Chapter-3

Improved Efficiency Measure through Directional Distance Formulation of DEA

3.1 Introduction:

In this chapter, we propose an improved model for efficiency measure through directional distance formulation by modifying ADDM. Similar to efficiency defined in slack based model (SBM) as a product of input and output efficiencies, here objective function is considered as the product of input and output efficiencies with directional distance approach. This model gives a different approach for measuring efficiency through directional distance function of DEA formulation. The proposed model in this chapter is simple, intuitive and can be easily put into practice. The measure of efficiency obtained using the proposed model is more precise compared to the efficiency measure using ADDM discussed in Chapter-2. The dual of the proposed model is quite useful to give the economic interpretation. It is possible to examine the efficiency levels of DMUs even when both inputs and outputs consist of positive and negative values. It measures maximum non-radial contraction of inputs and expansion of the outputs consistent with technical feasibility in the direction of an ideal DMU. Advantage of directional distance function is that it allows the evaluation of the degree of efficiency in any direction from the observation points. The proposed model allows us to alter the variable weights. Decision maker's value judgment can be incorporated to measure the inefficiency in this model. The improved efficiency measure (IEM) is a superior model; its variant models and relation to equivalent basic DEA models are established in this chapter.

Unlike most data envelopment analysis (DEA) studies, IEM measures maximum non-radial contraction of inputs and expansion of the outputs in the direction of ideal DMU consistent with technical feasibility. Another advantage of directional distance function is that it allows the evaluation of the degree of efficiency in any direction from the observation points. Hence it is possible to measure the efficiency in different direction by changing the value of direction vectors. Here we consider the direction vector as range of possible improvement as defined in Portela et al. (2004). The dual problem of this model gives the economic interpretation as virtual profit maximization / cost minimization problem. This model in particular cases reduces to basic DEA models such as Charnes-Cooper-Rhodes (CCR) developed by Charnes *et al.* (1978) , Banker-Charnes-Cooper (BCC) extended by Banker *et al.* (1984) and slack based model (SBM) developed by Tone (2001). Moreover this model satisfies all the properties put down by Cooper *et al.* (1999) and Tone (2001).

3.2 Improved efficiency measure through directional distance formulation of DEA:

In this section we develop a generalized model for measuring the Pareto-Koopmans efficiency based on non-oriented and non-radial model with superior discrimination ability. This model is applicable for production technology involving either input variables or output variables or both possibly are having negative values. Let '*n*' DMUs with $x \in R^m$ denote inputs and $y \in R^s$ denote outputs. Under the standard assumption of convexity and free disposability of inputs and outputs, the production possibility set is

$$T = \left\{ (x,y) : (x,y) : x \geq \sum_{j=1}^{n} \lambda_j x_j \text{ and } y \leq \sum_{j=1}^{n} \lambda_j y_j ; \sum_{j=1}^{n} \lambda_j = 1;\ \lambda_j \geq 0; (j = 1,2,...,n) \right\} \quad (E\ 3.1)$$

In order to estimate the efficiency of DMU_0 we develop an improved efficiency measure through directional distance formulation of DEA–minimization model by formulating the following fractional program, referred to as IEM for the production possibility set defined in (E 3.1) is

$$\left.\begin{array}{lll}
 & \min S(x_o, y_o) = \eta = \dfrac{1 - \sum_{i=1}^{m} W_i \beta_{io}^-}{1 + \sum_{r=1}^{s} Z_r \beta_{ro}^+} & (3.1\text{a}) \\
\textit{subject to} & \sum_{j=1}^{n} \lambda_j y_{rj} - \beta_{ro}^+ R_{ro}^+ \geq y_{ro} & (3.1\text{b}) \\
 & \sum_{j=1}^{n} \lambda_j x_{ij} + \beta_{io}^- R_{io}^- \leq x_{io} & (3.1\text{c}) \\
 & \sum_{j=1}^{n} \lambda_j = 1 & (3.1\text{d}) \\
 & \sum_{i=1}^{m} W_i = 1, \qquad \sum_{r=1}^{s} Z_r = 1 & (3.1\text{e}) \\
 & \lambda_j \geq 0,\ j = 1,2,\cdots,n;\ i = 1,2,...m;\ r = 1,2,...k & \\
 & \beta_{ro}^+, \beta_{io}^- \text{ are unrestricted.} &
\end{array}\right\} \langle M3.1 \rangle$$

The directional vectors R_{ro}^+ and R_{io}^- shown in $\langle M3.1 \rangle$ are as stated in (E 2.1) and $(E\ 2.2)$ respectively. In the numerator of equation (3.1a) β_{io}^- is the relative reduction rate in the input '*i*' for i= 1, 2, ...m, and $\sum_{i=1}^{m} W_i \beta_{io}^-$ is the possible weighted mean reduction of inputs or it is the input inefficiency and $1-\sum_{i=1}^{m} W_i \beta_{io}^{-}$ is the efficiency of inputs. β_{ro}^{+} in denominator evaluates the expansion of output 'r' and $\sum_{r=1}^{s} Z_r \beta_{ro}^{+}$ is the possible weighted mean expansion of outputs and the inverse $\left(1+\sum_{r=1}^{s} Z_r \beta_{ro}^{+}\right)^{-1}$ measures the output efficiency. So 'η ' is the product of input and output efficiencies. It measures maximum non-radial contraction of inputs and expansion of the outputs consistent with technical feasibility in the direction of ideal DMU. This model allows us to alter the variable weights. Decision maker's value judgment can be incorporated to measure the inefficiency in this model. Here W_i and Z_r are the weights of input and output variables respectively derived based on the auxiliary information available on the importance of the variable. If such information is not available, the weights of the variables can also be derived from the data by taking inversely proportional to the coefficient of variation for each input and output variables or an equal weight for all inputs and outputs. Efficiency of the IEM lies between zero and one. A DMU_o is said to be Pareto-Koopmans efficient if $\eta^{*}=1$. This is equivalent to all $\beta_{io}^{-}=0$ and $\beta_{ro}^{+}=0$. This implies that there is no excess input and output shortfalls in the optimal solution relative to other DMUs.

Applying the Charnes and Cooper (1962) transformation to fractional programming of IEM, it can be converted into following linear programming as in the Tone (2001). Multiply a scalar variable 't'(>0) to both denominator and numerator of (3.1a). This does not cause any change in the value of η. Further, we adjust 't', so that the denominator becomes one. The adjusted expression of the denominator term is included as a constraint.

$$\left.\begin{array}{lll} & Min\ \tau = t - \sum_{i=1}^{m} W_i B_{io}^{-} & (3.2a) \\ subject\ to & t + \sum_{r=1}^{s} Z_r B_{ro}^{+} = 1 & (3.2b) \\ & \sum_{j=1}^{n} \Lambda_j y_{rj} - B_{ro}^{+} R_{ro}^{+} \geq t y_{ro} & (3.2c) \\ & \sum_{j=1}^{n} \Lambda_j x_{ij} + B_{io}^{-} R_{io}^{-} \leq t x_{io} & (3.2d) \\ & \sum_{j=1}^{n} \Lambda_j = t & (3.2e) \\ & \sum_{i=1}^{m} W_i = 1, \quad \sum_{r=1}^{s} Z_r = 1 & (3.2f) \\ & \Lambda_j \geq 0,\ j=1,2,\cdots,n;\ i=1,2,\ldots m;\ r=1,2,\ldots k & \\ & B_{io}^{-}, B_{ro}^{+} \text{ are unrestricted} & \end{array}\right\} \quad (M\ 3.2)$$

Let the optimal solution for $\langle M\ 3.2\rangle$ be $(\tau^*, t^*, \Lambda^*, B_{io}^{-*}, B_{ro}^{-*})$, then we have the optimal solution of IEM defined as $\left(\eta^* = \tau^*, \lambda^* = \Lambda^*/t^*, \beta_{io}^{-*} = B_{io}^{-*}/t^*, \beta_{ro}^{+*} = B_{ro}^{+*}/t^*\right)$. Based on the optimal solution we can determine the efficiency of the DMUs. A DMU_o is Pareto-Koopmans efficient if $\eta^* = 1$ otherwise it is inefficient. Pareto-Koopmans efficient input-output projection for an inefficient DMU_o for $\langle M\ 3.1\rangle$ is

$$x_{io}^* = x_{io} - \beta_{io}^- R_{io}^- = \sum_{j=1}^{n} \lambda_j^* x_{ij} \le x_{io} \quad \text{for all } 'i'$$

$$y_{ro}^* = y_{ro} + \beta_{ro}^+ R_{ro}^+ = \sum_{j=1}^{n} \lambda_j^* y_{ij} \ge y_{ro} \quad \text{for all } 'r'$$

A DMU_o is Pareto-Koopmans efficiency if and only if $\beta_{ro}^+ = 0$ for each output $'r'$ and

$\beta_{io}^- = 0$ for each input $'t'$ implies $\eta^* = 1$.

The IEM can be considered by output maximization approach by maximizing an objective function defined in (E 3.2) and constraints are shown in $\langle M\ 3.1\rangle$. This is an alternative model that maximizes the ratio of output efficiency to input efficiency.

$$\text{Max} \quad \delta = \frac{1 + \sum_{r=1}^{s} Z_r \beta_r^+}{1 - \sum_{i=1}^{m} W_i \beta_i^-} \qquad (E\ 3.2)$$

The process of converting fractional programming to linear programming with the constraint $c - \sum_{i=1}^{m} W_i \phi_i^- = 1$ the maximization problem reduces to $\langle M\ 3.2\rangle$. The relation between solutions of these two approaches is $\eta^* = 1/\delta^*$.

3.2.1 The dual program of IEM:

The dual model for the linear programming problem defined in the $\langle M\ 3.2\rangle$ can be formulated with dual variable $\hbar, \upsilon_0 \in R,\ v \in R^m,\ u \in R^s$ as follows

$$
\left.\begin{array}{lll}
 & Max\ \hbar & (3.3a) \\
subject\ to & \hbar - u_r y_{ro} + v_i x_{io} - \upsilon_0 = 1 & (3.3b) \\
 & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \upsilon_0 \le 0;\ j = 1,\cdots,n & (3.3c) \\
 & -u_r + \dfrac{Z_r \hbar}{R_{ro}^{+}} \le 0 & (3.3d) \\
 & -v_i \le -\dfrac{W_i}{R_{io}^{-}} & (3.3e) \\
 & \hbar \text{ and } \upsilon_0 \text{ are unrestricted.} &
\end{array}\right\} \langle M\ 3.3 \rangle
$$

Taking $\hbar = 1 + u_r y_{ro} - v_i x_{io} + \upsilon_0$ as in the equation (3.3a) and consider this value in the model $\langle M\ 3.3 \rangle$ we can re arrange the model as

$$
\left.\begin{array}{lll}
 & Max\ \ 1 + u_r y_{ro} - v_i x_{io} + \upsilon_0 & (3.4a) \\
subject\ to & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \upsilon_0 \le 0;\ j = 1,\cdots,n & (3.4b) \\
 & -u_r + \dfrac{Z_r u_r y_{ro}}{R_{ro}^{+}} - \dfrac{Z_r v_i x_{io}}{R_{ro}^{+}} + \dfrac{Z_r \upsilon_0}{R_{ro}^{+}} \le \dfrac{-Z_r}{R_{ro}^{+}} & (3.4c) \\
 & -v_i \le -\dfrac{W_i}{R_{io}^{-}} & (3.4d) \\
 & \upsilon_0 \text{ is unrestricted} &
\end{array}\right\} \langle M\ 3.4 \rangle
$$

υ_0 is a scalar and free in sign may take the value positive , negative or zero. This dual variable is associated with constraint $\sum_{j=1}^{n} \lambda_j = 1$ of the envelopment model and corresponds to variable returns to scale. In $\langle M\ 3.4 \rangle$ the objective function $u_r y_{ro} - v_i x_{io} + \upsilon_0$ is to maximize the virtual profit by finding the optimal virtual prices and costs of a DMU_0 in comparison with virtual profit of remaining DMUs does not exceed zero i.e., as can be seen in the constraints $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \upsilon_0 \le 0$. Here, in the dual problem, our intention is to find the virtual prices of input and output variables relative to other DMUs to maximize the profit.

3.3 Desirable properties of the IEM:

Fare and Lovell (1978) proposed a set of desirable properties for measures of technical efficiency. They restricted these properties to input based technical efficiency. Ali and Seiford (1990) studied translation invariance property of DEA models and limited the scope of their study to non negative input and output values. Pastor (1996) took on the generalization of translation invariance property by allowing inputs and outputs to take negative values. Lovell and Pastor (1995) discussed the desirable properties of unit and translation invariance and they introduced normalized weighted BCC DEA model, which satisfies both properties. Borger *et al.* (1998) listed similar requirements for the efficiency model. Pastor *et al.* (1999) added few more properties in the context of enhanced Russell graph efficiency measure. Cooper *et al.* (1999) have specified that a good measure of efficiency should be a single real number and be capable of identifying all sources of inefficiencies. It should be readily interpretable in managerial contexts and easily for implementable. Tone (2001) considered units and translation invariant, monotonic property and reference set dependent technology are the good properties of an efficiency measure. Here we study these properties for the proposed efficiency measure.

a. Efficiency of the DMUs lies between zero and one $0 \le \eta \le 1$:

Proof: At the optimal solution to the IEM, constraints of the model become

$$x^{*}_{io} = x_{io} - \beta^{-}_{io} R^{+}_{ro} \quad \Rightarrow \beta^{*}_{io} = \frac{x_{io} - x^{*}_{io}}{R^{-}_{io}} = \frac{x_{io} - x^{*}_{io}}{x_{io} - Min\{x_{ij}\}} \quad \text{and} \quad y^{*}_{ro} = y_{ro} + \beta^{+}_{ro} R^{+}_{ro} \Rightarrow$$

$$\beta^{*}_{ro} = \frac{y^{*}_{ro} - y_{ro}}{R^{+}_{ro}} = \frac{y^{*}_{ro} - y_{ro}}{Max\{y_{rj}\} - y_{ro}}$$ i.e β^{-*}_{io} and β^{+*}_{ro} equals to the ratio of optimal slack to the maximum slack. This implies $0 \le \beta^{-}_{io}, \beta^{+}_{ro} \le 1$. By the definition of objective function η lies between zero and one.

b. Efficiency measure is translation invariant:

Proof: Let an amount k_r is added to each output and c_i to each input, then the constraints in the model becomes

$$\sum_{j=1}^{n} \lambda_j (y_{rj} + k_r) - \beta^{+}_{ro} R^{+}_{ro} \ge (y_{ro} + k_r) \; and$$

$$\sum_{j=1}^{n} \lambda_j (x_{ij} + c_i) + \beta^{-}_{io} R^{-}_{io} \le (x_{io} + c_i)$$

The range of the variable does not change

$$\sum_{j=1}^{n} \lambda_j y_{rj} + k_r \ge y_{ro} + k_r + \beta_{ro}^{+} R_{ro}^{+} \text{ and}$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} + c_i \le x_{io} + c_i - \beta_{io}^{-} R_{io}^{-} \quad \because \sum_{j=1}^{n} \lambda_j = 1$$

c. Efficiency measure is unit invariance:

Let an amount α_r is multiplied to each output and γ_i to each input, then the constraints in the model becomes

$$\sum_{j=1}^{n} \lambda_j \alpha_r y_{rj} \ge \alpha_r y_{ro} + \beta_{ro}^{+} \alpha_r R_y \text{ and}$$

$$\sum_{j=1}^{n} \lambda_j \gamma_i x_{ij} \le \gamma_i x_{io} - \beta_{io}^{-} \gamma_i R_x$$

These constraints are reduce to IEM of DEA formulation, whose solution therefore does not change when the unit of measurement changes.

d. $S(x_o, y_o)$ is weakly monotonic in inputs and outputs and also satisfies the following relationships:

i) $S((\gamma x_o^+; \frac{-x_o^-}{\gamma}), (y_o^+; -y_o^-)) \le \frac{1}{\gamma} S((x_o^+; -x_o^-), (y_o^+; -y_o^-))$ if $\gamma > 1$

ii) $S((\gamma x_o^+; \frac{-x_o^-}{\gamma}), (y_o^+; -y_o^-)) \ge \frac{1}{\gamma} S((x_o^+; -x_o^-), (y_o^+; -y_o^-))$ if $\gamma < 1$

iii) $S((x_o^+; -x_o^-), (\mu y_o^+; \frac{-y_o^-}{\mu})) \ge \mu S((x_o^+; -x_o^-), (y_o^+; -y_o^-))$ if $\mu > 1$

iv) $S((x_o^+; -x_o^-), (\mu y_o^+; \frac{-y_o^-}{\mu})) \le \mu S((x_o^+; -x_o^-), (y_o^+; -y_o^-))$ if $\mu < 1$

v) $S((\kappa x_o^+; \frac{-x_o^-}{\kappa}), (\frac{y_o^+}{\kappa}; -\kappa y_o^-)) \ge \frac{1}{\kappa^2} S((x_o^+; -x_o^-), (y_o^+; -y_o^-))$ if $\kappa < 1$

vi) $S((\kappa x_o^+; \frac{-x_o^-}{\kappa}), (\frac{y_o^+}{\kappa}; -\kappa y_o^-)) \le \frac{1}{\kappa^2} S((x_o^+; -x_o^-), (y_o^+; -y_o^-))$ if $\kappa > 1$

The interpretation of the notions of $-x_o^-$ is that these are inputs which are desirable to be used as much as possible. For example, in the pollutant disposal plant, it is desirable to use sewage sludge as inputs. Such inputs are reflected by assigning them negative values. More the DMU uses this kind of inputs, better for the efficiency. Similarly $-y_o^-$ are the undesirable outputs. In a production system, undesirable outputs are restricted. Lesser the DMU produces undesirable outputs, better it is for the efficiency evaluation. Proof is similar to that of ADDM.

e. Efficiency measure depends on reference set

For an inefficient DMU_o, we define the reference set E_o corresponding to the positive λ_j's at the optimal solution. Therefore the reference set E_o is

$E_o=\{j \mid \lambda_j^* > 0\}$, j=1,2,...n

Let (x_o, y_o) be input and output vectors represented using reference set by

$$x_{io} = \sum_{j\in E_o} x_{ij}\lambda_j^* + \beta_{io}^{-*}$$
$$y_{ro} = \sum_{j\in E_o} y_{rj}\lambda_j^* - \beta_{ro}^{+*}$$

Efficiency measure η^* in IEM depends only on β_{io}^{-*}, β_{ro}^{+*}. Hence this measure depends on reference set dependent values and not on the remaining DMUs in the data set.

3.4 Special cases of IEM:

3.4.1 Non-oriented radial IEM:

Let us consider i) Equal weights for all the variables, ii) $\beta_i^- = \beta^-$ and $\beta_r^+ = \beta^+$ in IEM and adding slack variables then model becomes

$$\left.\begin{aligned}
&Min\ \tau = \frac{1-\beta^-}{1+\beta^+} && (3.5a)\\
&subject\ to\ \sum_{j=1}^{n}\lambda_j y_{rj} - \beta^+ R_{ro}^+ - sl_r^+ = y_{ro} && (3.5b)\\
&\sum_{j=1}^{n}\lambda_j x_{ij} + \beta^- R_{io}^- + sl_i^- = x_{io} && (3.5c)\\
&\sum_{j=1}^{n}\lambda_j = 1 && (3.5d)\\
&\lambda_j \geq 0,\ j=1,2,\cdots,n;\\
&sl_r^+, sl_i^- \geq 0\ i=1,2,...m;\ r=1,2,...k\\
&\beta^-, \beta^+ \text{ are unrestricted}
\end{aligned}\right\} \quad \langle M\ 3.5\rangle$$

Model $\langle M\ 3.5\rangle$ is called the radial IEM model. A DMU$_o$ under evaluated through this model is said to be Pareto-Koopmans efficient if i) $\beta^- = 0, \beta^+ = 0$, and ii) $sl_r^+ = sl_i^- = 0$ i.e all slacks are zero, otherwise the DMU$_o$ is said to be inefficient.

3.4.2 Slack based model is derived from IEM:

Let us substitute the values by considering $\lambda_j = \partial_j$, $\beta_{io}^- = s_i^- / R_{io}^-$, $\beta_{ro}^+ = s_r^+ / R_{ro}^+$ in $\langle M\ 3.1\rangle$, we will get

$$\left.\begin{array}{ll}
\min S(x_o, y_o) = \varphi = \dfrac{1 - \sum_{i=1}^{m} W_i \left(s_i^- / R_{io}^-\right)}{1 + \sum_{r=1}^{s} Z_r \left(s_r^+ / R_{ro}^+\right)} & (3.6a) \\
subject\ to\ \sum_{j=1}^{n} \partial_j y_{rj} - s_i^- \geq y_{ro} & (3.6b) \\
\sum_{j=1}^{n} \partial_j x_{ij} + s_r^+ \leq x_{io} & (3.6c) \\
\sum_{j=1}^{n} \partial_j = 1 & (3.6d) \\
\partial_j \geq 0,\ j = 1, 2, \cdots, n; & \\
s_i^-, s_r^+ \geq 0;\ i = 1, 2, \ldots m;\ r = 1, 2, \ldots k &
\end{array}\right\} \langle M\ 3.6\rangle$$

Under optimality, the constraints of the $\langle M\ 3.6\rangle$ equality conditions hold and results in

$$\left.\begin{array}{ll}
\min S(x_o, y_o) = \chi^* = \dfrac{1 - \sum_{i=1}^{m} W_i \left(s_i^{-*} / R_{io}^-\right)}{1 + \sum_{r=1}^{s} Z_r \left(s_r^{+*} / R_{ro}^+\right)} & (3.7a) \\
subject\ to\ \sum_{j=1}^{n} \partial_j^* y_{rj} - s_i^{-*} = y_{ro} & (3.7b) \\
\sum_{j=1}^{n} \partial_j^* x_{ij} + s_r^{+*} = x_{io} & (3.7c) \\
\sum_{j=1}^{n} \partial_j^* = 1 & (3.7d) \\
\partial_j \geq 0,\ j = 1, 2, \cdots, n;\ i = 1, 2, \ldots m;\ r = 1, 2, \ldots k & \\
s_i^-, s_r^+ \geq 0 &
\end{array}\right\} \langle M\ 3.7\rangle$$

When inputs and outputs are non-negative and if i) Equal weights are given for all the variables and ii) Directional vectors are the observed input and output variables in

IEM given in $\langle M\ 3.1\rangle$, then solving this we can observe that IEM is equivalent to SBM

$$\left.\begin{array}{lll}
\min S(x_o,y_o)=\varphi^*=\dfrac{1-\frac{1}{m}\sum_{i=1}^{m}\left(s_i^{-*}/x_{io}\right)}{1+\frac{1}{s}\sum_{r=1}^{s}\left(s_r^{+*}/y_{ro}\right)} & & (3.8a)\\
\text{subject to} & \sum_{j=1}^{n}\partial_j^* y_{rj}-s_i^{-*}=y_{ro} & (3.8b)\\
 & \sum_{j=1}^{n}\partial_j^* x_{ij}+s_r^{+*}=x_{io} & (3.8c)\\
 & \sum_{j=1}^{n}\partial_j^*=1 & (3.8d)\\
 & \partial_j\geq 0,\ j=1,2,\cdots,n; & \\
 & s_i^-,s_r^+\geq 0;\quad i=1,2,...,m;\ r=1,2,...,k &
\end{array}\right\}\quad \langle M\ 3.8\rangle$$

3.4.3 BCC model is special case of IEM:

When data are non-negative, let us consider i) Equal weights for all the variables. ii) Directional vector for input is observed input and output is zero iii) $\beta_{io}^- = \beta$, $\beta_{ro}^+ = 0$. iv) Adding slack variables.

Apply above conditions in IEM given in $\langle M\ 3.1\rangle$, then model becomes

$$\left.\begin{array}{lll}
 & Min\ \tau = 1-\beta & (3.9a)\\
\text{subject to} & \sum_{j=1}^{n}\lambda_j y_{rj}-sl_r^+=y_{ro} & (3.9b)\\
 & \sum_{j=1}^{n}\lambda_j x_{ij}+\beta x_{io}+sl_i^-=x_{io} & (3.9c)\\
 & \sum_{j=1}^{n}\lambda_j=1 & (3.9d)\\
 & \lambda_j\geq 0;\quad j=1,2,\cdots,n & \\
 & sl_i^-,sl_r^+\geq 0;\ i=1,2,...,m;\ r=1,2,...,k & \\
 & \beta \text{ is unrestricted} &
\end{array}\right\}\quad \langle M\ 3.9\rangle$$

This model is called input oriented radial IEM. A DMU_0 evaluated through $\langle M\ 3.9\rangle$ is said to be Pareto-Koopmans efficient if i) $(1-\beta)=\theta$ ii) $sl_i^- = sl_r^+ = 0$ i.e all slacks are zero. Substitute

Let us consider the input oriented BCC model as $(1-\beta)=\theta_B$, $\lambda_j=\mu_j$, $sl_r^+=s_r^+$, $sl_i^-=s_i^-$ in $\langle M\ 3.9\rangle$, then

$$\left.\begin{array}{lr} Min\ \ \theta_B & (3.10a) \\ subject\ to\ \sum_{j=1}^{n}\mu_j x_{ij}+s^-=\theta_B x_{io} & (3.10b) \\ \sum_{j=1}^{n}\mu_j y_{rj}-s^+=y_{ro} & (3.10c) \\ \sum_{j=1}^{n}\mu_j=1 & (3.10d) \\ \mu_j\geq 0,\ s^-\geq 0,\ s^+\geq 0 & \end{array}\right\}\quad \langle M\ 3.10\rangle$$

The model $\langle M\ 3.10\rangle$ is same as BCC model. Hence IEM is special case of BCC model under given conditions.

3.4.4 Results:

a. Let η^* *and* θ_B^* be the optimal solution of IEM $\langle M\ 3.1\rangle$ and VRS DEA $\langle M\ 3.10\rangle$, then $\eta^*\leq\theta_B^*$.

Let $\left(\theta_B^*,\mu^*,s^{-^*},s^{+^*}\right)$ be the optimal solution for the VRS DEA model $\langle M\ 3.10\rangle$. For an inefficient DMU_o, projection of X_o and Y_o becomes

$$x_{io}=\sum_{j=1}^{n}\mu_j^* x_{ij}+(1-\theta_B)x_{io}+s^{-^*} \qquad (E\ 3.3)$$

$$y_{ro}=\sum_{j=1}^{n}\mu_j^* y_{rj}-s^{+^*} \qquad (E\ 3.4)$$

Let us define the

$$\lambda=\mu^*,\ \beta_i^-=s^{-^*}+(1-\theta_B^*)x_{io}\big/R_{io}^-,\ \beta_r^+=s^{+^*}\big/R_{ro}^+$$

Let $\left(\lambda,\beta_i^-,\ \beta_r^+\right)$ be the feasible solution of the $\langle M\ 3.1\rangle$ and its objective function is

$$\eta = 1 - \left\{ \sum_{i=1}^{m} W_i \left(\frac{s^{-*} + (1-\theta_B^*) x_{io}}{R_{io}^-} \right) \right\} \Big/ \left[1 + \sum_{r=1}^{s} Z_r \left(\frac{s^{+*}}{R_{ro}^+} \right) \right]$$

Consider numerator

$$1 - \left[\sum_{i=1}^{m} W_i \left(\frac{s^{-*}}{R_{io}^-} + (1-\theta_B^*) \frac{x_{io}}{R_{io}^-} \right) \right]$$

$$1 - \left[\sum_{i=1}^{m} W_i \left(\frac{s^{-*}}{R_{io}^-} \right) + (1-\theta_B^*) \sum_{i=1}^{m} W_i \left(\frac{x_{io}}{R_{io}^-} \right) \right]$$

DEA is applicable when both inputs and outputs have non-negative values. So, we compare IEM with VRS DEA models when all the inputs and outputs are non-negative.

Since, $R_{io}^- = x_{io} - \underset{j}{Min}\{x_{ij}\} \Rightarrow \frac{x_{io}}{R_{io}^-} \geq 1$ *and* $0 < \theta_B \leq 1$

$$\Rightarrow \quad \sum_{i=1}^{m} W_i \left(\frac{x_{io}}{R_{io}^-} \right) \geq 1$$

$$\Rightarrow \quad (1-\theta_B) \sum_{i=1}^{m} W_i \left(\frac{x_{io}}{R_{io}^-} \right) \geq (1-\theta_B)$$

$$1 - \left[\sum_{i=1}^{m} W_i \left(\frac{s^{-*}}{R_{io}^-} \right) + (1-\theta_B^*) \sum_{i=1}^{m} W_i \left(\frac{x_{io}}{R_{io}^-} \right) \right] \leq 1 - \left[\sum_{i=1}^{m} W_i \left(\frac{s^{-*}}{R_{io}^-} \right) + (1-\theta_B^*) \right] = \left[\theta_B^* - \sum_{i=1}^{m} W_i \left(\frac{s^{-*}}{R_{io}^-} \right) \right] \leq \theta_B^*$$

$$\eta = 1 - \left\{ \sum_{i=1}^{m} W_i \left(\frac{s^{-*} + (1-\theta_B^*) x_{io}}{R_{io}^-} \right) \right\} \Big/ 1 + \sum_{r=1}^{s} Z_r \left(\frac{s^{+*}}{R_{ro}^+} \right) \quad \leq \theta_B^* \qquad (E\ 3.5)$$

Hence $\eta^* \leq \eta \leq \theta_B^*$

b. A DMU_0 is BCC efficient if and only if it is IEM efficient

We can show this by similar arguments as in Tone (2001). Suppose a DMU_0 is IEM inefficient, then by definition it holds that $(\beta_i^-, \beta_r^+) \neq 0$.

Let $\lambda^*, \beta_i^{-*}, \beta_r^{+*}$ be the optimal solution of IEM in $\langle M\ 3.1\rangle$. For an inefficient DMU$_o$ projection of x_o and y_o becomes

$$x_{io} = \sum_{j=1}^{n} \lambda_j^* x_{ij} + \beta_i^{-*} R_{io}^-$$

$$y_{ro} = \sum_{j=1}^{n} \lambda_j^* y_{rj} - \beta_r^{+*} R_{ro}^+$$

Let us define the

$$\mu = \lambda^*,\ s^- = \beta_i^{-*} R_{io}^- + (\theta_B - 1)x_{io},\ s^+ = \beta_r^{+*} R_{ro}^+ \qquad (E\ 3.6)$$

Now let us transform the constraints of $\langle M\ 3.10\rangle$ using equation $(E\ 3.6)$

$$\theta_B x_{io} = \sum_{j=1}^{n} \lambda_j^* x_{ij} + \beta_i^{-*} R_{io}^- + (\theta_B - 1)x_{io}\ ,\ y_{ro} = \sum_{j=1}^{n} \lambda_j^* y_{rj} - \beta_r^{+*} R_{ro}^+\ ,\ \sum_{j=1}^{n} \lambda_j^* = 1$$

This solution is feasible for VRS DEA model provided $\beta_i^{-*} R_{io}^- + (\theta_B - 1)x_{io} \geq 1$, if $\theta_B = 1$ then $\left(s^+ = \beta_r^+ R_{ro}^+, s^- = \beta_i^- R_{io}^-\right) \neq 0$. Then, the VRS DEA model is inefficient, otherwise $\theta_B < 1$. In this case also VRS DEA model is inefficient.

Suppose that a DMU$_o$ is BCC inefficient, then we have either $\theta_B^* < 1$ or $\theta_B^* = 1$ and $\left(s^+, s^-\right) \neq 0$, in both the cases from the equation $\left(\text{E } 3.5\right)$ $\eta < 1$ for a feasible solution of IEM. Hence DMU$_o$ is IEM inefficient. Therefore BCC inefficiency is equivalent to IEM inefficiency.

3.5 Numerical example:

In order to illustrate the performance of our new model, we have taken the data pertaining to the pollutant processing system analyzed in Sharp *et al.* (2007). These data contains 13 DMUs which have one positive input (cost), one negative input (effluent), one positive output (saleable output) and two negative outputs (methane and CO_2) shown in Table-1 as observed inputs and outputs. We have applied four different models namely IEM$\langle M\ 3.1\rangle$, non-oriented RDM $\langle \text{M } 2.1\rangle$ and non-oriented radial IEM$\langle M\ 3.5\rangle$ as shown in the column-2 of Table-3.1. It can be seen that five efficient DMUs (3, 7, 8, 11, 13) are efficient for all models. IEM simultaneously maximizes outputs and minimizes the inputs by considering individual

DMU	Models	Projection of input and output values					Reference Set	Efficiency	Slacks				
		Inputs		Outputs					Inputs		Outputs		
		Cost	Effluent	Saleable	CO_2	Methane			Cost	Effluent	Saleable	CO_2	Methane
DMU-1	observed	1.03	-0.05	0.56	-0.09	-0.44							
	IEM	0.97	-0.17	0.82	-0.08	-0.43	7	0.4492					
	RDM	1.03	-0.18	0.88	-0.09	-0.42	3,7	0.9649		0.0462			0.0003
	IEM-RADIAL	1.03	-0.18	0.87	-0.09	-0.42	3,7,13	0.9126					0.0003
DMU-2	observed	1.75	-0.17	0.74	-0.24	-0.31							
	IEM	1.18	-0.17	0.74	-0.06	-0.31	3,7,11	0.5093					
	RDM	1.69	-0.35	1.46	-0.22	-0.28	3,7,8,11,13	0.9181					
	IEM-RADIAL	1.66	-0.42	1.15	-0.23	-0.28	3,8,11	0.8453					0.0124
DMU-3	**observed**	**1.44**	**-0.56**	**1.37**	**-0.35**	**-0.21**							
	IEM	**1.44**	**-0.56**	**1.37**	**-0.35**	**-0.21**	**3**	**1**					
	RDM	**1.44**	**-0.56**	**1.37**	**-0.35**	**-0.21**	**3**	**1**					
	IEM-RADIAL	**1.44**	**-0.56**	**1.37**	**-0.35**	**-0.21**	**3**	**1**					
DMU-4	observed	10.80	-0.22	5.61	-0.98	-3.79							
	IEM	8.06	-1.41	5.61	-0.98	-1.09	3,8,13	0.4667					
	RDM	8.20	-0.78	6.66	-0.72	-0.60	3,7,8,13	0.7352					2.1888
	IEM-RADIAL	8.05	-0.81	6.58	-0.74	-0.55	3,8,13	0.5779					2.3131
DMU-5	observed	1.30	-0.07	0.49	-1.08	-0.34							
	IEM	1.12	-0.14	0.70	-0.04	-0.34	7,11	0.5374					
	RDM	1.28	-0.42	1.18	-0.26	-0.29	3,7	0.9243		0.1827		0.743	0.027
	IEM-RADIAL	1.25	-0.40	1.15	-0.24	-0.30	3,7	0.7942				0.76	0.0169
DMU-6	observed	1.98	-0.10	1.61	-0.44	-0.34							
	IEM	1.70	-0.55	1.61	-0.36	-0.20	3,13	0.6354					
	RDM	1.95	-0.54	1.84	-0.36	-0.20	3,13	0.9708		0.3722		0.0639	0.1322
	IEM-RADIAL	1.76	-0.58	1.61	-0.37	-0.23	3,8,13	0.7836				0.0678	0.1062
DMU-7	**observed**	**0.97**	**-0.17**	**0.82**	**-0.08**	**-0.43**							
	IEM	**0.97**	**-0.17**	**0.82**	**-0.08**	**-0.43**	7	1					
	RDM	**0.97**	**-0.17**	**0.82**	**-0.08**	**-0.43**	7	**1**					

	IEM-RADIAL	**0.97**	**-0.17**	**0.82**	**-0.08**	**-0.43**	**7**	**1**					
DMU-8	**observed**	**9.82**	**-2.32**	**5.61**	**-1.42**	**-1.94**							
	IEM	**9.82**	**-2.32**	**5.61**	**-1.42**	**-1.94**	**8**	1					
	RDM	**9.82**	**-2.32**	**5.61**	**-1.42**	**-1.94**	**8**	**1**					
	IEM-RADIAL	**9.82**	**-2.32**	**5.61**	**-1.42**	**-1.94**	**8**	**1**					
DMU-9	observed	1.59	0.00	0.52	0.00	-0.37							
	IEM	1.29	-0.11	0.57	0.00	-0.24	11	0.6563					
	RDM	1.29	-0.11	0.57	0.00	-0.24	11	0.9945	0.30	0.10			0.128
	IEM-RADIAL	1.29	-0.11	0.57	0	-0.24	11	0.9473	0.2706				0.128
DMU-10	observed	5.96	-0.15	2.14	-0.52	-0.18							
	IEM	2.71	-0.50	2.54	-0.38	-0.18	3,13	0.5359					
	RDM	3.77	-0.45	3.53	-0.41	-0.15	3,13	0.8596	1.49		0.3438	0.0364	
	IEM-RADIAL	5.88	-0.18	5.22	-0.33	-0.11	3,11,13	0.7231			0.398		
DMU-11	**observed**	**1.29**	**-0.11**	**0.57**	**0**	**-0.24**							
	IEM	**1.29**	**-0.11**	**0.57**	**0**	**-0.24**	**11**	**1**					
	RDM	**1.29**	**-0.11**	**0.57**	**0**	**-0.24**	**11**	**1**					
	IEM-RADIAL	**1.29**	**-0.11**	**0.57**	**0**	**-0.24**	**11**	**1**					
DMU-12	observed	2.38	-0.25	0.57	-0.67	-0.43							
	IEM	1.07	-0.25	0.93	-0.14	-0.38	3,7	0.4064					
	RDM	2.16	-0.57	1.96	-0.39	-0.23	3,8	0.8448				0.1799	0.1309
	IEM-RADIAL	2.08	-0.69	1.69	-0.43	-0.34	3,8	0.6983				0.1589	0.0347
DMU-13	**observed**	**10.3**	**-0.16**	**9.56**	**-0.58**	**0**							
	IEM	**10.3**	**-0.16**	**9.56**	**-0.58**	**0**	**13**	**1**					
	RDM	**10.3**	**-0.16**	**9.56**	**-0.58**	**0**	**13**	**1**					
	IEM-RADIAL	**10.3**	**-0.16**	**9.56**	**-0.58**	**0**	**13**	**1**					

Table-3.1: Efficiency score, projection and slacks of input and output variables for inefficient DMUs by different non oriented models.

variations in inputs and outputs. Non-oriented RDM and non-oriented radial IEM overstates efficiency measure by keeping the slacks at the optimal solution and slacks for these two models are shown in the last five columns of Table-3.1. Models $\langle \mathrm{M}\ 2.1 \rangle$ and $\langle M\ 3.5 \rangle$ yields almost identical targets and reference DMUs for inefficient DMUs. A program is developed using MATLAB 7.7 to solve this problem and it can be seen in Appendix, Program-3.

In order to verify the efficiencies of different models developed in this chapter and to compare these models with the standard DEA models, we have taken the non-negative data by translating values of variables given in the Sharp *et al.* (2007). Columns of the Table-3.2 shows the results of different models namely IEM$\langle M\ 3.1 \rangle$, slack based model (SBM), IEM (considering directional vectors as observed input and output) $\langle M\ 3.8 \rangle$, non-oriented RDM $\langle M\ 2.1 \rangle$, radial IEM $\langle M\ 3.5 \rangle$, radial IEM (directional vector as observed input) $\langle M\ 3.9 \rangle$, VRS DEA $\langle M\ 3.10 \rangle$ respectively.

$\langle M\ 3.1 \rangle$	**SBM**	$\langle M\ 3.8 \rangle$	$\langle M\ 2.1 \rangle$	$\langle M\ 3.5 \rangle$	$\langle M\ 3.9 \rangle$	$\langle M\ 3.10 \rangle$
0.4492	0.9476	0.9476	0.9649	0.9126	0.983	0.983
0.5093	0.8683	0.8683	0.9181	0.8453	0.8974	0.8974
1	1	1	1	1	1	1
0.4667	0.2805	0.2805	0.7352	0.5779	0.7158	0.7158
0.5374	0.8154	0.8154	0.9243	0.7942	0.9552	0.9552
0.6354	0.8601	0.8601	0.9708	0.7836	0.9374	0.9374
1	1	1	1	1	1	1
1	1	1	1	1	1	1
0.6563	0.9285	0.9285	0.9945	0.9473	0.956	0.956
0.5359	0.7102	0.7102	0.8596	0.7231	0.8498	0.8498
1	1	1	1	1	1	1
0.4064	0.76	0.76	0.8448	0.6983	0.8451	0.8451
1	1	1	1	1	1	1

Table-3.2: Comparison of efficiency score by different models.

3.6. Application of canonical correlation:

There are situations in which number of inputs and outputs are significantly more as compared to number of DMUs and this leads to majority of DMUs comes out to be efficient. There are different rules for minimum number of units required for efficiency measurement. Nunamaker (1985) argued that number of units in the study should be at least three times greater than the sum of the number of inputs and outputs. But many applications of DEA violate this restriction and majority of DMUs comes out to be efficient in the analysis. Our study focused to address this problem by reducing dimension of inputs/outputs through canonical correlation. Friedman and Sinuany-Stern (1997) applied the canonical correlation for scaling DMUs. They used one set of positive canonical variates and their purpose was to generate a common set of weights for the exercise of ranking units on the common scale. But in many practical situations more than one significant canonical correlation may exist and also both positive and negative canonical variates may also be prevalent. We address this problem by using the improved efficiency measure through directional distance formulation of DEA approach to measure the efficiency developed in Section-3.2. This approach is application of directional distance formulation of DEA to a modified problem through canonical correlation.

3.6.1. Canonical correlation analysis (CCA):

Canonical correlation analysis (CCA) uses multiple inputs and outputs and focuses on the correlation between a linear combination of the variables in one set and a linear combination of the variables in another set. The idea is, first to determine linear combinations of variables in the sets that have maximum correlation. Then a subsequent linear combination in each set is sought such that the correlation between these is maximum of correlation between such linear combinations that are uncorrelated with the earlier linear combinations. The pairs of linear combinations are called the canonical variables and their correlations are called canonical correlations. Usually, first a few canonical correlations will be significant and remaining canonical correlations are insignificant. Canonical variates corresponding to insignificant canonical correlations are dropped. This will reduce the dimension of variables. CCA measures linear relationship between two groups of canonical variables V and U. The linear combination of the variables V and U is defined as

$$\begin{aligned} Z_j &= V_1 x_{1j} + V_2 x_{2j} + \cdots + V_m x_{mj} \\ W_j &= U_1 y_{1j} + U_2 y_{2j} + \cdots + U_s y_{sj} \\ & j = 1,2,\cdots,n \end{aligned} \qquad \langle E\ 3.7 \rangle$$

The aim of CCA is to identify and quantify the relations between m-dimensional random variable X and s-dimensional random variable Y. The coefficients V_i,

i=1,2,...m and U_r, r= 1,2,...,s must be such that the square of the correlation between Z and W will be maximum.

$$Max\ r_{zw} = V'S_{xy}U \Big/ \sqrt{(V'S_{xx}V)\,(U'S_{yy}U)}$$

$$subject\ to \quad V'S_{xx}V = 1 \qquad \langle E\ 3.8\rangle$$

$$U'S_{yy}U = 1$$

It is assumed that the variables of the two groups are linearly independent i.e. the rank X_{mxn}=m and the rank Y_{sxn}=s. In the case $m \le s$ the optimal solution for V' in this problem is the eigenvector corresponding to the largest eigenvalue of the quadruple matrix product $S_{xx}^{-1} S_{xy} S_{yy}^{-1} S_{yx}$. Its rank is less than or equal to m.

$$(S_{xx}^{-1} S_{xy} S_{yy}^{-1} S_{yx} - \lambda I)V = 0 \qquad \langle E\ 3.9\rangle$$

λ is the eigenvalue of the above quadruple matrix product and its eigenvector is V. There are at most 'm' solutions for λ and the largest eigenvalue λ_1 gives the square of maximum r_{zw} and the eigenvector V_1 provide the weights by which the set of the inputs should be linearly combined in order to achieve the maximal correlation. The vector of the combined weights of the outputs U_1 is obtained by

$$U_1 = \frac{S_{yy}^{-1} S_{yx}}{\sqrt{\lambda_1}} V_1 \qquad \langle E\ 3.10\rangle$$

Friedman and Sinuany-Stern (1997) used the CCA method by defining scaling ratio score, T as a ratio of linear combinations of inputs and outputs. Then they utilize the common weights for the linear combinations that are drawn from the largest eigenvalue of the CCA method, as shown below.

$$T_j = \frac{W_j}{Z_j} = \frac{\sum_{r=1}^{s} U_{1r} Y_{rj}}{\sum_{i=1}^{m} V_{1i} X_{ij}} \qquad j = 1,2,\cdots,n \qquad \langle E\ 3.11\rangle$$

While DEA efficiency ratio is bounded above by 1, the scaling ratio T_j of the CCA is unbounded.

The rank of the product matrices $S_{xx}^{-1} S_{xy} S_{yy}^{-1} S_{yx}$ and $S_{yy}^{-1} S_{yx} S_{xx}^{-1} S_{xy}$ is min (m,s). Therefore at most m ($m \le s$) canonical variates can be extracted. λ_1 refers to the first and largest eigenvalue of the matrix $S_{xx}^{-1} S_{xy} S_{yy}^{-1} S_{yx}$, V_1 and U_1 are the corresponding eigenvectors associated with λ_1. Successive canonical variates are extracted so that the second pair is the second most highly correlated pair out of all possible linear combinations that are uncorrelated with the first canonical variate pair, the third pair is the third most highly correlated pair out of all possible linear combinations that are mutually uncorrelated with the first and second canonical variate and so on. These properties are applied to the subsequent canonical variates and altogether '*m*' pairs of canonical variates are generated. The '*m*' canonical variates associated with the *Y*'s are uncorrelated and as the case of *X*'s. The correlation between the j^{th} canonical variates for the *X*'s and the k^{th} $(k \neq j)$ canonical variate for the *Y*'s is zero.

The approach we are suggesting in this chapter is the integration of directional distance formulation of DEA and canonical correlation. The canonical model selects linear functions that have maximum covariances between set of variables and again to restrictions on orthogonality. The techniques may therefore be loosely characterized as a sort of double barreled principal components analysis. It identifies the components of one set of variables that are highly linearly related to the components of the other set of variables.

Canonical correlation has been used to measure efficiency by Friedman and Sinuany-Stern (1997) but on the full set of DMUs. Their purpose was to generate a common set of weights for the purpose of ranking units on the common scale. They pointed out that negative set of weights could arise and that would be problematic. Their advice for dealing with negative set of weights was that first check if an input had been misclassified as an output or vice versa. If this is not the case, they suggest removing those variables. But in many practical situations, more than one significant canonical correlation exists and also both positive and negative canonical variates may be prevalent. This problem can be addressed by using the directional distance function approach to measure efficiency. Here, discussion is made when two significant canonical correlations are present. However, this could be easily generalized when more than two canonical correlations are significant. The method of canonical correlation constructs the composite inputs and outputs by choosing optimal weights of the inputs and outputs. Since DEA model can be viewed as an alternative approach for selecting suitable weights for combining various inputs and outputs, integration would enhance the potential applicability of directional distance formulation of DEA models. In the integration approach, first, find the significant canonical correlation by maximizing the correlation between bundle of inputs and outputs. Find the canonical variant for the inputs and outputs that come from the largest and significant eigenvalue of the CCA. Then utilize the common weights which have come from all the units to find the linear combinations of the outputs (W_{kj}) and inputs (Z_{kj}) for all significant canonical correlations. Afterwards, use the IEM model to measure the efficiency of DMUs.

3.6.2. An illustrative example:

In order to illustrate the integration of CCA and directional distance formulation of DEA, we have taken data set of 30 private domestic banks for the year 1998. It consists of four output and four input variables. The inputs are deposits (D), borrowings (B), labour (L), fixed assets (FA). The outputs are net interest margin (NIM), non-interest income (NII), credits (C) and investments (I). First we applied DEA-VRS model to the entire data set. According to this model 23 banks were found to be efficient. In the second stage we apply CCA to all the DMUs and the values of the input and output weights were obtained. The values of the input and output weights V_1,V_2,V_3,V_4 and U_1,U_2,U_3 and U_4 are given in Table-3.3. In this example, two canonical correlation are found to be significant as tested by the Bartlett's test with the p-value less than 0.01.

Canonical Variate	Canonical correlation	V1	V2	V3	V4	U1	U2	U3	U4
1	0.9979	-0.9941	-0.0196	-0.0098	0.1067	0.0711	-0.0668	-0.5907	-0.3544
2	0.8645	-0.3120	0.4087	-0.5863	0.6261	0.1880	-1.0121	-0.0665	0.7591

Table-3.3: Significant canonical correlation and their canonical variates.

To each banks weighted output w_{kj} and weighted input z_{kj} were calculated. When we calculated the composite input and output for each canonical variate and for each DMUs, we have standardized the variables. These standardized variables give better insights to relative importance of each variable.

VRS-DEA efficiency score are recorded in column-2 of Table-3.4. Further, we applied the improved directional distance model developed in Section-3.2 to the input and output canonical variates. Here the number of efficient DMUs will be reduced to 9 as shown in column-7 of Table-3.4. Moreover all efficient DMUs in the IEM model are also efficient VRS-DEA model.

Bank Name	DEA Efficiency	Z_{1j}	Z_{2j}	W_{1j}	W_{2j}	IEM Efficiency
BANK OF RAJASTHAN	0.9491	-1.5415	0.4781	-1.6100	1.5542	0.9296
CATHOLIC SYRIAN BANK	0.9176	-0.9939	0.7578	-0.9762	0.9940	0.9730
BANK OF PUNJAB	1	-0.8061	-0.6447	-0.8759	0.2059	0.9589
BHARAT OVERSEAS BANK	1	-0.6410	0.3575	-0.5586	0.7004	1
DEVELOPMENT CREDIT BANK	1	-0.8732	-0.4852	-1.0069	-0.1368	0.9279
CITY UNION BANK	0.9786	-0.5701	0.27891	-0.55081	0.4277	0.9811
GLOBAL TRUST BANK	1	-2.0616	-2.4970	-1.9963	-0.1599	1
HDFC BANK	1	-1.3474	-0.6570	-1.3721	0.6626	0.9643
CENTURION BANK	1	-0.7914	-0.2363	-0.7077	0.4274	1
FEDERAL BANK	1	-3.7463	1.2103	-3.8365	3.6228	1
DHANALAKSHMI BANK	0.9565	-0.5818	0.30948	-0.5424	0.6404	0.9890
GANESH BANK OF KURUNDWAD	1	- 0.04725	0.004151	-0.03955	0.066873	1
INDUSIND BANK	1	-2.6801	-1.3082	-2.7142	0.1626	0.8803
ICICI BANK	0.9061	-1.6833	-0.4281	-1.4803	0.3518	0.9823
KARUR VYSYA BANK	1	-1.2173	0.6836	-1.1967	0.4801	0.9537
LAKSHMI VILAS BANK	0.9289	-0.7951	0.4791	-0.7821	0.4842	0.9709
KARNATAKA BANK	1	-1.9453	0.7441	-1.8276	2.2199	1
LORD KRISHNA BANK	1	-0.3959	0.3568	-0.3902	0.2982	0.9822
JAMMU & KASHMIR BANK	1	-2.7024	0.9026	-2.6599	3.4466	1
IDBI BANK	1	-1.1928	0.3762	-1.1251	1.2695	0.9911
NEDUNGADI BANK	1	-0.4317	0.4199	-0.4589	0.4071	0.9725
RATNAKAR BANK	1	-0.1456	0.131434	-0.14539	0.183391	0.9901
SANGLI BANK	1	-0.5480	0.2410	-0.5441	0.4215	0.9773
TAMILNAD MERCANTILE BANK	1	-0.9028	0.5455	-0.9343	0.6441	0.9540
NAINITAL BANK	1	-0.1702	0.1138	-0.1256	0.2910	1
SOUTH INDIAN BANK	1	-1.5080	0.7380	-1.4965	1.9590	0.9923
SBI COMMERCIAL AND INTERNATIONAL	1	-0.2858	-0.1790	-0.2615	0.0729	0.9987
UTI BANK	1	-1.7377	0.2311	-1.6564	0.8951	0.9594
VYSYA BANK	1	-3.3013	-0.3191	-3.1418	1.7429	1
UNITED WESTERN BANK	0.8935	-1.4971	0.3110	-1.4772	0.6376	0.9419

Table-3.4: Efficiency measurement through application of CCA.

3.7 Conclusions:

This chapter gives a new approach for measuring efficiency through directional distance function of DEA formulation. Method proposed in this chapter is simple, intuitive and can be easily put into practice. Moreover, this method could be applicable to DMUs with either negative data or both (positive and negative data). The RDM measure of inefficiency proposed by Portela *et al.* (2004) appears to be an attractive way to deal with cases where both negative inputs and outputs occur. However, its inefficiency contains the slacks in the variable. The new model presented overcomes this problem. It measures maximum non-radial contraction of inputs and expansion of the outputs consistent with technical feasibility in the direction of ideal DMU. Advantage of directional distance function is that it allows the evaluation of the degree of efficiency in any direction from the observation points. The new model allows us to alter the variable weights. Decision maker's value judgment can be incorporated to measure the inefficiency in this model. The dual problem of this model gives the economic interpretation as virtual profit maximization / cost minimization problem. The IEM is a more generalized model; its variant models and relation to equivalent basic DEA models are established in this chapter.

In this model, efficiencies take into account the individual input and output variables. It represents the solution for the model with non-zero slacks when measuring efficiency. Hence, it leads to the Pareto-Koopmans measure of technical efficiency. Efficiency measure is bounded by zero and one, monotonically decreases for any increase in outputs and/or reduction in inputs or increases monotonically for decrease in outputs and or increase in inputs. This model is translation and unit invariant. This measure of efficiency is easily interpreted for use in a variety of managerial and scientific contexts. Although we assumed VRS in the proposed model, this can be trivially adopted to the constant returns to scale (CRS) case. Apart from the translation property, all other properties of the model holds good for the CRS case.

Further, canonical correlation analysis is integrated with the improved efficiency measure through directional distance formulation of DEA. This is particularly useful when number of inputs and outputs are more as compared to number of DMUs. Application of canonical correlation will reduce the dimension of inputs and outputs, therefore, the method has more discriminative power.