and homogeneous, formed from identical elements of finite size. The
behaviour is described in terms of stress, strain and their volume element derivatives.

v) A solid offers resistance to external tensile and compressive forces; therefore internal forces of attraction and repulsion are capable of being generated within the material.

Any two particles in a material are kept at their equilibrium distance apart such that the electrostatic forces of attraction and repulsion between the particles, and the externally applied force (if any), are kept in balance. That is, the resultant force between the two particles must be zero. Fig 5 is a diagram which relates interparticle force to particle spacing. When no external loads are applied, the distance between the particles is \( r_0 \), at which point the electrostatic attractive force just equals the electrostatic repulsive force, and the resultant interparticle force \( F(r_0) = 0 \). If an external load \( P \) is applied, the new equilibrium spacing is represented by \( r \), and now \( F(r) = P \). \( F_{\text{max}} \) corresponds to the 'yield' point between the particles, while the slope of the line \( ab \), which is tangential to \( F(r) \) at \( a \), represents the instantaneous modulus of elasticity of the material.
The potential energy created in the system by moving the two particles from their equilibrium positions is indicated by the corresponding area under the $F(r)$ curve in Fig 5.

In systems of fixed energy, the equilibrium configuration of particles is one of the maximum entropy. On the other hand, in systems of fixed entropy, the equilibrium configuration is one of minimum energy. Therefore, the equilibrium interparticle distance, when $r = r_0$ and $F(r) = 0$, corresponds to a position of minimum potential energy in the system.

2.2. The Materials Concepts Required For The Engineer

As far as the civil engineer is concerned, the conceptual knowledge that he must have about the structure of materials can be summarised as follows:

1) The 'building blocks' of all real materials are:

   atomic particles
   atoms
   aggregations of atoms
   molecules
   aggregations of molecules
crystals
fibres
aggregates
chunks
blocks.

(The generic term 'aggregations' is sometimes used in this text, in the sense of 'constituent particles or building blocks'.

The term 'particles' is also used in the same sense, unless specifically indicated to mean the fundamental particles which constitute the atoms.)

ii) At each stage some type of cohesive or bonding force binds the constituents together. Furthermore, at each stage some type of cohesive or bonding force binds a particular group of aggregations to similar groups, and to groups of a larger aggregation. Thus there are inter-particle forces which bind the atomic particles together to form atoms. The atoms themselves bond together by means of interatomic forces, to form molecules. So on through intermolecular forces, intercrystalline forces, interaggregate forces, etc. With the larger aggregations, mechanical friction forces also become of consequence. These friction forces, however, have to be mobilised by some means such as by gravity, by the confining effects of a container, by the hydrostatic compression effect of a binding matrix, etc.)
2.3 Observations And Deductions Pertaining To Interparticle Bonding Forces

1) It is to be observed that mechanical properties of the material depend solely on the type, strength and orientation of each of the various types of bonding forces that are exercised within the material, and on the quantum of that type of bonding force which is of significance at the particular point under consideration.

2) From the relationship between interparticle force and particle spacing, as illustrated in Fig 5, three important points are deduced:

   a) very much more force is required to push the particles together from the equilibrium position \( r=r_0 \) than is required to pull them apart.

   b) the force/spacing relationship is NOT linear.

   c) the tangent modulus of elasticity in tension and compression has the same value only at zero load. Thereafter the moduli have different values, not only in tension and in compression, but also from load to load.
2.4 Fundamentals Of The Mechanical Properties Of Solid Materials

Mechanical Property

2.4.1

1) The term 'property' in respect of some given cause, is that which relates cause to effect. The Hookean spring provides the simplest illustration. Thus, if a steel coiled spring extends linearly by a distance D under a load L, an equation may be set up connecting this particular cause (load) with the resulting effect (extension), to give

\[ D = C \times L. \]

The constant C relates cause to effect, and is the relevant property of the spring. In this instance, it constitutes a mechanical property.

ii) It is to be observed that, due to the circumstances of the method of test and of the geometry of the spring, C is the property of the spring, and not of the material (steel) of which the spring is manufactured.

iii) a) Real materials may be classified, inter alia, from the point of view of their mechanical properties. One such classification is based on the
relation that exists between an applied stress and the resulting strain, or between the stress and the rate of strain, depending on which is applicable.

b) Thus, a material may be:
   linearly elastic (Hookean substance)
   non-linearly elastic (Mooney substance)
   viscous (Newtonian substance)
   elasto-plastic (St. Venant substance)
   firmo-viscous (Kelvin, or Voigt, substance).
   elasto-viscous (Maxwell substance)
   visco-plastic (Bingham substance)
   etc.

Real materials do not, in general, possess characteristics which correspond to only one of the classifications given above, but exhibit properties which are common to more than one of the categories. The classical bodies, however, provide a convenient means whereby the behaviour of real materials can be compared and predicted.
Elasticity, Plasticity, Fracture, Plastic Flow

3.4.3

The fundamentals of elasticity and plasticity of materials may be summed up thus:

i) Most of the knowledge on materials pertaining to the properties of elasticity and plasticity has been obtained by what have been termed 'uniaxial load tests' on bars, cubes, prisms and cylinders. The manner in which the results are usually quoted implies that they refer to uniaxial states of stress or strain in the test specimen.

ii) When a material in the shape of, say, a bar, is tensed under a uniaxial load, the behaviour of the particles is as illustrated in Fig 6.1. The particles draw further apart in the direction of load, and come closer together in the direction transverse to the load (due to the action of the interparticle bonding forces), thus giving rise to Poisson's effect. The ratio of applied external stress to longitudinal strain is the conventional modulus of elasticity (E). E is thus a measure of the stiffness of the material, but not of its failure strength. The ratio of lateral strain to longitudinal strain is, of course, Poisson's ratio (\( \nu \)).
iii) Much the same effects are noticed if the bar is loaded in uniaxial compression, except for an obvious reversal of strains. The particles come closer together in the longitudinal direction and move apart in the transverse direction.

iv) In compression, however, the test results vary depending on the slenderness ratio of the specimen (i.e., on the ratio of the height to the lateral dimension), on the load contact area, etc. The specimen sometimes behaves as if it were compressing, and sometimes as if it were bending and buckling. A 6-in cube of concrete has a higher unit-stress strength in compression than an 8-in cube, and a lower strength than a 4-in cube. A cylinder of concrete exhibits a lower crushing strength than a cube of the corresponding dimension. Yet it would be expected that, since the material is the same in all these cases, the unit-stress strength should be an invariant, which according to the results of the tests, it is not.

v) It has been stated that a given material has the same stress-strain curve and the same modulus of elasticity, in tension and in compression, especially if it is ductile. On the other hand, it has also been reported that a material has different stress-strain curves, and different moduli of elasticity, in tension and in compression, especially if it is brittle.
vi) Under pure hydrostatic compression, there is a decrease in volume. No ductility is apparent. No shear stress can develop. The ratio of the change in volume to the original volume is quoted as the bulk modulus \( K \) of the material.

vii) Under an applied shear stress, the material becomes distorted (as compared to the change in volume which occurs under normal stress). The angle through which distortion occurs is a measure of the rigidity \( G \) of the material.

viii) In visco-elastic media (as in fluids) the rate of deformation is of significance. Strain measurements are therefore replaced by measurements of rate of strain; and in place of the modus of rigidity, \( G \), the coefficient of viscosity is quoted.

ix) When certain materials are progressively loaded in uniaxial tension, a stage is reached when the stress becomes large enough to overcome the interparticle forces of attraction, and separation occurs. The material is then said to have exhibited brittle fracture in tension.

x) Much the same thing happens under shear forces, except that now a portion of the material slides over the adjoining portion without actually separating from it. (Fig 7.1, 7.2) The material is then said to
have yielded, or to have manifested plastic flow. Such materials are classed as ductile.

xi) Brittle Fracture strength and ductile yield point are obviously related to tensile strength and shear strength respectively. In addition to these two type of failure which are recognised as being possible in materials, most works also quote what is termed the 'compressive strength' of materials, thereby implying that just as materials can fail in tension or in shear, they can also fail in compression. It will be shown further on that this concept is invalid, and that compression, as such, cannot cause failure.

xii) In a monocrystalline body, it is possible to calculate the resistance that could theoretically be offered to brittle failure occurring under a tensile stress, or to plastic yielding occurring under shear. The calculation is based on the values of the electrostatic, interparticle forces of attraction, and on the structure (that is, the relative positions of the particles) of the crystal.
Such calculations have indicated that many of the common engineering materials should have strengths of the order of \(10^6\) lbs/in\(^2\), with corresponding high strains at failure.

In actual fact, of course, the materials have very much lower strengths than this.

The reason is that no real crystal is ever perfect. The defects that may be found in a crystal include vacancies, interstitials, edge dislocations, screw dislocations, impurities, mosaics, excess electrons, excitons, Schottk’s defect, and Frankel’s defect. As a result of these defects, the strength of the single crystal is reduced in the order of a hundred or a thousand times.

Real materials, however, are usually polycrystalline in structure. The numerous crystals which constitute the material take up a variety of positions in the body, along a number of different alignments. When plastic flow starts in any given crystal, therefore, it does not proceed calamitously, but soon becomes checked by some other crystal which lies along the path of flow, the second crystal itself having a different orientation to the first.
Obstacles to continuous plastic flow are also provided by the piling up of dislocations at grain boundaries, by impurities, by stress fields, by twinning, by the complex deformation modes of other grains, etc. As a result of these obstacles to plastic flow, the strength of a polycrystalline material becomes higher than the strength of the defective monocrystal, but still remains much lower than the theoretical strength of the non-defective crystal.

This increase in the strength of ductile materials under load results in the familiar work-hardening, or strain-hardening. Its effect is to raise the yield point, for if the material is now unloaded, and then reloaded, the new yield point is seen to occur at the point at which the material was unloaded.

xiii) As the aggregations become smaller in size, the modes of deformation of the grains and the obstacles in the path of failure become more complex. The strength of the material as a whole is therefore increased, so long as the particles do not become so small that the failure paths can go round them and so miss them completely.
xiv) Depending on the structure of the material, there are certain planes (glide planes), and certain directions (glide directions) along these planes, which are most susceptible to slip. Whether or not slip actually occurs along these planes, or along less favourable glide planes, is contingent on the level and the orientation of the shear component of the stress at that point.

xv) Brittle failure, in those materials which are susceptible to it, usually commences in a crack or flaw in the material. The larger the specimen, the more the chances of it containing flaws, and therefore the lower its strength. Crystals of a thickness of only 10 cms. have shown elastic strains, under load, of up to 10% without fracturing (21); while glass 'whiskers' have been made of such small thicknesses, and the flaws have consequently been so minimised, that the glass failed only at a load of about 500,000 lbs/in$^2$ (82). On the other hand, the mere drawing of a feather across the glass 'whisker' scratched it enough to cause it to fail at a much lower load.
xvi) Due to the preponderant effect that flaws have on the strength of brittle materials, and because flaws cannot be prevented from occurring haphazardly, the brittle strength of the same material can show a wide variation.

xvii) Strictly speaking, no material is homogeneous, and the mechanical properties of a real material are therefore not isotropic. In practice, however, and at the macroscopic, engineering level, the aggregations which constitute a material are usually so thoroughly randomly oriented (except for some materials whose grains display directional orientation) that the material can be considered isotropic for all practical purposes. It shows no preferred slip directions. The 36 elastic constants, which arise from the fact that the six components of stress at a point are each taken as being linearly proportional to each of the six components of strain, are therefore reduced to just two. These may be any two of the inter-related $E$, $G$ and $U$.

xviii) Under certain circumstances, a material which ordinarily shows ductile yielding may become
embrittled, and fail by brittle fracture. The ordinarily lower yield point becomes raised, and as a result the brittle fracture strength now becomes operative before yield can occur. The circumstances which embrittle a material are: the occurrence of notches, grooves, holes, fillets, keyways, screwthreads, and other such stress raisers or stress concentrations, which increase the triaxiality of the stress at the concerned point by means of elastic or plastic restraint; low temperatures, which inhibit plastic flow; and increased rates of loading (eg; impact loads), which possibly act by allowing the dislocations little or no time to move.

Discussion On Contemporary Failure Theories

2.4.3

The more important theories on the failure of materials are now briefly described and examined.

1. Maximum Principal Stress Theory

a) This theory considers that the maximum or the minimum principal stress constitutes the criterion for failure. If the material is ductile, the yield is supposed to commence when the yield point, as obtained from a simple, uniaxial tension test, is reached by the maximum principal stress, or when the minimum principal
stress reaches the yield point value as obtained from a simple, compression test.

b) i) The theory disregards the effects of the triaxiality of stress.

ii) Again, in simple tension, sliding occurs along planes inclined at 45° to the applied load, along which planes neither the tensile stress nor the compressive stress is a maximum.

iii) Furthermore, even though this theory predicts that a material is weak in compression, it has been observed that such materials have in fact shown a very high strength when subjected to hydrostatic compression.

.2 Maximum Principal Strain Theory

a) This concept assumes that a ductile material begins to yield either when the maximum strain (elongation) equals the yield point strain in simple tension, or when the minimum strain (shortening) equals the yield point strain in simple compression.
b) i) The theory breaks down while considering a plate subjected to equal tensions in two perpendicular directions. The theory predicts that the plate should sustain a greater stress now than it would sustain in single-way tension. This does not occur in reality.

ii) The theory also indicates a much lower strength under hydrostatic stress than actually occurs.

3 Maximum Shear Stress Theory

a) The theory, as conceived by Coulomb, postulates that the material will yield when the maximum shear stress equals the maximum shear stress at yield as obtained from a simple tensile test.

b) i) The theory is more suited to ductile materials.

ii) It implies that the strengths as obtained from simple tension and compression tests are the same, when, in fact, they are not so.

c) Navier modified the theory, but his concept implies that the compressive strength of the material is always greater than the tensile strength.
d) Tresca theorized that yield occurred when the magnitude of the maximum shear stress had a value equal to half the yield stress in simple tension. This theory therefore also implies that a material possesses equal yield strength in compression and in tension. This is not true in practice.

e) Nadai used the concept of the octahedral shear stress at yield being equal to \( \frac{2}{3} \) yield stress in simple tension, or equal to \( \frac{2}{3} \) yield stress in pure shear. This criterion, however, does not apply to the fracture of brittle materials.

4 Maximum Strain Energy Theory

a) This theory utilizes the value of the strain energy per unit volume as the basis for determining failure. Beltrami and Higa\(^{34}\) developed the concept; but it fails in that it predicts yielding under hydrostatic compression, which in reality cannot occur. The theory disregards the difference in the behavior of the material in tension and in compression.

5 Maximum Distorsional Strain Energy

Von Mises plastic failure theory, which was based on the concept that the invariant of stress
deviation gave a constant for the material which in uniaxial tension was the yield stress) was developed into the distortional strain energy theory by Huber & Hencky. This theory admits of no strain hardening, which, in practice, actually occurs.

6.6 Griffith's Hypothesis

Griffith's crack theory pertains mainly to brittle materials. It considers that imperfections on the surface of the specimen act as stress concentrations, so that at the tip of the crack the stress is above the fracture stress, even though the average stress is low. The crack extends, and the stress becomes concentrated at the new tip. Fracture continues, and grows indefinitely. In ductile materials, plastic flow occurs just ahead of the crack. This serves to dissipate the energy, and prevents further growth of the crack.
Mohr's Theory

a) This theory stipulates that, at failure, the shear stress is a function of the principal stress.

b) The theory gives good results for brittle materials, but it implies that fracture will always take place along planes passing through the direction of the intermediate principal stress, which is not consistent with actual test results.

2.5 Observations And Deductions Pertaining To Elasticity, Plasticity and Failure: The Critical Strain Criteria For Failure

The Evaluation Of A 'Property': 'Strength'

2.5.1

1. Property

a) The tests that are commonly used by engineers for ascertaining the property of materials
do not evaluate a fundamental property of the
material so much as they evaluate a property of the
specimen which is being tested.

b) Thus, to use the simple example of
the Hookean coiled spring: $C$ in the relation $D = CxL$
(see subsection 2.4.1) is a property of the
spring, and not of the steel. The property $C$ is
an amalgam of two parts: one part relates to the
geometry of the spring, and depends on the diameter of
the wire, on the diameter of the coil, on the length
of the coil, and on the pitch of the turns; the
second part relates to the material, and depends on
its elastic properties in shear and in tension.
The overall property $C$, as obtained, does not
indicate the separate identities and values of the
two constituent parts.

42 Strength

a) The term 'strength' of a material'
must be used with care. It has already been shown
that the 'strength' of a monocrystal of a given
material is different from the 'strength' of the
polycrystalline body, which itself is different
from the 'strength' of a shaped structural member, the substance being the same in all cases.

Furthermore, consider the crushing strength test that is carried out on concrete. Other things being equal a wide range of crushing strengths for the same concrete is obtained, depending on whether cubes, prisms, or cylinders are being tested: depending on the contact surface dimensions of the specimen: depending on the height: lateral dimension ratio of the specimen, etc. The wide difference in results is due to the varying effects that the shape of each type of specimen has on the stresses that are being induced. That is, the properties accruing from the geometry of the specimen are also being evaluated, although unwittingly. It is therefore as absurd to crush a 6-in cube, and then to define the crushing load as the 'compressive strength of the concrete', as it would be to quote the value C, at failure, (in the equation for the spring given previously) as the 'tensile failure strength of steel'. The correct reporting of a cube test would be something as follows: 'Strength of a 6-in cube under a uniaxially applied compressive load: X lbs'.
b) A column transmits load to the ground in one of two ways: either by becoming compressed, as in a short column, or by bending, as in a long column. The mode chosen is that which is the 'easiest' under the circumstances, and depends on the slenderness ratio of the column, i.e., on its geometrical characteristics. The failure load is thus related to the mode which is operative, and therefore to the geometrical property. Consequently, the failure load is not an intrinsic, particular property of the material, but is a compound of the characteristics of the material and of the column shape.

53 Testing For Property

a) Finally, it may be seen that the ideal test should specifically evaluate the particular property of the material that is under investigation, independently of the shape of the material. In practice this is seldom achieved. The results therefore are a compound of the fundamental mechanical property of the material, of its shape and environment, of the type and pattern of load, of the time factor, etc. The results should therefore be quoted in a manner that would make this clear: and the interpretation
of the results must be done keeping all these factors in mind.

b) In quality control tests, however, where the aim is to ascertain whether the material is of the required standard, (as distinct from evaluating a fundamental mechanical property of the material), methods of test in which the results also depend on the shape of the material (as in the concrete cube test) are certainly acceptable. These arbitrary, empirical tests serve as a means of comparison, and as such can be applied to quality control work.
The Induced Stress-Strain Response

2.5.2

1 Tension

i) The aggregations in an unstressed solid body retain their equilibrium positions due to a balance being maintained between the interparticle forces of attraction and repulsion.

ii) An applied load can act to pull the particles apart; in which case, and within limits, the particles move apart such that the net interparticle force of attraction that is generated is sufficient to just balance the load, in accordance with the relationship of Fig 5. At the same time the particles move inward in the lateral direction, giving rise to Poisson's effect (Fig 6.1) A lateral strain and stress is thus induced in the body. These induced strains and stresses, together with the applied external tensile load, must set up a triaxial state of strain and stress in the material, even through the external load is uniaxial. That is, an induced state of uniaxial stress is not possible in real materials. As a result of these triaxial
stresses, each point in the body becomes strained. These strains can, of course, be resolved into the principal strain components.

4.2 Compression

1) An applied load can act to push the particles together; in which case, and within limits, the particles come closer together such that the net interparticle force of repulsion that is generated is sufficient to balance the load, in accordance with the relationship of Fig 5. At the same time the particles move outward in the lateral direction, giving rise to Poisson's effect (Fig 6.1). The generation of triaxial stress, and the consequent straining of each point in the body, occurs as described for the tensile force in ii) preceding.

4.3 Shear

1) An applied load can act to slide a layer of the particles past an adjoining layer. It would appear that the shear resistance that is generated is not due to a separate, specific and fundamental interparticle 'shear force' as such, but is simply the result of the interplay of the fundamental interparticle forces of attraction and repulsion.
ii) This concept is illustrated in Fig 7(i). When a 'shearing' force is applied, the interparticle forces of repulsion prevent the layers from moving closer together, while the interparticle forces of attraction prevent the layers from moving further apart. Layer 1 therefore tends to slide over Layer 2. By the time the particles of Layer 1 have slid through a distance equal to one particle spacing, the values of the interparticle forces have passed through a maximum. The bonds between the new neighbouring particles at the interface of the layers, in the new slipped position, will reform or not depending on the nature of the material and on whether the interparticle bonds have become ruptured in passing through the maximum value. (It will be shown later that, in any case, it is only the interparticle bonds of attraction that can become broken.)

4 Strain

i) So long as the strain under normal loads is not so large that the interparticle bonds become broken; or so long as the slip strain under shear loads is not so large that the bonds become
broken or become reformed in the new, slipped position: then the strains are elastic and will disappear on the removal of the load.

ii) a) The elastic strains due to normal loads cause a change in the volume of the body.

b) The elastic strains due to shear load cause an elastic distortion in the body, but the volume remains constant.

c) The non-elastic strains due to shear load cause a permanent distortion in the body, but the volume remains constant.

d) Shear stresses are therefore in the nature of deviator stresses, while hydrostatic stresses, which are normal stresses, are in the nature of non-deviator stresses. (Fig 6.2)

5 Poisson's Effect

Poisson's effect, as measured by Poisson's Ratio (U) is a measure of the structure of the material and of the directional effects of the interparticle bonds. Thus, if a body consisted of
chains of single particles, with no bonding existing between chains, the load coming on any one chain would strain only that chain, no transverse transmission of stress would occur, and the body would show no lateral straining. The manner in which the axial strain under an applied load is transformed into lateral strain is therefore indicated by the value of Poisson's ratio.

Failure of Materials
2.5.3

1 Tensile Fracture

Consider a body which is under the action of a hydrostatic, tensile force. No forces can develop in the body except for normal tension, which acts against the interparticle forces of attraction and causes elastic tensile strain in the body. As the force is increased, a stage is reached when it becomes too large to be balanced by the interparticle bond. (The point at which the interparticle force can no longer contain the applied force can easily be observed in Fig 5.) The bond is then ruptured, and separation
occurs in the material. The material is then said to have failed in tensile fracture.

2 Shear Plastic Flow

Consider a body which is under the action of a shear force. It has been shown in subsection 2.5.2.3 that as the shear strain increases under increasing load, the elastic state is passed and the layers tend to slide into new positions with respect to each other, the new positions being progressively spaced one particle spacing apart. It has also been stated that, depending on the nature of the material, the bonds between the new neighbouring particles on either side of the glide plane may or may not reform. If the bonds reform, the material remains unseparated and as whole as it was before; all that has happened is that it has yielded, or exhibited plastic flow. But it has not separated, and in this sense it has not failed (though, of course, under increasing loads plastic flow may develop to such an extent that the section of the body decreases calamitously, so that, when carried to an extreme, a material can fail due to plastic flow. See Fig 7(11)).
3 MISCONCEPTIONS REGARDING MODES OF FAILURE

1) Now, most of the work that has been done relating to the behaviour of engineering materials has described only the effects of plastic flow when discussing the action of shear forces. As a result, the ways in which a material can fail have been stated as:

   a) in compression, the compressive strength of the material having been overcome.

   b) in tension, the material fracturing in a brittle manner.

   c) in shear, the material yielding ductilely due to plastic flow.

ii) It will be shown that the first mode of failure is wrongly defined and that no material can fail in compression, as such.

iii) It will also be shown that a second response, leading to a different mode of failure, may occur under shearing forces.
4 Shear Fracture

i) In subsection 2.5.2.3, it was stated that when a layer of particles slides past another layer under the action of a shearing force, the interparticle bonds may reform in the new slipped position, or they may rupture, depending on the nature of the material. A little consideration will show that rupture of the bonds is in fact a real possibility. Materials in which the aggregations are held together by a bond such as the metallic bond will no doubt exhibit a reforming of bonds in the successive slip positions. But if the particles were held by a force similar to that which results in the 'diamond bond' type of crystals, the bonds will rupture and remain ruptured, when the shear strain has increased to beyond the elastic limit.

ii) Take the case of a cylindrical stick of blackboard chalk. If sufficient torsion is given to the cylinder, it will suddenly fail because of separation occurring along a helical path. (Fig 7(iii)) Due to the direction and nature of the torsional shear forces, maximum tension develops on planes at 45° to the direction of shear, and separation occurs along a
helix, corresponding to the progressive 45° angle orientation. This separation is due to tensile fracture.

On the other hand, consider a similar cylinder of chalk which is acted on by a shearing force of the type as illustrated in Fig 7(iv). Failure by sliding separation occurs in the manner depicted, and after very little relative movement has occurred between the two portions. (Contrariwise, if a cylinder of a ductile material such as lead was being similarly loaded, considerable relative movement would occur between the two portions, which would still remain unseparated. Fig 7(v)). This separation is obviously due solely to shear, there being no other force acting in the direction of separation. It is a characteristic of brittle materials.

iii) The science of geology differentiates between tension faults, shear faults and shear slides, the distinction between a fault and a slide being that only in the former does separation occur.
iv) Therefore, in addition to the occurrence of plastic flow under shear, which is the only shear failure that has been usually described, the mechanism of shear fracture is also seen to be operative in some materials.

5 'Failure In Compression'

The question of 'failure in compression' will now be discussed.

1) Consider a body acted upon by a hydrostatic, compression force (Fig 6.2). It will be seen that, as the force increases, all that can happen is that the aggregations of which the body is composed come closer together, the interparticle forces of repulsion becoming increased (in accordance with the relationship shown in Fig 5) so as to balance the external applied force. No separation into parts can occur. The material can therefore not fail. In fact, compression aids nature, in keeping the particles together. Thus, in large-grained materials like soil and concrete, where friction is of significance, compression aids in increasing the frictional resistance of the aggregations. Fig 5 also illustrates how the interparticle force of repulsion increases.
greatly with decreased spacing, and shows no indication of yielding; this is in contradistinction to the behaviour of the interparticle force of attraction. Ultimately, and if the applied force were inordinately high, it is conceivable that the particles would be so pushed against and even into each other that a form of atomic fusion would occur, in which case a new substance would be formed. But this takes the discussion beyond the scope of an engineering problem.)

For example: place a sheet of paper, or a piece of twine, or a very thin sheet of aluminium, between the platens of a compression testing machine and load the machine to capacity. It will be observed that, except for some slight flattening, no failure by separation has occurred in any of the specimens.

For example: half fill a very thick steel container, open at one end, with any powder, and load the powder from the open end in a compression testing machine. The powder becomes compressed, but does not 'fail,' enveloped as it is by hydrostatic 'containing' forces.
For example: Kaplan reports that brittle marble cylinders showed no signs of fracture as such when subjected to even extremely high hydrostatic compression loads.

For example: compress a cylindrical specimen of a ductile material such as lead. The cylinder flattens out into a disc, without fracturing. The term 'compressive strength' thus has no obvious meaning when applied to ductile materials.

ii) A distinction must be made between a material 'failing under the action of externally applied compressive loads' and 'failing in compression', which latter implies that the material has a definite compressive strength. The phrases mean two very different things. While a material cannot fail in compression (that is, it cannot fail at any particular point within itself due to the particles being pushed closer together at that point), it can fail at those points where shear and tension develop consequent to the application of an external compressive force.

6 Definition Of Failure

The term 'failure' of a material now needs to be clearly defined.
As far as the effects of the application of stress to an engineering material are concerned, one or more of the following responses can be observed:

- Elastic Strain  
  (elastic volume change)

- Elastic deviator strain  
  (elastic distortion, constant volume)

- Plastic deviator strain  
  (yield, plastic flow, permanent distortion, constant volume)

- Elastic instability  
  (as in the buckling of long columns)

- Plastic instability  
  (as in the formation of the three plastic hinges in a fixed beam)

- Fracture  
  (separation)

'Failure' depends on the level of the particular response or combination of responses, and is itself a relative term.

Thus, if deflections in a railroad bridge are to be controlled so that an engine does not become derailed, 'failure' may be said to have occurred if the
deflections become just moderately large, even though the material is well within the elastic strain state.

On the other hand, a prestressed concrete pavement slab is allowed to crack under load. 'Failure', therefore, cannot be said to have occurred even though separation of a type has taken place.

Therefore, whenever 'failure' is mentioned, it must be made clear, even if only by positive implication, in what type of response the material is deemed to have failed.

.7 The Modes Of Failure

From what has been observed so far relating to the meaning and the modes of failure, the following deductions are made:

i) Failure by separation can occur through the mechanism of tensile fracture.

ii) Failure by separation can occur through the mechanism of shear fracture.

iii) Failure by inordinate yielding can occur through the mechanism of shear plastic flow.
iv) Failure by compression of the particles is not possible.

v) A given material can fail by only one of the two shear modes, which are shear fracture and shear plastic flow. The mode of shear failure to which it is subject is a characteristic or property of that material, and depends on its nature (that is, as its structure). If a ductile material should fail in shear, it would do so by the mechanism of shear plastic flow. If a brittle material should fail in shear, it would do so by the mechanism of shear fracture.

vi) A given material therefore has two failure strengths, (in the sense of fracture or of yield).

These strengths are:

- its tensile fracture strength
- its shear fracture strength
- its shear plastic flow strength (whichever is applicable)
(For the sake of convenience, a generalization is made of the term 'shear strength', it being explicitly understood that the term covers both, shear fracture strength and shear flow strength, only one of which strengths is applicable to any given material.)

vii) A given body under given conditions of external loading may fail at some point within itself, and will do so either in tension or in shear. The location of the point at which failure will occur, the direction of failure, and the manner in which the material fails, depend on the following: a) the values and directions of the shear and normal tension components of the resultant stress that is induced at that point under the action of the external loads; (b) the relative values of the shear and tensile strengths of the material.

Thus it may occur that, at a particular point, the shear component of the induced, resultant stress has a lower value than the normal tensile component; but if the shear strength of the material is lower than the tensile fracture strength, the material will fail, in shear, at the lower shear stress.
It is assumed, of course, that at the engineering level, materials are isotropic with respect to all mechanical properties.

**Principles Governing The Induced Response**

2.5.4

(i) **The Aims Of Analysis**

The ultimate aims of analysis can be summed up in the following questions:

i) due to the applied external loads, what are the induced external loads?

ii) due to the external loads, what are the components of the induced stresses in the body?

iii) what are the components of the induced strains in the body?

iv) will the material fail?

The computations for obtaining the induced external forces constitute a problem in statics or in hyperstatics, and will not be considered here. The remaining three questions pertain to the induced response in the material.
3.2 Principles Governing The Induced Response

i) It will be observed that the development of the induced stresses in the body depends on certain principles, which are:

a) when the external forces are transmitted through a medium, the transmission takes place in as direct a manner as possible so that the energy of the system is kept to a minimum.

b) the forces can only be transmitted along those paths which are themselves capable of resisting the force.

c) the actual state of induced stress at any point in the medium will depend on the manner in which the external forces have been 'routed' along the various available paths, and the way in which they have been resolved and transformed into their various components before they reach that point. This in turn depends on the pattern of interparticle bonding forces, and on the geometry of the body. (It is to be noted that, as a result, the induced stresses in a body will vary from tension to compression to shear throughout the body, even though the body as a whole may be 'pure' tension or compression.)
d) The transmitted forces will tend to follow those paths which are more capable of resisting them.

e) The computations for evaluating strain have to be integrated into stress computations at all stages since each follows one from the other. This is of special significance in problems involving finite strain, where the geometrical consequences of the deformations affect the problem in a manner not considered by the ordinary theory of elasticity.

(Note: As an aid to envisaging the concept of routing, it is convenient to consider the lines of interparticle force as tension-compression springs. The nodes where the springs intersect constitute the centre of the particle).

3.3 Microscopic/Macroscopic Evaluation Of The Stress-Strain Relationship

i) Any continuous medium has to satisfy exactly the four basic laws (i.e., Newton's Law, the 1st and 2nd laws of Thermodynamics, and the Conservation of Mass Law). The property relating stress to strain is not a basic law, however, but is a constitutive
property which depends on the nature of the material. The stress-strain relationship can therefore be obtained only by observation and measurement, and is thus only as precise as is the scale of observation.

ii) All mechanical testing has to obtain the property of materials has necessarily been done at the macroscopic level, and it is from the results of these tests that the microscopic theories of behaviour have been developed. It is only in recent times that tests at the microscopic level have been attempted. It would appear more elegant to be able to analyse and design, at the engineering level, from the results of tests conducted on atoms and molecules. At the moment, however, tests at the engineering level itself, and interpretations therefore, are certainly more realistic.

iii) The principles stated in subsection 2.5.4.2 preceding, particularly pertain to materials in the non-homogeneous, particle state, where aggregations and force paths are distinctly separate. At this level, particle displacements will occur in a series of 'jumps'. At the engineering level,
of course, where observations are statistically averaged out over finite dimensions, the stress-strain relationship can be taken to be continuous.

4 Practical Demonstration of Induced Routing of Stress

1) The validity of the observation that the induced stresses in the material depends on the manner in which the external forces are routed and resolved into components as they are transmitted through the body, which in turn is contingent partly on the geometry of the specimen, and partly on the inter-particle bonding patterns, may be simply demonstrated.

2) A single sheet of paper, when compressed, shows no sign of failure. This is because the thickness dimension is so small that only very slight routing and resolving of the applied force into shear and tension components is possible. Most of the stress induced in the sheet remains as compressive as the applied external load. No failure can therefore occur.

Now make a stack of paper sheets, in the form of a cube or prism. Load the stack in compression. It will be found that, if the load is
large enough, the paper cube fails suddenly and explosively, with a tensile-cum-shear mode of failure. (See S. No. 1 of Table T-2, and Photograph Nos. 21, 12, 16, 18 & 19). It would appear that, due to the geometry of the specimen, and due to the friction that develops between the leaves of paper (analogous to interparticle force), the externally applied compression load becomes 'transformed' into, and induces, tension and shear stresses (in addition to compression stresses) at various points throughout the specimen. The paper sheets then tear in tension, or shear across a tightly compressed wad of leaves, when the respective stresses at those points reach the failure strength of the material.

The Critical Strain Criterion For Failure

2.5.6

i) Having determined the stresses and strains that would be induced in a material under external loads, it remains to be ascertained whether or not the material would fail.

ii) Failure is now taken to mean fracture or inordinate plastic flow, in which case the three modes of failure as established in subsection 2.5.3 are operative.
iii) From the engineering point of view, it would appear to be preferrable to work in terms of strains rather than of stresses, since, in practice strains are more easily and more positively capable of being evaluated.

iv) The development of critical strains shall therefore be taken to be the criterion for failure; and a material that is under load shall be deemed to be about to fail at a particular location if either the normal tensile strain, or the shear strain, at that location, becomes respectively equal to the normal tensile strain or the shear strain, as it developed at those points in a test specimen where tensile fracture, or shear failure, respectively occurred.

v) No matter how involved the stresses at a point may be, the components of the net resultant strain provide a true indication of how far apart the aggregations are being forced. Critical strains therefore provide a convenient means of predicting the failure of a material.
vi) It is to be noted that compression strains can never be critical. The individual strains which develop at a particular location may therefore be combined into two components: 'hydrostatic' strain (which may be 'tensile' or 'compressive') and deviator strain (which is due to shear). Only tensile 'hydrostatic' strain, and deviator strain, can be critical.

vii) The ease of application of this critical strain concept is demonstrated in the last portion of this paper, where critical strains are employed to predict the cracking of prestressed concrete pavement slabs.
2.6 Observations On Methods Of Test

i) What the engineer would need to know about the stress-strain response of the material he is considering using may be summarized thus:

a) What is the strain response of the material measured in the line of action of a tensile/compressive uniaxial load?

b) What is the strain response of the material measured transverse to the line of action of a tensile/compressive uniaxial load?

c) What is the strain response of the material in the line of action of a shearing load?

d) How much of the strain is recovered, instantaneously and with the passage of time, once the load is removed?

e) What is the critical strain for failure to occur by tensile fracture?

f) When it fails in shear, does it fail by shear fracture or by shear plastic flow?
g) What is the critical strain for failure to occur in shear?

The methods of test must be devised accordingly.

ii) The other mechanical properties such as hardness, abrasion resistance, toughness, fatigue limit, impact resistance, etc. also need to be evaluated, of course, but as far as the basic stress-strain response is concerned the points enumerated in (i) preceding are applicable.

iii) A hydrostatic compression test would be eminently suited for obtaining some of the properties sought, especially in that the stresses and strains are purely normal. The bulk modulus in compression \( K_{\text{comp.}} \) would be obtained from this test.

iv) A hydrostatic tensile test, if it could be set up, would enable what may be termed the bulk modulus in tension \( K_{\text{tens.}} \), to be obtained. The strain characteristics of the material at tensile fracture could also be evaluated.
v) A precisely instrumented test in simple shear would enable the modulus of rigidity (G) to be obtained directly.

vi) Using these fundamental properties of $K_{\text{tens.}}$, $K_{\text{comp.}}$, and $G$, the moduli of elasticity in tension and compression ($E_{\text{tens.}}$, $E_{\text{comp.}}$), and Poisson's Ratio ($\nu$) can be obtained from the usual relationships established by the theory of elasticity, assuming that at the engineering level the material is isotropic to all mechanical properties.

Due to the 'normal strain' characteristics of the hydrostatic tests, the stress-strain responses obtained from these tests can be taken as being in the line of the load. Poisson's ratio $\nu$ will indicate the nature of the response transverse to the line of the load.

vii) The triaxial testing of specimens would also be an eminently suitable way of investigating the properties of a material. The hydrostatic stresses and deviator stresses could be varied so as to obtain the failure modes under succession of different conditions. The patterns of response and of failure could then be evaluated.
viii) a) The testing in compression of cylinders and prisms would provide some indication of the behaviour of the material under uniaxially applied compressive loads. The height of the specimen must be large enough so that the triaxial effects of end friction restraint are not felt over the central portion of the height dimension. (The specimen must not be too long, either, or bending would occur). However, the induced stresses at the section where observations are being made would still be triaxial.

b) Cube tests should not be adopted. The effects of the restraint brought by the friction which develops at the interfaces of the cube and the platens of the compression testing machine, are exercised throughout the height of the cube, due to its comparatively small dimension. As a result, a complicated state of triaxiality of stress develops throughout the cube, and no part of it is in 'simple compression'.

ix) Tensile split tests on cylindrical specimens are also advocated. They provide a very good indication of the degree of tensile stress which
a material can resist before it fails in tensile fracture. The splitting is usually even enough for the assumption to be made that the tension is uniformly distributed over the split faces.

x) The cutting of grooves in cylindrical or prismatic specimens sets up a triaxial state of stress under external axial loadings. The testing of such specimens may furnish important data on the characteristics of the material, but no clear pattern has emerged from such tests as have been carried out by the author, (see S.Nos 17, 18, 19 of Table T-2), probably due to incomplete observations.

xi) Torsion tests will enable the effects of pure shear to be evaluated. The tensile fracture failures that usually occur in brittle materials which subjected to torsion are conveniently studied by means of these tests.

xii) The behaviour of materials under impact loads is not being considered in this paper. Impact, however, is a powerful tool in the study of the response of materials to load, involving as it does the concept of applied energy. In the main, impact serves to embrittle a material.
xiii) Flexure tests are considered a very good means of obtaining the critical strain tensile fracture of brittle materials, especially if the third-point method of test is adopted wherein a significant portion of the specimen beam is under a state of pure bending. The value obtained for the critical strain may vary somewhat from that obtained from pure tension tests; but where flexural tension needs to be considered, as when analysing pavement slabs, the critical tensile strain as ascertained from flexure tests could be made use of with justification.

2.7 Observations On Experimental Strength Tests

Scope Of Work

2.7.1 A number of tests were conducted on a variety of specimens with a view to studying the stress-strain response of an elastic material, and to investigating the modes of failure.

.1 Stress-Strain Response Tests

The tests for the investigations of the response to load were as given below. (In all cases,
a light elastic material in the shape of a slab was used)

i) The specimen was supported completely over its bottom surface. A compressive load surface was applied from the top, the loadprint completely covering the top surface.

ii) The specimen was supported centrally over a small portion of its length, the support running through the full width.

A compressive load was applied from the top, the loadprint being positioned directly over the support. (Photograph No 126)

iii) The specimen was completely supported over its bottom surface on a rigid base. A compressive load was applied at the top, through a centrally placed loadprint running through the full width. (Photograph No 127)

iv) The set-up for the test was as in (iii) preceding, except that the loadprint was very much wider, though still less than the length of the specimen. (Photograph No 129)
v) The set-up for the test was as in (iii) preceding, except that the loadprint was very much narrower. (Photograph No 130),

vi) The set-up for the test was as in (iii) preceding, except that the specimen was supported over a material having similar elastic characteristics. (Photograph No 128)

vii) The specimen was supported on a rigid base. A compressive load was applied eccentrically, towards one end. (Photograph No 131)

#2 Failure Mechanism Investigations

The tests that were conducted to investigate the failure mechanism of certain materials were as given below -

1) Compression test on a 4 in x 4 in x 4 in (at failure) cube of paper sheets.

(Photographs Nos 21, 12, 16, 18, 19)
ii) Compression test on a 4 in x 4 in x 1.9 in (at failure) prism of paper sheets.

iii) Compression test on a 4 in x 4 in x 4 in (at failure) prism of oil-soaked paper sheets.

(Photographs No II)

iv) Compression test on a 6 in x 6 in prism of paper sheets.

v) Compression test on a 4 in cube of 9-days old concrete (Photographs Nos 7, 5, 8, 9, 132, 4, 3, 92, 1).

vi) Compression test on a 4 in cube of 9-days old concrete, with friction reducing oiled-paper layers at the ends (Photographs Nos 2, 125)

vii) Compression test on a 4 in cube of 9-days old concrete, with friction reducing oil layers at the ends.

viii) Compression test on a 6 in dia x 2 in disc of 9 days old concrete.

ix) Compression test on a 6 in dia x 2 in disc, of 9-days old concrete with friction reducing oiled-paper layers at the ends.
x) Compression test on a 6-in cube of green concrete.

xi) Compression test on a 6-in dia x 2-in disc of green concrete (Photograph No 135).

xii) Compression test on a 6-in dia x 12-in cylinder of green concrete (Photograph No 134).

xiii) Compression test on a 4-in dia x 8-in cylinder of green concrete, with friction reducing layers at the ends.

xiv) Compression test on a 6-in cube of 28-day old concrete.

xv) Compression test on a 4-in cube of 28-day old concrete.

xvi) Compression test on a 4-in dia x 8-in high cylinder of 28 day old concrete. (Photograph No 97)

xvii) Compression test on a 6-in, grooved cube of concrete (Photograph No 91, 93)

xviii) Compression test on a 4-in, grooved cube of concrete.

xix) Compression test on a 4-in dia x 8-in, grooved cylinder of concrete (Photograph No 95, 96)
xx) Tensile split test on a 4-in dia x 8-in cylinder of concrete (Photograph No 98).

xxi) Shear test on a 1-in dia x 10 in cylinder of concrete (Photograph No 99).

xxii) Flexure test on a 2-in x 2-in beam of concrete (Photograph No 100).

xxiii) Compression test on a 6-in dia x 1½-in disc of concrete (Photograph No 103).

xxiv) Punching shear test on a 6-in dia x 1½-in disc of concrete. (Photograph No 104)
The punch was 2-in in dia.

xxv) 'Subgrade-supported punching shear' test on a 6-in dia x 1½ in disc of concrete.
The disc was supported over subgrade material (Photograph No 101.) and loaded over a 2-in dia punch.
Observations And Inferences

2.7.3

1. Stress-Strain Response Test

The observations that were made and the inferences that were drawn are set out in Table T-1.1 to 1.6

2. Failure Mechanism Investigations

The observations that were made and the inferences that were drawn are set out in Table T-2.1 to 2.18.