Chapter-1

Introduction

1.1 Data envelopment analysis:

Data envelopment analysis (DEA) is a relatively new "data oriented" approach for evaluating the performances of a set of entities called decision making units (DMUs) with multiple inputs and outputs. DEA confines attention to production activities that can be characterized as a deterministic process of transforming quantifiable and homogenous inputs to quantifiable and homogenous outputs. That is, any stochastic variations in the process are assumed non-existing, negligible for the purposes of the analysis, or correctable by means of some kind of data pre-processing. Over the past three decades, DEA has become increasingly popular for measuring comparative efficiency of organizations. DEA has evaluated the performances of different kinds of entities engaged in different kinds of activities in different contexts. It has opened up possibilities for use in cases which have been resistant to other approaches because of the complex and often unknown nature of the relations between multiple inputs and outputs involved in many of the activities. Recent years have seen variety of applications of DEA that have used DMUs of various forms to evaluate the efficiency of entities such as hospitals, airlines, universities, cities, courts, business firms, countries, regions etc. (see Cooper *et al.* (2004), Ramanathan (2001) and Thanassoulis (2001)).

Efficiency measurement has been a subject of enormous significance as organizations are under pressure to improve their productivity. There is an increasing concern among organizations to study the level of efficiency with which they work, relative to its competitors. The public sector organizations such as educational institutions, public health, public transport, local bodies, university departments, airlines etc., are having extremely complicated production / service systems. These organizations render plenty of diverse services by using a set of resources. Their activity is frequently measured on the basis of only a few parameters of productivity reflecting some aspects of activities of organization under study. The efficiency of public sector organizations is more often evaluated only by its outcomes irrespective of resources utilized. Hence, traditional efficiency measurement system provides a very unbalanced picture of performance that can lead the decision makers to miss important opportunities for improvement. The most commonly used method of comparison for efficiency evaluation was regression analysis and finding performance index. These measures are often inadequate due to the multiple inputs and outputs related to different resources, activities and exogenous factors. The various measures that can be used to assess efficiency are technical, allocative, and cost efficiency.

The technical efficiency of an organization depends on its level of productivity, or the ratio of its outputs to inputs. This could lead to productivity improvements through adoption of technological advances or increase in efficiency through better management or change in the operating environment in which production occurs. All units use a range of inputs including labor, capital, land, fuel and materials to produce services. If a unit is not using its inputs in a technically efficient manner, it is possible to increase the quantities of outputs without increasing inputs, or to reduce the inputs being used to produce same quantity and quality. It is necessary to look at the different concepts of efficiency. The most common efficiency concept is technical efficiency: the conversion of physical inputs into outputs relative to the best practice. An organization operating at best practice is said to be 100 per cent technically efficient. If operating below best practice levels, then the organization's technical efficiency is expressed as a percentage of best practice. Managerial practices and the scale or size of operations affect technical efficiency which is based on operating relationships but not on prices and costs.

Allocative efficiency refers to whether inputs, for a given level of output and set of input prices are chosen to minimize the cost of production, assuming that the organization being examined is already fully technically efficient. Allocative efficiency is also expressed as a percentage score, with a score of 100 per cent indicating that the organization is using its inputs in the proportions which would minimize costs. An organization that is operating at best practice in operating terms could still be allocatively inefficient because it is not using inputs in the proportions which minimize its costs, given relative input prices.

Finally, cost efficiency refers to the combination of technical and allocative efficiency. An organization will only be cost efficient if it is both technically and allocatively efficient. Cost efficiency is calculated as the product of the technical and allocative efficiency scores, so an organization can only achieve a 100 per cent score in cost efficiency if it has achieved 100 per cent in both technical and allocative efficiency.

Data envelopment analysis is a one of the methods that measures technical efficiency. It provides a means of calculating apparent efficiency levels within a group of organizations. The efficiency of an organization is calculated relative to the groups observed best practice. Typically using linear programming, DEA measures the efficiency of an organization within a group, relative to observed best practice within that group. The organizations can be whole agencies, separate entities within the agency or disaggregated business units within the separate entities. DEA may help identify possible benchmarks towards which performance can be targeted. The weighted combinations of peers and the peers themselves may provide benchmarks for relatively less efficient organizations. The actual levels of input use or output production of efficient organizations (or a combination of efficient organizations) can serve as specific targets for less efficient organizations aiming to improve performance. The ability of DEA to identify possible peers or role models as well as simple efficiency scores gives it an edge over other measures such as total factor productivity indices. DEA is a non-parametric technique of frontier estimation that determines both the relative efficiency of a number of DMUs and the targets for their improvement. DMUs can represent any set of organizations or departments that perform fundamentally the same task with the same set of variables.

Based on a data set, a DMU is said to be efficient if its performance relative to other DMUs cannot be improved is the main theme behind this methodology. DEA neither assumes a specific functional form for the production function nor the inefficiency distribution, in contrast to parametric statistical approaches. Since DEA in its present form was first introduced by Charnes *et al.* (1978), researchers in a number of fields have quickly recognized that it is an excellent and easily applicable methodology for modeling operational process for performance evaluations. Since its inception, according to Ali *et al.* (2008), more than 4000 research papers have appeared in the top twenty refereed journals till the end of 2006. Such a rapid growth in last three decades witnessing acceptance of the methodology of DEA is a testimony of its strengths and applicability. The main advantages of DEA are that it can readily incorporate multiple inputs and outputs to calculate technical efficiency. By identifying the 'peers' for organizations which are not observed to be efficient, it provides a set of potential role models that an organization can look to, in the first instance, for ways to improve its operations. However, like any empirical technique, DEA is based on a number of simplifying assumptions that need to be acknowledged when interpreting the results of DEA studies. There are different types of data envelopment models developed in the literature in order to measure efficiency.

1.2 Literature review:

The measurement of technical efficiency started with the works of Debrau (1951) and Koopmans (1951), followed by Farrell (1957). Farrell implemented the first measure of technical efficiency. It leads to two different notions of technical efficiency that emerged to measure the efficiency of decision making units. Debrau-Farrell technical efficiency seeks maximum equi-proportionate increase in all outputs or equi-proportionate reduction in all inputs. Initially, Farrell's measure was implemented in the linear programming formulation which gave rise to the DEA models; the CCR (Charnes, Cooper and Rhodes) developed by Charnes *et al.* (1978) with more precision and suggested a way of dealing with efficiency in practice. They defined efficiency and justified the necessity for a relative measure rather than absolute measure of efficiency. Further, the DEA model is extended to variable returns to scale (VRS) by Banker *et al* (1984) which is termed as the BCC (Banker, Charnes and Cooper) model. The returns to scale (RTS) classification of DMUs has been the subject of study by numerous authors, including Banker (1984) using free variable; and Fare *et al.* (1999) applying scale efficiency index method. Zhu and Shen (1995) suggested a method for the CCR RTS method under multiple optima. Seiford and Zhu (1997) reviewed the various methods and suggested computationally simple methods to characterize RTS. Due to the radial nature of CCR and BCC models, efficiency scores obtained from these models overstate efficiency when nonzero slacks are present because they do not account for non-radial inefficiency of the slacks. In this model, efficient radial projection of an observed input-output bundle onto frontier does not necessarily exhaust the potential for expansion in all outputs or potential reduction in all inputs. Projected point may have input or output slacks. This measure of efficiency is called "weak efficiency" or also called "Farrell efficiency" [see Cooper *et al.* (2007)].

In order to overcome these drawbacks, second notion of technical efficiency i.e Pareto-Koopmans efficiency based on the additive model of the DEA developed by Charnes *et al.* (1985) accounts for all sources of inefficiency. A DMU_o is Pareto-Koopmans efficient if efficiency is equal to one and all the slacks are zero otherwise the DMU_0 is inefficient. A DMU_0 is fully efficient if and only if none of its inputs or outputs can be improved without worsening some of its other outputs. In the Debrau-Farrell measures of technical efficiency, slacks may remain at optimal solution. A DMU_0 is Pareto-Koopmans efficient if there is no slack in the optimal solution. The additive DEA model does not directly provide an efficiency measure and the measure is unit variant. To sort out these problems, several DEA models to measure the inefficiency have been discussed in the last few years. Tone (2001) introduced slacks-based measure (SBM) which is invariant to the units of measurement. Fare and Lovell (1978) introduced the input oriented non-radial measure of technical efficiency, which they called the Russell measure. This model is restricted to input oriented. The input (output) oriented non-radial measures ignore output (input) slacks present at the optimal solution. Only the Pareto-Koopmans measure of efficiency ensures that neither input nor output slacks will be present at the optimal solution of the relevant DEA model. The combination of the input and output Russell measures of technical efficiency is developed by Fare *et al.* (1985). Further, Pastor *et al.* (1999) revised Russell measure and referred it as the enhanced Russell measure. This is equivalent to Tone's SBM as discussed in Cooper *et al.* (2006). Range adjusted measure (RAM) model of Cooper *et al.* (1999) is similar to the additive model with additional feature that the efficiency score lies between zero and one. There are also non-radial models employed as a second stage in a two stage efficiency analysis. For more details, see Cooper *et al.* (2001), Portela and Thanassoulis (2007), Portela *et al.* (2003) and others.

There are areas of research within DEA literature aimed at addressing the problem of unacceptable weights for the variables in the efficiency measurement. Charnes *et al.* (1990), in their study on large industrial banks, recognized that undesirable weighting schemes are the natural outcome in many DEA applications. To provide more realistic multipliers, they imposed a set of linear restrictions that define a convex cone. Dyson and Thanassoulis (1988) and Roll *et al.* (1991) used the absolute lower and upper bounds on input and output multipliers. Thomson *et al.* (1986) introduced assurance region (AR) a special case of cone ratio idea. Many applications of the AR form of various DEA models can be found in the literature. Various generalization on the AR concept appeared in the literature such as Thomson *et al.* (1990). Allen *et al.* (1997) and Thanassoulis and Allen (1998) present a global type of restriction on the weighted values for each DMU. Cook and Zhu (2008) present a convex dependent assurance region DEA. Bessent *et al.* (1988) were the first to introduce the idea of constrained facet analysis. Lang *et al.* (1995) improved on these ideas by adopting a two-stage approach. Similar approaches have been suggested by Green *et al.* (1996), Olesen and Petersen (1996).

In many applications of DEA, some of the input variables may not be under the control of management. Banker and Morey (1986a) first introduced the DEA model that allowed for non-discretionary inputs by modifying the input constraints. Ruggiero (1996) has pointed out in some cases, the Banker and Morey (1986a) model can overestimate technical efficiency by allowing production impossibilities into reference set. Ruggiero's approach restricts weights to zero for production possibilities with higher levels of the non-discretionary inputs. Recent simulation analysis in Syrjanen (2004) and Muniz *et al.* (2006) demonstrates that Ruggiero's method performs relatively well in evaluating efficiency. Also, there are situations in which inputs and outputs of DMUs fall into natural categories. Initially, Banker and Morey (1986b) incorporated the categorical variable into DEA formulation, to which Kamakura (1988) contributed a variation to its mixed integer linear programming formulation (MILP). Subsequently, Rousseau and Semple (1993) improved the shortcomings in Kamakura's method. In recent times, Lozano and Villa (2006) pioneered work on integer inputs and outputs in DEA by incorporating integrality constraints to DEA framework and developed MILP procedure for solving DEA with integer variables. However, their model violated the DEA properties of convexity, free disposability and returns to scale. As a consequence, this model was not in agreement with minimum extrapolation principle of Banker *et al.* (1984) and it also overestimates the efficiency of evaluated DMU. In order to succeed over these limitations, Kuosmanen and Kazemi Matin (2009) developed axiomatic approach to DEA formulation in constant returns to scale. Further, Kazemi Matin and Kuosmanen (2009) extended to variable returns to scale.

The usual choice of input and output variables in DEA performance measurement is such that higher value of variable is better for outputs and lower value of variable is better for inputs. In some cases, a factor can behave opposite to this, for example in production of pollutant plants. This type of issue has been addressed by number of authors in practice. Scheel (2001), Seiford and Zhu (2002), Fare and Grosskopf (2004) and Hua and Bin (2007). In the literature, directional distance functions approach is used to address this type of problem. This method was originated by Luenberger (1992) in a way to introduce distance function as benefit function followed by Chambers *et al.* (1996) who developed directional distance function. The standard directional distance function formulation of DEA to measure the technical efficiency is developed by Chambers *et al.* (1998) and by Fare and Grosskopf (2000). These models are restricted to non-negative data. Further, Portela *et al.* (2004) developed the range directional model to handle unrestricted data.

Another branch of discussion made in the thesis is ranking of efficient DMUs. The main drawback of DEA is that substantial number of DMUs comes out to be efficient, particularly when the number of DMUs is less compared to number of inputs and outputs. There are several ranking methods in DEA literature. Sexton *et al.* (1986) started the subject of ranking in DEA by finding cross efficiency. Further, Doyle and Green (1994) developed the idea of maverick index within the scope of cross efficiency. Andersen and Petersen (1993) applied super-efficient method to discriminate efficient units. Hashimoto (1997) developed a DEA super-efficient model with assurance regions. Torgersen *et al.* (1996) computed a complete ranking of efficient DMUs by measuring their importance as a benchmark for inefficient DMUs. Sinuany-Stern *et al.* (1994) used linear discriminant analysis to find a score function that ranks DMUs.

Since inception of DEA by Charnes *et al.* (1978), it has been used to evaluate performance in a variety of problems. The schools performance is compared by Ramanathan (2001), health care systems by Roberts *et al.* (2004) and very rare applications like performance of portfolio by Joro and Paul (2006), assess the financial stability of air fleet participants by Bowlin (2004) based on financial ratios, performance of auto component industry by Saranga (2009). Furthermore, DEA was used by Cielen *et al.* (2004) and recently Premachandra *et al.* (2009) to predict bankruptcy.

In health care efficiency measurement, various methods of assessment are used. Mention a few; stochastic frontier, fixed effects, regression models and simple ratios of health outcomes and DEA. Among these, non-parametric approach DEA has gained more importance (see Cooper *et al.* 2004). The application of DEA in health system was initiated by Nunamaker (1983), but the use of DEA in health care research gained popularity in 1990. Hollingsworth *et al.* (1999) reviewed 91 studies of DEA applications on healthcare. These studies were carried on measured efficiency of health districts, hospitals, nursing homes, primary health care, etc. More recently, Mirmirani *et al.* (2008) applied DEA technique for measuring health care efficiencies of translation economies country. Bhat *et al.* (2001) conducted the study by analysing the hospital efficiency of district level government and grant-in-aid hospitals in Gujarat, comparing their relative efficiency using DEA approach. In another study carried out by Dash *et al.* (2007), DEA was applied to determine the efficiency of a set of district hospitals in Tamilnadu.

DEA approach was also applied to measure the human development index (HDI) and benchmark poor performing countries against the best practices of countries by Mahlberg and Obersteiner (2001). Despotis (2004) reassessed the HDI by introducing the transformation paradigm. Arcelus *et al.* (2005) applied DEA to assess each country's resource allocation policies to produce an HDI. Recently, Lozano and Gutierrez (2008) applied RAM of DEA model to measure HDI. A few basic DEA models are discussed in the following sections in greater detail.

1.3 Constant returns to scale or Charnes, Cooper, Rhodes (CCR) Model:

DEA is a linear programming based technique for measuring the relative performance of homogenous organizational units where the presence of multiple inputs and outputs makes comparison difficult. The efficiency score in the presence of multiple input and output factors is defined as

$$Efficiency = \frac{\text{weighted sum of outputs}}{\text{weighted sum of inputs}} \qquad (E\ 1.1)$$

1.3.1 CCR multiplier Model:

Assume that there are a set of 'n' DMUs, each operating with 'm' inputs and 's' outputs. Let y_{rj} be the amount of r^{th} output and x_{ij} be the amount of its i^{th} input from the DMU$_j$. Let $x_j = (x_{1j}, x_{2j}, \dots x_{nj})$ be the vector of inputs for j^{th} DMU and $y_j = (y_{1j}, y_{2j}, \dots y_{nj})$ be the vector of outputs for j^{th} DMU for $j = 1,2, \dots n$. All data are assumed to be non-negative but at least one component of every input and output vector is positive and is given by $x_{ij} \geq 0$, for all $i = 1,2,\dots,m$ and $x_j \neq 0$ for all $j = 1,2,\dots,n$; $y_{rj} \geq 0$, for all $r = 1,2,\dots,s$ and $y_j \neq 0$ for all $j = 1,2,\dots,n$. Then the set of feasible activities called the production possibility set is defined as

$$T_{CRS}=\left\{(x,y):x\geq\sum_{j=1}^{n}\lambda_j x_j \text{ and } y\leq\sum_{j=1}^{n}\lambda_j y_j;\lambda_j\geq 0 \text{ for } j=1,2,...,n.\right\} \quad (E\ 1.2)$$

Then the relative efficiency score of DMU_0 which is one of the DMUs among j=1,2,...n is evaluated by solving the following model proposed by Charnes *et al.* (1978). To solve the model, a linear programming formulation, which is known as CCR (Charnes, Cooper, Rhodes) model is used. This maximizes ratio of weighted average of outputs to weighted averages of inputs, subject to constrains on output and input variables.

$$\left.\begin{aligned} &Max\ \theta = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\ &subject\ to \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad j=1,\cdots,n \\ &v_i, u_r \geq 0 \end{aligned}\right\} \quad \langle M\ 1.1\rangle$$

The ratio of outputs to inputs is used to measure the relative efficiency of the DMU_0 to be evaluated relative to the ratios of all the DMUs. The construction of CCR model makes reduction of multiple outputs/multiple inputs situation to that of a single virtual output/input for each DMU under evaluation. Thus ratio provides a measure of efficiency. In a mathematical programming this ratio is to be maximized that forms the objective function. Ratios of virtual output to virtual input should not exceed one for every DMU forms the constraints. By virtue of these constraints, the optimal objective value of θ^* is at most one. Applying Charnes and Cooper (1962) transformation for linear fractional programming can be converted to equivalent linear programming problem and results in

$$\left.\begin{aligned} &Max\ \ \theta_{CCR} = \sum_{r=1}^{s} u_r y_{ro} \\ &subject\ to\ \ \sum_{i=1}^{m} v_i x_{io} = 1 \\ &\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \ \ j=1,\cdots,n \\ &v_i \geq 0,\ \ i=1,2,\cdots,m;\ u_r \geq 0,\ \ r=1,2,\cdots,s \end{aligned}\right\} \quad \langle M\ 1.2\rangle$$

This model is called the CCR multiplier model.

1.3.2 CCR Envelopment Model:

The dual of the $\langle M\ 1.2 \rangle$ is given below and the objective function is expressed as real variable θ and a nonnegative vector $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)$ as the decision variables.

$$\left.\begin{aligned} &\theta^{*}_{CCR} = \min\ \theta \\ \textit{subject to}\ &\sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io} \quad i = 1,2,\cdots,m \\ &\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} \quad r=1,2,\cdots,s \\ &\lambda_j \ge 0,\ j=1,2,\cdots,n \end{aligned}\right\} \quad \langle M\ 1.3 \rangle$$

The model $\langle M\ 1.3 \rangle$ has a feasible solution $\theta = 1$, $\lambda_o = 1, \lambda_j = 0\ (j \ne o)$. Hence optimal θ, θ^{*}_{CCR}, is not greater than one as the objective is to minimize θ. The constraints of $\langle M\ 1.3 \rangle$ require the activity $(\theta x_{io}, y_{ro})$ to belong to T_{CRS}, while the objective seeks minimum θ that reduces the input vector x_{io} radially to θx_{io}. It seeks for an activity in T_{CRS} that guarantees reference outputs at least at the level y_{ro} of DMU_o in all components and reducing the input vector proportionally to a value as small as possible. So, $\left(\sum_{j=1}^{n} \lambda_j x_{ij}, \sum_{j=1}^{n} \lambda_j y_{rj}\right)$ outperforms $(\theta x_{io}, y_{ro})$ when $\theta < 1$. The model $\langle M\ 1.3 \rangle$ is referred as CCR input oriented envelop model. Sometimes this model also referred as the "Farrell model" because it is used in the activity analysis by Farrell (1957). This model ignores the presence of non-zero slacks and in DEA literature this is termed as "weak efficiency".

1.3.3 Slack adjusted CCR model:

The slack adjusted CCR model includes the input and output slacks in the model and can be given as

$$\left.\begin{aligned}
&\theta^* = \min\ \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right) \\
&\textit{subject to} \\
&\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{io} \quad i = 1,2,\cdots,m \\
&\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \quad \mathrm{r}=1,2,\cdots,\mathrm{s} \\
&\lambda_j, s_i^-, s_r^+ \geq 0, \;\; \mathrm{j}=1,2,\cdots,\mathrm{n}
\end{aligned}\right\} \qquad \langle M\ 1.4\rangle$$

where ε is a non-Archimedean infinitesimal, is an arbitrary small positive real number and is problem specific. Solution to the model is carried out in two stages. Optimizing on θ in the first stage, with this value fixed, a second stage is then used to maximize the slacks. If any slacks are not zero, the associated $u*$ or v * in the dual (multiplier) model will then also involve ε with these slacks values as coefficients. Here a $\mathrm{DMU_0}$ is efficient if and only if $\theta^* = 1$ and $s_i^- = s_r^+ = 0$ for all '*i*' and '*r*'. $\mathrm{DMU_0}$ is weekly efficient if $\theta^* = 1$ and $s_i^- \neq 0$ and/or $s_r^+ \neq 0$ for some '*i*' and '*r*'. If an optimal solution $\left(\theta^*, \lambda^*, s_i^{-*}, s_r^{+*}\right)$ of the model $\langle \mathrm{M}\ 1.4\rangle$ satisfies $\theta^* = 1$ and $s_i^- = s_r^+ = 0$ then $\mathrm{DMU_0}$ is called CCR efficient. Otherwise, the $\mathrm{DMU_0}$ is called CCR inefficient.

The condition $\theta^* = 1$ is referred to as radial efficiency. $\theta^* < 1$ means that all inputs can be simultaneously reduced without altering the mix of radial and non-radial reduction. $\left(1 - \theta^*\right)$ is the maximal proportionate reduction of inputs allowed. Any further reductions associated with nonzero slacks will necessarily change the input proportions. The inefficiencies associated with nonzero slacks are termed as mix inefficiencies. A DMU is Pareto-Koopmans efficient if and only if it is not possible to improve any input or output without worsening some other input or output. For an inefficient $\mathrm{DMU_0}$, one can define its reference set E_o, corresponding to the positive λ_j's at the optimal solution. Therefore the reference set E_o is

$E_o = \left\{ j \mid \lambda_j^* > 0, j \in 1,2,...\mathrm{n} \right\}$.

1.3.4 Output oriented CCR model:

There is another type of model that attempts to maximize the outputs for a given level of inputs. This is referred as the output oriented model and can be formulated as

$$\left.\begin{aligned} &\delta^* = \max\ \delta \\ \textit{subject to}\quad &\sum_{j=1}^{n} \mu_j x_{ij} \le x_{io} \quad ;\ i = 1,2,\cdots,m \\ &\sum_{j=1}^{n} \mu_j y_{rj} \ge \delta y_{ro} \ ;\ \ r = 1,2,\cdots,s \\ &\mu_j \ge 0 \qquad\qquad ;\ j = 1,2,\cdots,n \end{aligned}\right\} \qquad \langle M\ 1.5\rangle$$

In the model $\langle \text{M } 1.5\rangle$ the higher the value of δ^*, the less efficient the DMU and δ^* expresses the output enlargement rate, θ^* describes the input reduction rate. An optimal solution of the model $\langle \text{M } 1.5\rangle$ can be directly derived from the optimal solution of input oriented CCR model given in $\langle \text{M } 1.3\rangle$ as follows $\lambda = 1/\mu,\ \theta = 1/\delta$. Therefore, an input oriented CCR model will be efficient for any DMU if and only if it is also efficient when performance is evaluated under the output oriented CCR model.

1.4 Variable Returns to Scale or Banker, Charnes, Cooper (BCC) Model:

Variable returns mean that we might get different levels of output due to reduced performance or economics of scale. The constant returns to scale operation is the most limiting factor for the use of DEA. Many economists viewed this assumption as over restrictive and Banker *et al.* (1984) proposed variation in the DEA model, which we call Banker, Charnes, Cooper (BCC) model. They made simple but remarkable modification to the CCR DEA model in order to handle variable returns to scale. The efficiency computed from BCC model is pure technical efficiency, whereas the one from CCR model is the aggregate measure of technical and scale efficiency. If we add a variable to the model, we can construct a DEA model with the variable return to scale. The production possibility set for variable returns to scale is defined as

$$T_{VRS} = \left\{ (x,y) : x \geq \sum_{j=1}^{n} \lambda_j x_j \text{ and } y \leq \sum_{j=1}^{n} \lambda_j y_j ; \sum_{j=1}^{n} \lambda_j = 1;\ \lambda_j \geq 0,\ j = 1,2,...n. \right\} \quad (E\ 1.3)$$

1.4.1 BCC multiplier Model:

The input oriented VRS multiplier model for evaluating the efficiency of the DMUs as developed by Banker *et al.* (1984) is given by

$$\left.\begin{aligned}
& Max \quad \theta_{BCC} = \sum_{r=1}^{s} \mathrm{u}_r y_{ro} + t \\
\text{subject to} \quad & \sum_{i=1}^{m} \mathrm{v}_i x_{io} = 1 \\
& \sum_{r=1}^{s} \mathrm{u}_r y_{rj} - \sum_{i=1}^{m} \mathrm{v}_i x_{ij} + t \leq 0, \quad j = 1, \cdots, n \\
& \mathrm{v}_i \geq 0 \quad i = 1,2,\cdots,m \\
& \mathrm{u}_r \geq 0 \quad r = 1,2,\cdots,s \\
& t \text{ is unrestricted}
\end{aligned}\right\} \quad \langle M\ 1.6 \rangle$$

1.4.2 BCC Envelopment Model:

The dual of the $\langle M\ 1.6 \rangle$ is expressed as real variable θ and a nonnegative vector $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)$ of the variables as follows

$$\left.\begin{aligned}
& \theta^*_{BCC} = \min\ \theta \\
\text{subject to} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io} \quad ; i = 1,2,\cdots,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} \quad ; r = 1,2,\cdots,s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0 \quad ; j = 1,2,\cdots,n
\end{aligned}\right\} \quad \langle M\ 1.7 \rangle$$

DMU_0 is BCC efficient if $\theta^*_{BCC} = 1$ and all the slacks are zero, otherwise the DMU is inefficient. A DMU_0 is CCR efficient if it is BCC efficient but converse is not true. The efficiency computed from BCC model is pure technical efficiency, whereas the efficiency from CCR model is the aggregate measure of technical and scale efficiency. Therefore, pure scale efficiency can be defined as the ratio of CCR efficiency over BCC efficiency. The other oriented BCC models are similar to CCR model.

1.5 Alternative DEA models:

The DEA models cited so far can be termed as basic DEA models. Several variations of the basic DEA model have been proposed in the literature over the years. The literatures on modification of the basic DEA models are extensive and growing. We shall restrict our discussion to additive DEA model, and slack based model (SBM) in this chapter.

1.5.1 Additive DEA model:

So far we have discussed input-oriented and output-oriented DEA models. Now, we combine both orientations in a single model called additive model to evaluate the technical inefficiencies developed by Charnes *et al.* (1985) and is given by

$$\left.\begin{aligned} & Max \quad \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \\ \text{subject to} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} \quad ; \quad i = 1,2,\cdots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \quad ; \quad r = 1,2,\cdots,s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j, s_i^-, s_r^+ \geq 0; \ \forall\, i,j,r; \ j = 1,2,\cdots,n \end{aligned}\right\} \qquad \langle M\ 1.8 \rangle$$

The objective function of the model $\langle$M 1.8$\rangle$ simultaneously maximizes outputs and minimizes inputs. This model uses the metric that differs from one used in the radial measure model. Here, a DMU_0 is efficient if and only if all the slacks are zero in an optimum solution, otherwise the DMU_0 is inefficient. This model is translation invariant but not unit invariant. Although this model can discriminate efficient from the inefficient DMUs, it has no means of gauging the depth of inefficiency.

1.5.2 Slack based model (SBM):

Tone (2001) developed non-radial, non-oriented and slack based model (SBM) by elaborating the additive DEA model. This model is translation and unit invariant.

The efficiency measure of this model is ranging between zero and one. The SBM model can be formulated as follows

$$\left.\begin{aligned}
&\min S(x_o, y_o) = \varphi^* = \frac{1 - \frac{1}{m}\sum_{i=1}^{m}\left(s_i^{-*} / x_{io}\right)}{1 + \frac{1}{s}\sum_{r=1}^{s}\left(s_r^{+*} / y_{ro}\right)} && (1.9a)\\
&\textit{subject to} \quad \sum_{j=1}^{n} \lambda_j^* y_{rj} - s_r^{+*} = y_{ro} && (1.9b)\\
&\sum_{j=1}^{n} \lambda_j^* x_{ij} + s_i^{-*} = x_{io} && (1.9c)\\
&\sum_{j=1}^{n} \lambda_j^* = 1 && (1.9d)\\
&\lambda_j \geq 0,\ s_r^+, s_i^- \geq 0\\
&j = 1, 2, \cdots, n;\ i = 1, 2, \ldots, m;\ r = 1, 2, \ldots, s
\end{aligned}\right\} \quad \langle M\ 1.9\rangle$$

The SBM can be transformed into a linear program using Charnes-Cooper transformation in the similar way as in the CCR model. A DMU_o is SBM efficient if $s_i^{-*} = 0$ and $s_r^{+*} = 0$, for all i=1,2,...,m and for all r=1,2,...,s. This is equivalent to $\varphi^* = 1$ otherwise the DMU_o is inefficient.

1.6 Directional distance formulation of DEA:

Over the last few years, DEA has been used through directional distance formulation in the measurement of efficiency. Specifically, this approach is used where performance measurement of the production units involves negative outputs and inputs. In the presence of negative data, the use of radial measures of efficiency traditionally used in DEA is problematic. Positive radial expansion factors applied to negative data lead in the opposite direction to the one would wish to improve the performance. The treatment of undesirable outputs has similarities with the treatment of negative outputs since both should be contracted rather than expanded. Any output which incurs cost instead of revenue can also be considered as undesirable outputs. Several approaches exist to deal with undesirable outputs as can be seen in the literature. Chambers *et al.* (1996) introduced the directional distance function based on Luenberger benefit function to obtain a measure of technical efficiency reflecting the potential for increasing the outputs while reducing the inputs simultaneously. The directional distance function measures the amount that one can translate an input and/or output of a DMU radially from itself to the technology frontier in a pre-assigned direction. This issue is addressed in Chung *et al.* (1997), they resolve it by solving two BCC models; one where the objective function is maximized and another where it is minimized. This approach is, however, only valid when all the units have negative values on a variable. Later Scheel (2001) extended this approach for the case of undesirable outputs. One of the approaches is based on the directional distance function and was first proposed by Chung *et al.* (1997).

To illustrate the point, consider the example in Figure-1.1 where two outputs are taken and all DMUs have the unit input. Assessing the efficiency of unit 'C' using the radial output oriented BCC model (Banker *et al.* (1984)), implies an expansion of both outputs by a multiple greater than one. This however, implies a movement of the inefficient unit 'C' to the frontier in the direction shown in Figure-1.1. This movement is not desired since the negative output is being expanded making it even worse. Clearly positive radial expansion factors applied to negative data lead in the opposite direction to the one we would wish to follow to improve the performance.

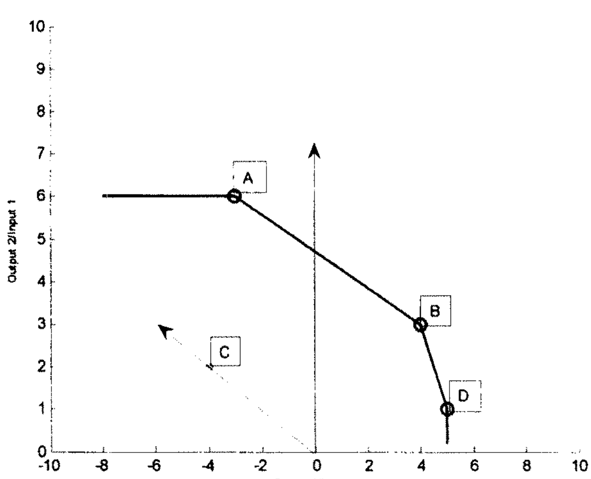

Figure-1.1: Illustration with negative data.

Translation invariant of the additive DEA model is subject to it being specified under VRS. Banker *et al.* (1984) showed that CRS models are not translation invariant because they do not impose the sum of the intensity variables (weights corresponding to reference DMU) equal to one i.e. dual decision variables $\sum_{j=1}^{n}\lambda_j = 1$. In the presence of negative data the CRS model cannot be used. This model assumes any activity can be radially expanded or contracted to form other feasible activities.

Consider a set of units j=1,2,...,n, with input levels x_{ij} , i=1,2,...,m and output levels y_{rj}, r=1,2,...s, and DMU$_o$ which is to be assessed. Let $x_j \in R^m$ denote inputs and $y_j \in R^s$ outputs $i = 1,2,...,m;\ r = 1,2,...,s; j = 1,2,...,n$. The production technology under the assumption of convexity and free disposability of inputs and outputs is given by as in $(E\ 1.3)$

Consider a pair of input-output bundle (x_o, y_o) and the reference input and output bundle be (g_x, g_y). Then with reference to some production possibility set, T, the directional distance function can be defined as

$$\vec{D}(x_o, y_o; g_x, g_y) = Max\ \beta : (x_o - \beta g_x, y_o + \beta g_y) \in T$$
$$if\ (x_o - \beta g_x, y_o + \beta g_y) \in T\ for\ some\ \beta \qquad (E\ 1.5)$$

The directional distance function evaluated at any specific input-output bundle will depend on reference input-output bundle ($-g_x$, g_y) i.e the direction vector. The directional distance function measures the amount that one can translate an input and or output of a DMU radially from itself to the technology frontier in a pre assigned direction.

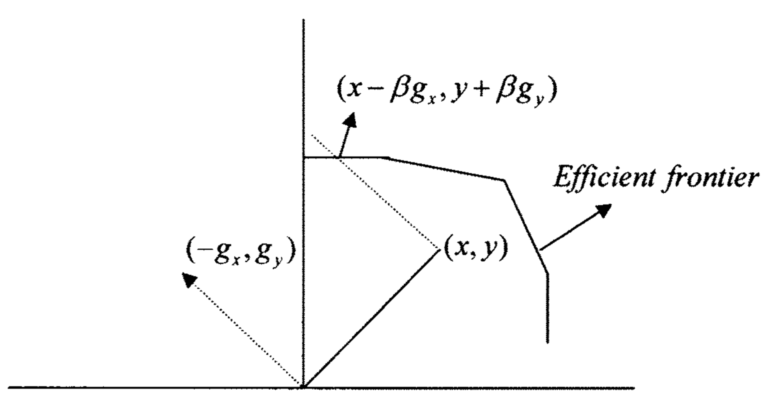

Figure-1.2: Single input and output directional distance function.

In other words, we seek to increase the output and reduce the input by the proportion of β. This is illustrated diagrammatically in Figure-1.2.

Distance functions viewed as applied tools in performance measurement. In DEA one frequently distinguishes between input and output orientations i.e between input and output distance functions. These functions are reciprocal to each other. The input distance function is associated with input minimization and output distance function is associated with output maximization. The direction in which the traditional input and output distance functions evaluate efficiency is determined by the input-output data from an observation or DMU itself. For the directional distance functions the direction in which DMUs are to be evaluated is a choice variable. These distance functions allow the researcher to choose the direction based on certain criteria to estimate technical efficiency. This may be an important consideration in some applications, e.g. when desirable and undesirable outputs are jointly produced and in the case of negative variables.

The VRS DEA formulation for the directional distance function for the production possibility set is developed by Chambers *et al.* (1998) is

$$\left.\begin{aligned}
&\qquad Max\ \beta \\
&subject\ to\ \sum_{j=1}^{n}\lambda_j y_{rj} - \beta y_{ro} \ge y_{ro} && (1.10a)\\
&\qquad \sum_{j=1}^{n}\lambda_j x_{ij} + \beta x_{io} \le x_{io}\ ; && (1.10b)\\
&\qquad \sum_{j=1}^{n}\lambda_j = 1, \qquad \lambda_j \ge 0; && (1.10c)\\
&\qquad x_{io}, y_{ro} \ge 0 && (1.10d)\\
&\qquad \beta \text{ unrestricted}\\
&\qquad i = 1,2,\cdots,m;\ r = 1,2,\cdots,s;\ j = 1,2,\cdots,n
\end{aligned}\right\} \quad \langle M\ 1.10\rangle$$

The factor β is the measure of technical inefficiency of the unit and its efficiency is $(1-\beta)$.

1.7 Ranking in DEA:

The main drawback of DEA is that substantial number of DMUs comes out to be efficient, particularly when the number of DMUs is less compared to number of inputs and outputs. Several researchers worked to address this problem by way of discriminating the efficient DMUs. There are several ranking methods in DEA literature. Adler *et al.* (2002) have reviewed six groups of methods to fully rank both efficient and inefficient DMUs. The first group evaluates cross efficiencies (CE) by which DMUs are self and peer evaluated. The second group measures the super efficiency (SE) method in which the evaluated unit is excluded from the reference set. The third group is based on benchmarking, in which a DMU is highly ranked if it is chosen as a benchmark for many other inefficient DMUs. The fourth group utilizes multivariate statistical techniques, such as discriminant analysis and canonical correlation analysis. The fifth group ranks the DMUs through the proportional measure of inefficiency. The sixth group ranks the DMUs based on multiple criteria decision methodologies with DEA approach. However, it may be noticed that each of these techniques may be useful in a specific area while no one methodology of ranking is a complete solution in itself to the ranking methods. We will discuss a few methods of ranking efficient DMUs in detail in Chapter-5 and a new approach of ranking efficient DMUs is introduced.

1.8 Summary of the thesis:

Chapter-2 discusses developing a model known as Aggregate Directional Distance Formulation of Data Envelopment Model (ADDM). The radial measure of efficiency overestimates technical efficiency when there are nonzero slacks in the constraints. Proposed ADDM model takes into account of individual input and output variables for inefficiency measure. We defined a non-proportional, non-oriented efficiency measure. Additionally, we establish some properties of this model. Further, integer directional distance function is introduced in the chapter. Usual directional distance formulation of DEA and ADDM projection of efficient targets for inefficient DMUs generally have non-integer values. ADDM is modified and mixed integer aggregate directional distance formulation of DEA (MIADDM) is proposed. This model guarantees integer targets for inefficient DMUs and is applicable to measure efficiency where input and output variables are negative integer values.

In Chapter-3, we propose an improved model for efficiency measure through directional distance formulation by modifying ADDM. The model considers the objective function as product of input and output efficiencies. It measures the weighted average inefficiency of inputs and outputs. This model measures more precise efficiency score. It is possible to measure the efficiency in different direction by changing the value of direction vectors. The dual problem of this model gives the economic interpretation as virtual profit maximization / cost minimization problem. A few properties of the model were established. Some of the existing well known DEA models are deduced as special case of this model. Further, in this chapter by integrating DEA and canonical correlation, the efficient DMUs are discriminated.

Chapter-4 proposes modification in the improved efficiency measure through directional distance formulation of DEA discussed in Chapter-3 for predicting bankruptcy of firms. The conventional DEA or directional distance function measures the efficiencies of DMUs, and identifies the best performances of DMUs to determine an efficiency frontier. Contrary to the best relative efficiencies, we developed a model to measure worst relative efficiency within the range of zero and one. This identifies the worst performing DMUs and determines an inefficiency frontier. This idea of bankruptcy prediction is in consensus with economic theory. In this chapter we apply this method as a case study to information technology (IT) firms in India.

Chapter-5 contains ranking of efficient DMUs both for conventional DEA method by single virtual DMU and improved efficiency measure through directional distance formulation of DEA. In our proposed model, we create a virtual DMU whose inputs and outputs are averages of corresponding inputs and outputs of all the DMUs. Efficiency of this virtual DMU is the same as the efficiency of total outputs and inputs of corresponding variables in the CRS model. The inputs and outputs of the virtual DMU are not affected by extreme values and are also based on all the inputs and outputs of all the DMUs. Hence, in these cases, ranking the efficient DMUs with reference to average DMU is more meaningful. Using this idea a model is developed. In addition, certain properties of the model are also established..

Besides above approaches, a method to rank efficient DMUs in a system that involves some of the inputs and outputs represented by negative data is developed. In the presence of negative inputs/outputs, existing methods of ranking efficient DMUs cannot be used for directional distance approach of DEA formulation. Ranking of efficient DMUs is carried out based on directional distance formulation of DEA by applying the changing reference set technology model. The main idea proposed in this model is to rank efficient DMUs by excluding efficient DMUs from the reference set one by one and calculate the efficiency of inefficient DMUs. Further, average these efficiencies for each of the efficient DMUs and rank according to averaged efficiencies. In the study it is seen that this method discriminates the efficient DMUs more effectively. Certain properties of the model are also proved.

Chapter-6 contains real life application to measure health performances of major states of India with modification in ADDM to include uncontrollable variables under study. The main intention of this study is how efficiently different states utilise their resources to achieve health outcomes with consideration of socio-economic status of respective states. In this study we consider health care system of various states and identify states having higher health care efficiency based on infant mortality rate (IMR) and life expectancy at birth (LEB) as outputs and health centre per million populations (HCPMP), literacy rate (LR), percentage of population below poverty line (BPL) and per capita health expenditure (PCHE) as inputs. This study shows that present level of health outcomes of states is not necessarily indicative of how efficiently they utilise their resources to maximise outcomes. Some of the states utilise more resources than those of their efficient counterparts. These resources can be reallocated to those states that are efficient but poor in their health outcomes, in order to improve the health outcome of the nation as a whole. Since resources are limited, it is suggested to modify the policies of the government for allocating the resources to the states to improve the health outcomes.

Chapter-7 comprises the application of improved directional distance formulation of data envelopment model with appropriate modification to measure human development of Indian states. To address the issue of regional disparities among the states in the countries in terms of human well beings a method to measure the extent of disparity based on optimum combination of the indicators is developed. In this work we reassess the HDI through directional distance formulation of data envelopment analysis (DEA) of Indian states. It is found that the new measure of human development and the original HDI are highly correlated.

Further, we proposed a procedure to improve the performance of backward states in a stepwise approach. This procedure suggests for computing targets aiming to reduce the regional disparities systematically. Each poor performing state is compared with best practice states and its reference states are determined in order to achieve efficiency. Multistage reference technology, a new procedure is applied and it gives guidelines to policy makers and planners to formulate suitable plans, resource allocation and fix goals. This study also guides to allocate the amount of resources to each state in different sectors such as education, healthcare or economy of the state. Hence, this will reduce the disparities among the states in phase wise approach and leads to gradual decrease in inequality of human development between the regions.