Chapter-5

Ranking of Efficient Decision Making Units

5.1 Introduction:

The main objective of this chapter is to rank the efficient DMUs both for conventional DEA method by single virtual DMU and improved efficiency measure through directional distance formulation of DEA by changing reference technology. DEA model tried to make the DMU as efficient as possible by assigning favourable weights to inputs and outputs. This lead to substantial number of DMUs to be efficient, particularly when the number of DMUs is less compared to number of inputs and outputs. This is one of the main limitations of DEA. Several researchers worked to address this problem by way of discriminating the efficient DMUs through ranking. There are several ranking methods in DEA literature. Adler *et al.* (2002) have reviewed six groups of methods to fully rank both efficient and inefficient DMUs. The review concluded that each of these techniques may be useful in a specific area while no one methodology of ranking is a complete solution in itself to the ranking methods.

In order to handle the problems with negative data, we discuss approach of directional distance formulation of DEA in Section-5.6 onwards. Over the last few years, directional distance formulation of DEA has been used in the measurement of efficiency. This method was originated by Luenberger (1992) in a way to introduce distance function as benefit function. Following this, Chambers *et al.* (1998) developed the directional distance formulation of DEA to measure the technical efficiency. As in the case of DEA, directional distance formulation of DEA also provides the basis for discriminate DMUs into categorical classification as efficient and inefficient DMUs. However, inefficient DMUs can be ranked based on their observed efficiency score. But, this is not possible for the DMUs that are efficient. Therefore, there is a need for ranking the efficient DMUs. To overcome this limitation, we developed a procedure for ranking efficient DMUs in directional distance formulation of DEA in later part of this chapter. Attempts have been made to rank efficient DMUs in directional distance function formulation of DEA by applying the changing reference set technology model. Average re-evaluated efficiency of all inefficient DMUs is calculated for each of the efficient DMU when it is excluded from the reference set. The model discriminates and ranks the efficient DMUs effectively in the order of their merit. First part of the work discussed in this chapter has been published (see Shetty and Pakkala (2010a))

5.2 Ranking in DEA:

In this part, we proposed a new method of ranking efficient DMUs by introducing a single virtual DMU in DEA. This is ranked through measuring the efficiency of the virtual DMU by deleting efficient DMU one by one. The virtual DMU depends on all DMUs including inefficient DMUs. The efficient DMU is one that influences highest on the efficient frontier to get farthest in relation to the virtual DMU. Further, this method is compared with methods of cross efficiency (CE), supper efficiency (SE), and a model recently developed by Gholam Reza Jahanshaloo *et al.* (2007) for ranking the DMUs. Comparing to other methods, this method is simple, robust and gives good results in certain situations. Here, we discuss a few methods of ranking efficient DMUs in detail.

5.2.1 Cross-efficiency (CE) ranking methods:

The cross-evaluation matrix was developed by Sexton *et al.* (1986) who pioneered the subject of ranking in DEA. The CE method simply calculates the efficiency score of each DMU '*n*' times, using optimal weights evaluated by the '*n*' LPs. The results of all the DEA cross-efficiency can be summarized in a cross-efficiency matrix as shown below

$$e_{kj} = \frac{\sum_{r=1}^{s} u_{rk} y_{rj}}{\sum_{i=1}^{m} v_{ik} x_{ij}}, \quad k = 1,\cdots,n; \quad j = 1,\cdots,n \qquad (E\ 5.1)$$

e_{kj} represents the score given to unit '*j*' in the DEA run of unit '*k*' i.e. unit '*j*' is evaluated by the weights of unit k. All the elements in the matrix are between 0 and 1, $0 \le e_{kj} \le 1$ and elements in the diagonal e_{kk} represent the standard DEA efficiency score, $e_{kk} = 1$ for efficient units and, $e_{kk} < 1$ for inefficient units. The cross efficiency ranking method in the DEA context utilizes the results of the cross efficiency matrix e_{kj} in order to rank scale the units.

The average cross efficiency score given to unit '*k*' is

$$\bar{e}_k = \frac{\sum_{j=1}^{n} e_{kj}}{n} \qquad (E\ 5.2)$$

$\bar{e}_k$ is an equivalent or is more representative than e_{kk}, since all the elements of the cross efficiency matrix are considered. Furthermore, the optimal solution obtaining from linear programming is not unique. This implies that the weights produced by efficient DMUs in DEA formulation are arbitrary. Consequently, cross efficiencies so obtained could also differ and hence be quite misleading. To overcome this limitation, goal programming technique can be applied to choose between optimal solutions. Secondary goals may either be aggressive or benevolent formulations. The aggressive formulation assumes that given a choice among several optimal alternative solutions, it will choose weight such that its own efficiency rating is maintained and minimizes cross efficiency of other DMUs as much as possible. The benevolent formulation attempts to enhance cross efficiency ratings of other DMUs as much as possible.
The linearized surrogate model for benevolent cross efficiency formulation for the DMU$_o$ developed by Doyle and Green (1994) is given by

$$\left.\begin{array}{ll} Max & \sum_{r=1}^{s}\left(u_r \sum_{j\neq o}^{n} y_{rj}\right) \\ subject\ to & \sum_{i=1}^{m}\left(v_i \sum_{j\neq o}^{n} x_{ij}\right) = 1 \\ & \sum_{r=1}^{s} u_r y_{ro} - e_{jj}\sum_{i=1}^{m} v_i x_{io} = 0 \\ & e_{jo} \leq 1 \quad \text{for all } j \neq o \\ & u_r \text{ and } v_i \geq 0 \end{array}\right\} \qquad \langle M\ 5.1\rangle$$

Aggressive formulation of the cross efficiency model is the same as above model except objective function is to be minimized.

More recently, Liang *et al.* (2008) have extended the above model by developing three different secondary objective functions to determine the ultimate cross efficiencies. These three models which can be applied in different circumstances, give more or less same solutions. One of the alternative secondary goals is to minimize the total deviation. The benevolent cross efficiencies formulation model for the DMU$_d$ is given by

$$\left.\begin{aligned}
&\qquad Min \quad \sum_{j=1}^{n} \alpha_j' \\
&subject\ to \quad \sum_{r=1}^{s} u_r^d y_{rj} - \sum_{i=1}^{m} v_i^d x_{ij} + \alpha_j' = 0 \quad ; j = 1,2,\ldots,n \\
&\qquad \sum_{i=1}^{m} v_i^d x_{id} = 1 \\
&\qquad \sum_{r=1}^{s} u_r^d y_{rd} = 1 - \alpha_d' \\
&\qquad u_r^d, v_i^d, \alpha_j' \geq 0 \quad for\ all\ i,r,j \\
&\qquad \text{where } \alpha_d' \text{ is the CCR inefficiency of } DMU_d
\end{aligned}\right\} \quad \langle \text{M } 5.2 \rangle$$

The other two secondary goals are minimizing the maximum deviation efficiency score and minimizing the mean absolute deviation (see Liang *et al. (2008)*). These models can be used to know the stability of cross efficiency with respect to multiple DEA weights. These models with their different objective functions can be applied under different circumstances.

5.2.2 Super-efficiency ranking method (SE):

The Andersen and Petersen's super efficiency of input oriented Charnes, Cooper and Rhodes model (CCR) is given by

$$\left.\begin{aligned}
&\qquad Min \quad \theta_k \\
&subject\ to \quad \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j y_{rj} \geq y_{rk} \\
&\qquad \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j x_{ij} \leq \theta_k x_{ik} \\
&\qquad \lambda_j \geq 0, j = 1,2,\ldots n, \quad i = 1,2,\ldots m, \quad r = 1,2,\ldots s \\
&\qquad \theta_k \text{ unrestricted}
\end{aligned}\right\} \quad \langle \text{M } 5.3 \rangle$$

This method ranks all the efficient DMUs by super efficiency score greater than unity. The major drawbacks of this approach are the infeasibility and excessive high ranking of extreme DMUs. If infeasibility occurs, then the super efficient technique cannot provide a complete ranking of all DMUs. Thrall (1996) used this model to identify extreme efficient DMUs and has noted that super efficiency model may be infeasible.

5.2.3 Gholam Reza Jahanshaloo *et al.* ranking method (JJLA):

Gholam Reza Jahanshahloo *et al.* (2007) have proposed the method of ranking efficient DMUs based on the work of Hibiki and Sueyoshi (1999) on DEA cross reference model. Consider set {J} of all the DMUs, based on the conventional DEA model efficiencies of these DMUs are calculated and the efficient frontier is determined that identifies the efficient DMUs. These efficient DMUs are considered as reference set and are denoted by the set {B}. The main idea proposed in this model is to identify one of the extreme efficient DMUs by excluding efficient DMUs from the reference set one by one, calculate the efficiency of inefficient DMUs (set {J-B}), average these efficiencies for each of the efficient DMUs when it is excluded and rank them according to the average efficiencies. Extreme efficient DMU is one which comes closest to the remaining inefficient DMUs. The efficient DMU which influences the efficient frontier to get farthest from the remaining inefficient DMUs is classified as the best one.

In order to perform the above approach, Gholam Reza Jahanshaloo *et al.* (2007) modified DEA model as follows

$$\left.\begin{aligned}
&\min \delta_{a,b} = \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right) \\
\textit{subject to } &-\sum_{J\in j-b} \lambda_j x_{ij} + \theta\, x_{ia} - s_i^- = 0, \quad i = 1,2,\ldots,m \\
&\sum_{j\in J-b} \lambda_j y_{rj} - s_r^+ = y_{ra}, \qquad r = 1,2,\ldots,s \\
&\lambda_j \geq 0, s_i^- \geq 0, s_r^+ \geq 0, \qquad j = 1,2,\ldots,n \\
&\theta \text{ is unrestricted}
\end{aligned}\right\} \qquad \langle M\ 5.4\rangle$$

Where $J = \{1, 2,\ldots,n\}$ is the set of DMUs, $b \in \{B\}$ set of efficient DMUs, $a \in \{J\text{-}B\}$ set of inefficient DMUs.

5.3 Ranking efficient DMUs through virtual DMU in DEA (Proposed):

The users, like decisions makers/ managers/ social scientists of DEA methodology often prefer full ranking for the organizational units rather than simple dichotomic classification. There is a need to rank and scale the efficient units. Most of the applications of DEA are to measure the performance efficiency and to rank the

DMUs of non profit organization or commercial organizations. For example: comparative performance of schools, measurements of hospital efficiency, effect of environmental controls on productive efficiency of chemical industries, measurement of efficiency of various branches of the bank, efficiency measurement of different academic departments in a University, measurements of the efficiency of various municipalities and so on. In all these situations, DMUs fall under the control of the centralized decision makers who supervise these DMUs. The centralized decision maker provides the necessary resources as inputs in order to maximize the outputs. The decision-maker while interested in the efficiency of the each DMU, is also concerned about the overall production of outputs and consumption of the inputs.

In our proposed model, we create a virtual DMU whose inputs and outputs are averages of corresponding inputs and outputs of all the DMUs. Efficiency of this virtual DMU is the same as the efficiency of total outputs and inputs of corresponding variables in the CRS model. The inputs and outputs of the virtual DMU are not affected by extreme values and are also based on all the inputs and outputs of all the DMUs. Hence, in these cases, ranking the efficient DMUs with reference to average DMU is more meaningful. The main idea of our proposed model is to discriminate the efficient DMUs in such a way that when an efficient DMU is excluded from the reference set, the virtual DMU comes closer to the efficient frontier. The most efficient DMU is the one that influences highest on boosting the efficiency of virtual DMU. In order to rank the efficient DMUs and discriminate among these DMUs, we create a virtual DMU by averaging input and output variables. As stated by Wang and Yang (2007), such an imaginary DMU is called virtual DMU. Although the average DMU is a virtual DMU, it is a feasible DMU in practical production technology. We denote the virtual DMU as $x_{iAv}, \ldots i = 1,2 \ldots m$ and $y_{rAv}, \ldots r = 1,2,\ldots\ldots s$ are the average inputs and outputs of the all the DMU respectively.

$$x_{iAV} = \underset{j}{Average}(x_{ij}), \ldots i = 1,2,\ldots,m$$
$$y_{rAv} = \underset{j}{Average}(y_{rj}), \ldots r = 1,2,\ldots,s$$

Since the virtual DMU utilizes average inputs and outputs, if there exists at least one inefficient DMU, then virtual DMU falls farther from the efficient frontier. Hence it is inefficient.

Let x_{ij} and y_{rj} amounts of input *'i'* and output *'r'* respectively corresponding to DMU_j for i=1,2,...,m and r =1,2,...,s. Let $\bar{x}$ and $\bar{y}$ corresponding to vector columns for DMU_{Av}. The production possibility sets corresponding to CRS and VRS technologies are

$$T_{CRS} = \left\{ (x, y) : x \geq \sum_{i=1}^{n+1} \lambda_j x_j \; ; y \leq \sum_{i=1}^{n+1} \lambda_j y_j \; ; \lambda_j \geq 0 \; ; \; j = 1, 2, \ldots n+1 \right\} \qquad (E\ 5.3)$$

$$T_{VRS} = \left\{ (x, y) : x \geq \sum_{i=1}^{n+1} \lambda_j x_j \; ; y \leq \sum_{i=1}^{n+1} \lambda_j y_j \; ; \lambda_j \geq 0 \; ; \sum_{j=1}^{n+1} \lambda_j = 1 \; ; j = 1, 2, \ldots n+1 \right\} \qquad (E\ 5.4)$$

5.3.1 Constant Returns to Scale:

The input oriented constant returns to scale multiplier model for evaluating the efficiency of the efficient DMUs through single virtual DMU as stated by Shetty and Pakkala (2010a) for production possibility set defined in $(E\ 5.1)$ is

$$\left.\begin{array}{lll} & \delta_{Av,b} = Max \sum_{r=1}^{s} u_r y_{rAv} & (5.5a) \\ subject\ to & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j \in J - b & (5.5b) \\ & \sum_{r=1}^{s} u_r y_{rAv} - \sum_{i=1}^{m} v_i x_{iAv} \leq 0 & (5.5c) \\ & \sum_{i=1}^{m} v_i x_{iAv} = 1 & (5.5d) \\ & u_r \geq \varepsilon, \quad v_i \geq \varepsilon, \quad r = 1, 2, \ldots s; \quad i = 1, 2, \ldots m & (5.5e) \end{array}\right\} \quad \langle M\ 5.5 \rangle$$

Where $J = \{1, 2 \ldots n\}$ is the set of DMUs, $b \in \{E\}$ where $\{E\}$ is set of CRS efficient DMUs, Av corresponds to the inefficient virtual DMU. Our approach is to measure the impact of the efficient DMUs on the virtual DMU's efficiency, i.e. deviation of individual efficient DMU from the virtual DMU. An efficient $DMU_b \in \{E\}$ is deleted from the efficient frontier in order to find its effects on efficiency of single virtual DMU.

5.2.2 Variable Returns to Scale:

The input oriented variable returns to scale multiplier model for evaluating the efficiency of the efficient DMUs through single virtual DMU for production possibility set defined in (E 5.4) is given by

$$\left.\begin{array}{lll} & \delta_{Av,b} = Max\sum_{r=1}^{s} u_r y_{rAv} + t_b & (5.6a)\\ subject\ to & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + t_b \le 0, \quad j \in J-b & (5.6b)\\ & \sum_{r=1}^{s} u_r y_{rAv} - \sum_{i=1}^{m} v_i x_{iAv} + t_b \le 0 & (5.6c)\\ & \sum_{i=1}^{m} v_i x_{iAv} = 1 & (5.6d)\\ & u_r \ge \varepsilon, \ v_i \ge \varepsilon, \ r = 1,2,...s; \ i = 1,2,...m & (5.6e)\\ & t_b \ \ is\ unrestricted & \end{array}\right\} \quad \langle M\ 5.6 \rangle$$

Where $J = \{1, 2...n\}$ is the set of DMUs, $b \in \{E\}$ where $\{E\}$ is set of VRS efficient DMUs, Av corresponds to the inefficient virtual DMU.

5.2.3 Ranking procedure:

In our proposed approach, the basic idea is to compare the efficient units by evaluating the efficiency of virtual DMU with the linear combination of all the efficient units by excluding the efficient units one by one. Intuitively, this means unit 'b' is excluded from the frontier and $\delta_{Av,b}$ measures the distance of the virtual DMU from the new frontier. The original efficient frontier will change if the DMU_b is deleted and the new efficient frontier gets closer to the inefficient DMU. This score is always less than or equal to one. Hence based on the effects of efficient DMUs on virtual DMU, one can rank all the efficient DMUs.

Step 1: Find the efficiency of each DMU based on required model (either CRS $\langle M\ 1.2\rangle$ or VRS $\langle M\ 1.6\rangle$).

Step 2: Break the DMUs into two groups, the DMUs with efficiency score one i.e. efficient DMUs $E = \{1, 2,...e\}$ and DMUs with efficiency score less than one i.e. inefficient DMUs $I = \{e+1, e+2, ...n\}$

Step 3: Rank all the inefficient DMUs in descending order based on the efficiency score, so that maximum efficiency scored DMU receives the rank e+1 and the last one receives the rank 'n'.

Step 4: Rank the efficient DMUs by finding the efficiency score of virtual DMU when that efficient DMU is excluded from the reference set. Efficiency of virtual DMU is computed based on the proposed CRS or VRS model. Then efficient

DMUs are ranked according to efficiency score of virtual DMU. The efficient DMU that influences highest on the efficiency score of virtual DMU receives the rank 1 and which influences least receives the rank *'e'*. If there exists tie between efficiency score of virtual DMU when efficient DMU deleted from the reference set, these efficient DMUs can be discriminated by considering only tied efficient DMUs in the reference set and evaluating the efficiency of the virtual DMU.

Programs have been developed to find the efficiency scores for all the DMUs based on CRS or VRS model using the *MATLAB 7.7* and one main part of the program is given in the Appendix, Program-5.

5.4 Properties of the virtual DMU:

I. The virtual DMU is inefficient if there be at least one inefficient DMU in the set. The efficiency of the virtual DMU is less than unity i.e. $\delta^*_{Av} = Max\sum_{r=1}^{s} u_r y_{rAv} < 1$

Proof: The DEA model for efficiency measurement of virtual DMU is given below

$$\left.\begin{aligned}
&\delta^*_{Av} = Max\sum_{r=1}^{s} u_r y_{rAv} && (5.7a)\\
\textit{subject to}\quad &\sum_{i=1}^{m} v_i x_{iAv} = 1 && (5.7b)\\
&\sum_{r=1}^{s} u_r y_{rAv} - \sum_{i=1}^{m} v_i x_{iAv} \le 0 && (5.7c)\\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 && (5.7d)\\
&u_r \ge \varepsilon,\ v_i \ge \varepsilon && (5.7e)\\
&i = 1,2,\dots,m;\ r = 1,2,\dots,s;\ j = 1,2,\dots,n
\end{aligned}\right\} \quad \langle M5.7\rangle$$

The constraints guarantee that efficiency of all the DMUs bounded by unity,

$$e_o = Max\left(\sum_{r=1}^{s} u_r y_{ro} / \sum_{i=1}^{m} v_i x_{io}\right), \quad i = 1,\cdots,m;\ r = 1,\cdots s;$$

$$e_j = \left(\sum_{r=1}^{s} u_r y_{rj} / \sum_{i=1}^{m} v_i x_{ij}\right) \le 1, \qquad j = 1,\cdots,n$$

$$u_r \ge \varepsilon,\ v_i \ge \varepsilon,\ r = 1,2,\dots s;\ i = 1,2,\dots m$$

If at least one of the DMUs is inefficient i.e DMU_k then,

$$e_k < e_j = 1$$

This implies $e_1 + e_2 + \cdots + e_n < n$

The efficiency of DMU_{Av} is

$$e_{Av} = \left((e_1 + e_2 + \cdots + e_n)/n\right) < (n/n)$$
$$\Rightarrow e_{Av} < 1$$

Therefore the DMU_{Av} is inefficient.

II. For each efficient DMU $\{E\}$ $\delta^*_{Av,b} \geq \delta^*_{Av}$ $b \in E$

Proof: The feasible space of $\langle \text{M } 5.7 \rangle$ is a subset of the feasible space of $\langle \text{M } 5.5 \rangle$. Therefore the value of the optimal objective function of $\langle \text{M } 5.5 \rangle$ is greater than or equal to the optimal value of $\langle \text{M } 5.7 \rangle$. Hence by the fundamental theorem of duality implies $\delta^*_{Av,b} \geq \delta^*_{Av}$.

III. $\delta^*_{Av,b} = \delta^*_{Av}$, if and only if the dual model DMU_b is not the reference DMU for DMU_{Av}.

Proof: The optimal solution for the dual model of $\langle \text{M } 5.7 \rangle$, with $\lambda^*_b = 0$. This implies, in the dual model DMU_b is not the reference DMU for DMU_{Av} i.e. the optimal solution of $\langle \text{M } 5.7 \rangle$ corresponding to feasible solution of $\langle \text{M } 5.5 \rangle$. Delete the constraint $\sum_{r=1}^{s} u_r y_{rb} - \sum_{i=1}^{m} v_i x_{ib} \leq 0$ resulting in $\delta^*_{Av,b} \leq \delta^*_{Av}$. Now by the property II, defined above $\delta^*_{Av,b} = \delta^*_{Av}$.

Conversely, suppose $\delta^*_{Av,b} = \delta^*_{Av}$. We have to show that there is an optimal solution for equation $\langle \text{M } 5.7 \rangle$ with the dual model DMU_b is not the reference DMU for DMU_{Av} i.e. $\lambda^*_b = 0$. The optimal solution for model $\langle \text{M } 5.5 \rangle$ is the feasible solution for $\langle \text{M } 5.7 \rangle$ and $\delta^*_{Av,b} = \delta_{Av}$, where δ_{Av} is the objective function of equation $\langle \text{M } 5.7 \rangle$. Therefore $\delta_{Av} = \delta^*_{Av}$ from the assumption, this shows the latter solution is optimal for equation $\langle \text{M } 5.7 \rangle$. Then DMU_b is not reference for DMU_{Av}.

IV. The value of $\delta^*_{Av,b}$ is lies between 0 and 1

Proof: In $\langle \text{M } 5.7 \rangle$, the constraint $\langle 5.7\text{c} \rangle$ is

$$\sum_{r=1}^{s} u_r y_{rAv} - \sum_{i=1}^{m} v_i x_{iAv} \le 0,$$

$$\sum_{i=1}^{m} v_i x_{iAv} = 1 \text{ and } u_r \ge 0$$

At optimal solution, therefore

$$0 < \sum_{r=1}^{s} u_r y_{rAv} \le 1$$

Therefore from the fundamental theorem on LPP

$$0 < \delta^*_{Av,b} \le 1$$

V. If DMU$_b$ is not the reference set of DMU$_{Av}$. Then $\delta^*_{Av,b} < 1$

Proof: Since DMU$_b$ is not reference set of DMU$_{Av}$. Then $\lambda^*_b = 0$ for optimal solution of dual model each optimal solution of $\langle \text{M } 5.7 \rangle$corresponds to feasible solution of $\langle \text{M } 5.5 \rangle$.

Therefore $\delta^*_{Av,b} \le \delta^*_{Av}$.

Omitting λ^*_b from the feasible solutions $\langle \text{M } 5.7 \rangle$resulting to the above equation.

And from the property II $\delta^*_{Av,b} \ge \delta^*_{Av}$

This implies $\delta^*_{Av,b} = \delta^*_{Av}$, but from the property I, $\delta^*_{Av} < 1$

Therefore $\delta^*_{Av,b} < 1$

5.5 Numerical example:

Example 1: We now examine a numerical example using the model $\langle \text{M } 5.5 \rangle$ to illustrate the application of proposed model. Consider the example of production technology that utilizes two inputs to produce single output in Table-5.1.

DMU	1	2	3	4	5	6	7	AVERAGE
Input-1	4	7	8	4	2	10	3	5.43
Input-2	3	3	1	2	4	1	7	3
Output-1	1	1	1	1	1	1	1	1

Table-5.1: Data are taken from Cooper *et al.* (2007).

Applying CRS model $\langle M\ 1.2\rangle$ and VRS model $\langle M\ 1.6\rangle$ to above data set, it shows that DMU_3, DMU_4 and DMU_5 are efficient. After identifying the efficient DMUs, next step is to find the most efficient DMU among the efficient DMUs. Figure-5.1 shows the influence of original efficient frontier on the virtual DMU. It is evident from the Figure-5.1 that efficient DMU_4 influences much on the virtual inefficient DMU.

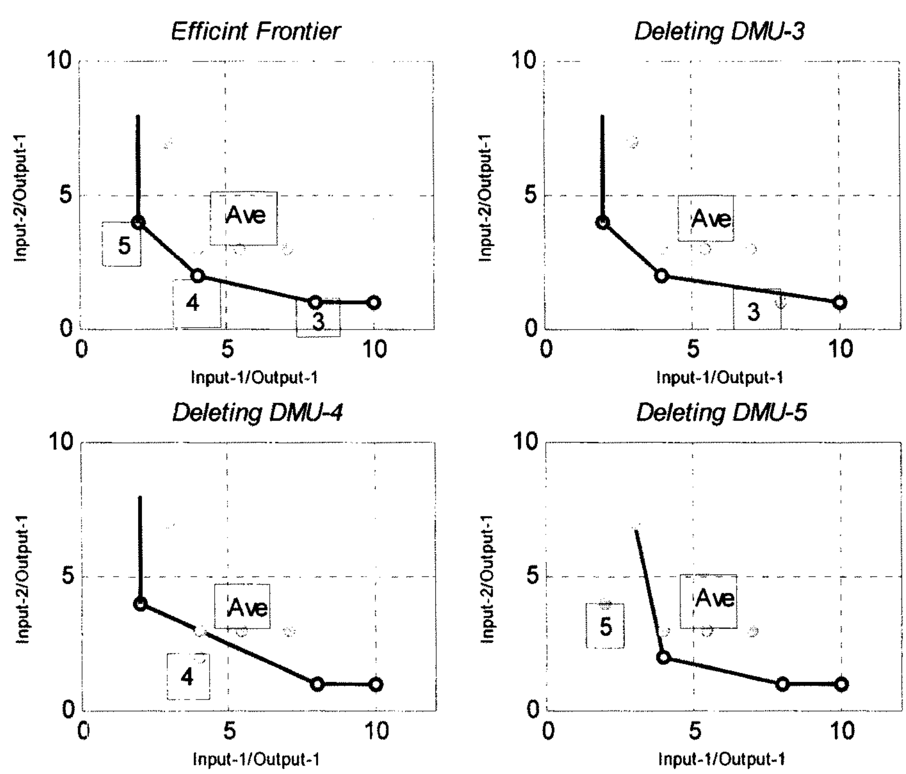

Figure-5.1: Influence of efficient frontier on virtual DMU.

Rank the efficient DMUs which influence more on virtual DMU as the best one. This proposed method is simpler and robust compared to the existing methods. It rightly identifies the efficient DMUs in order. The result of ranking the proposed method is compared with other methods (benevolent formulation of Doyle and Green

(1994) model $\langle M\ 5.1\rangle$ (Ben. D&G), benevolent formulation of Liang *et al.* (2008) model $\langle M\ 5.2\rangle$ (Ben. Liang *et al.*), aggressive formulation of Doyle and Green (1994) minimization of model $\langle M\ 5.1\rangle$ (Agg. D&G), Aggressive formulation of Liang *et al.* (2008) maximization of model $\langle M\ 5.2\rangle$ (Agg. Liang *et al.*), Andersen and Petersen (1993) super efficiency model $\langle M\ 5.3\rangle$ (SE), Gholam Reza Jahanshaloo *et al.* (2007) model $\langle M\ 5.4\rangle$ (JJLA), proposed method , model $\langle M\ 5.5\rangle$ (PROPOSED)] shown in Table-5.2 for CRS method.

	Ben. D&G		Ben. Liang *et al.*		Agg. D & G		Agg. Liang *et al.*		JJLA		SE		PROPOSED	
DMU	E	Rank	E	Rank	E	Rank	E	Rank	E	Rank	E	Rank	E	Rank
3	0.75	3	0.75	3	0.714	2	0.714	2	0.791	3	1.143	3	0.712	3
4	0.857	1	0.857	1	0.738	1	0.738	1	0.859	2	1.25	2	0.875	1
5	0.798	2	0.798	2	0.691	3	0.691	3	0.897	1	1.571	1	0.73	2

Table-5.2: Efficiency score of efficient DMUs and their ranking by different methods for CRS models.

Table-5.2 shows that the proposed model of ranking the efficient DMUs identifies DMU_4 as the most efficient in CRS model $\langle M\ 5.5\rangle$ and it also holds for VRS model $\langle M\ 5.6\rangle$. Different cross efficiency models shows the DMU_4 as the most efficient DMUs. SE and JJLA models identifying DMU_5 is most efficient by taking of higher weight on one particular variable into consideration and due to the effect of extreme point on the efficient frontier. In DEA, weight assigned to variables with respect to one DMU is often different from the other DMUs. Proposed approach of ranking efficient DMU correctly identifies the most stable and balancing efficient DMU as the best one and ranks the efficient DMUs in this order. Ranking of the efficient DMUs by SE method shows DMU_5 is most efficient DMU.

Example 2: In order to compare the proposed ranking method with the existing methods, we considered the example of Chinese cities (Adler and Revah (2008)). This problem was analyzed in a number of papers and consists of 35 cities and 6 variables. The three inputs all in 10,000 Renminbi (RNB), include Industrial Labour Force (ILF), Work Funds (WF) and Investment (INV). The three outputs include Groups of Industrial Output (GIOV) Profit and Taxes (P&T) and Retail Sales (RS) in RNB 1000000.

DMU	Ben. D&G		Ben. Liang *et al.*		Agg. D & G		Agg. Liang *et al.*		JJLA		SE		PROPOSED	
	E	Rank	E	Rank	E	Rank	E	Rank	E	Rank	E	Rank	E	Rank
Beijing	0.623	23	0.624	22	0.577	20	0.573	20	0.731	22	0.731	22	0.731	22
Changchum	0.742	9	0.737	10	0.676	10	0.672	10	0.872	12	0.872	12	0.872	12
Changsha	0.658	17	0.656	16	0.602	16	0.597	16	0.755	20	0.755	20	0.755	20
Chengdu	0.647	18	0.643	18	0.587	18	0.584	18	0.727	23	0.727	23	0.727	23
Chongqing	0.639	19	0.631	20	0.575	21	0.572	21	0.747	21	0.747	21	0.747	21
Dalian	0.627	21	0.625	21	0.575	22	0.571	22	0.673	27	0.673	27	0.673	27
Fuzhou	0.513	28	0.514	28	0.473	28	0.469	28	0.561	31	0.561	31	0.561	31
Guangzhou	0.677	13	0.678	13	0.626	13	0.62	13	0.791	16	0.791	16	0.791	16
Guiyang	0.533	26	0.53	26	0.488	26	0.485	26	0.591	29	0.591	29	0.591	29
Hangzhou	0.9	3	0.894	3	0.817	3	0.812	3	1	7	1.022	7	1	3
Harbin	0.627	22	0.623	23	0.567	23	0.564	23	0.772	18	0.772	18	0.772	18
Hefei	0.707	12	0.701	11	0.642	12	0.638	12	0.795	15	0.795	15	0.795	15
Hohot	0.527	27	0.525	27	0.483	27	0.48	27	0.571	30	0.571	30	0.571	30
Jinan	0.789	8	0.782	8	0.717	9	0.713	8	0.908	11	0.908	11	0.908	11
Kunming	0.605	24	0.602	24	0.553	24	0.549	24	0.652	28	0.652	28	0.652	28
Lanzhu	0.661	15	0.654	17	0.598	17	0.595	17	0.757	19	0.757	19	0.757	19
Lhasa	0.733	10	0.738	9	0.724	8	0.707	9	1	4	3.387	1	1	5
Nanchang	0.808	6	0.797	6	0.731	6	0.728	6	1	3	1.198	4	1	6
Nanjing	0.634	20	0.633	19	0.582	19	0.578	19	0.706	25	0.706	25	0.706	25

Nanning	0.835	4	0.835	4	0.797	4	0.793	4	1	5	1.977	2	1	4
Ningbo	0.971	1	0.97	1	0.894	1	0.887	1	1	2	1.183	6	1	2
Shanghai	0.963	2	0.958	2	0.881	2	0.875	2	1	1	1.48	3	1	1
Shenyang	0.672	14	0.665	14	0.606	14	0.603	14	0.777	17	0.777	17	0.777	17
Shenzen	0.367	35	0.346	35	0.339	33	0.342	33	0.965	8	0.965	8	0.965	8
Shijiazuhang	0.814	5	0.803	5	0.733	5	0.729	5	0.948	10	0.948	10	0.948	10
Taiyuan	0.455	29	0.452	29	0.411	31	0.408	30	0.529	32	0.529	32	0.529	32
Tainjin	0.66	16	0.658	15	0.605	15	0.601	15	0.815	13	0.815	13	0.815	13
Urumqi	0.359	33	0.36	33	0.333	34	0.33	34	0.484	33	0.484	33	0.484	33
Wuhan	0.802	7	0.795	7	0.728	7	0.724	7	0.948	9	0.948	9	0.948	9
Xiamen	0.427	30	0.432	30	0.413	30	0.407	31	0.718	24	0.718	24	0.718	24
Xian	0.603	25	0.598	25	0.543	25	0.54	25	0.701	26	0.701	26	0.701	26
Xining	0.401	32	0.4	32	0.365	32	0.362	32	0.472	34	0.472	34	0.472	34
Yinchuan	0.342	34	0.343	34	0.319	35	0.315	35	0.454	35	0.454	35	0.454	35
Zhengzhou	0.707	11	0.701	12	0.643	11	0.64	11	0.811	14	0.811	14	0.811	14
Zhuhai	0.423	31	0.419	31	0.414	29	0.414	29	1	6	1.193	5	1	7

Table-5.3: Efficiency score of efficient DMUs and their ranking by different methods of 35 cities for CRS models.

The DEA results obtained from the different models for CRS methods are illustrated in Table-5.3. Spearman's rank correlation are calculated for rankings based on different models for all DMUs (efficient and inefficient) and also for efficient DMUs are shown in Table-5.4 and Table-5.5 respectively.

Ranking Methods	Ben. Liang *et al.*	Agg. D & G	Agg. Liang *et al.*	JJLA	SE	PROPOSED
Ben. D&G	0.998	0.993	0.995	0.757	0.74	0.768
Ben. Liang *et al.*		0.996	0.997	0.758	0.742	0.769
Agg. D & G			0.999	0.798	0.783	0.808
Agg. Liang *et al.*				0.786	0.771	0.796
JJLA					0.994	0.997
SE						0.99

Table-5.4: Spearman's rank correlation coefficient for all (efficient and inefficient) DMUs.

Comparing the Spearman's rank correlation test statistics with the critical values for different models, correlations found to be significant (p-value < 0.01) for rankings all the DMUs in all models. Comparing the Spearman's rank correlation test statistics with the critical value for efficient DMUs, it is found that different CE methods are significantly (p<.024) correlated with the method proposed by us as shown in the Table-5.5. Cross-Efficiency DEA method cross evaluates each unit using the optimal weight of other units. This method results in the average evaluation of each DMU. The advantage of CE is that it avoids extreme weight variation of input and output variables of DMUs. Moreover, all the units are evaluated by the same set of weights and thus are comparable. The proposed method ranks the higher maverick indexed DMUs in the lower order of the efficient DMUs. The proposed method of ranking the efficient DMUs is highly correlated with CE. So, our proposed approach avoids the extreme weights problem and identifies the most stable DMU in higher order ranking.

Ranking Methods	Ben. Liang *et al.*	Agg. D & G	Agg. Liang *et al.*	JJLA	SE	PROPOSED
Ben. D&G	1	1	1	0.5	-0.357	0.857
Ben. Liang *et al.*		1	1	0.5	-0.357	0.857
Agg. D & G			1	0.5	-0.357	0.857
Agg. Liang *et al.*				0.5	-0.357	0.857
JJLA					0.286	0.643
SE						-0.286

Table-5.5: Spearman's rank correlation for efficient DMUs.

Here we compare the ratios of individual output to individual input as taken by Zhu(1998) for 9 combinations of two cities Hangzhou with Nanchang and Shanghai with Lhasa as given in Table-5.6.

i.e $d_{ir}^{j} = y_{rj} / x_{ij} \ (i = 1,2,\cdots m; r = 1,2,\cdots s)$

DMU	Ratios								
	d_{11}	d_{12}	d_{13}	d_{21}	d_{22}	d_{23}	d_{31}	d_{32}	d_{33}
Hangzhou	21158	3.4169	**6.0500**	4643.18	0.7498	**1.3276**	119.63	0.0193	**0.0342**
Nanchang	13612	2.4560	**7.4276**	2638.28	0.4760	**1.4396**	71.37	0.0129	**0.0389**
Shanghai	32619	4.2744	7.0210	9094	1.1917	1.9510	**88.5987**	**0.0116**	**0.0191**
Lhasa	3784	0.8726	0.2252	650	0.1499	0.0387	**904.5455**	**0.2086**	**0.0538**

Table-5.6: Ratios of individual output to individual input.

Unlike the DEA score, d_{ir}^{j} gives the ratio between every output and every input. The bigger the d_{ir}^{j}, the better the performance of DMU_j in terms of the r^{th} output and i^{th} input compared to other DMUs. Only three ratios of Nanchang are marginally greater than Hangzhou. In the remaining 6 ratios, Hangzhou overtakes Nanchang by a bigger margin. In CRS model, JJLA method of ranking identifies Nanchang as ranking 3 and Hangzhou is 7. But the proposed method identifies Hangzhou as ranking 3 and Nanchang as 6. In JJLA method, while measuring the efficiency of inefficient DMUs on deleting, Nanchang receives weights only for input 3 and zero weights for other inputs in most of the cases. This leads to extreme weight variations of inputs. As in the case of Lhasa and Shanghai, SE method gives the higher rank to Lhasa and lower rank to Shanghai, where only three ratios of Lhasa is greater than Shanghai. But in the case of proposed method, this problem is avoided by taking average weights for inputs.

5.6 Nerlove-Luenberger super-efficiency measure:

In this part of the chapter, we discuss ranking efficient DMUs based on directional distance formulation of DEA by applying the changing reference set technology. In the presence of undesirable inputs/outputs, existing methods of ranking efficient DMUs cannot be used for directional distance approach of DEA formulation. Ray (2008) introduced Nerlove-Luenberger super efficiency model to discriminate the efficient DMUs. This method is based on Andersen and Petersen (1993) procedure of super efficiency measure for efficient DMUs. Super efficiency measure is itself having the limitation of infeasibility and excessive high ranking of extreme DMUs. If infeasibility occurs, then the super efficient technique cannot provide a complete ranking of all DMUs. Thrall (1996) used the model to identify extreme efficient DMUs and noted that super efficiency model may be infeasible. Zhu (1996), Dula and Hickman (1997), Seiford and Zhu (1999) have established the conditions under which super efficient models are infeasible. The super efficient methodology can give an excessively high ranking to extreme DMUs. In order to overcome these limitations, Ray (2008) proposed an alternative procedure that uses directional distance function formulation of DEA model for computing the Nerlove-Luenberger super efficiency. The model to measure Nerlove-Luenberger super efficiency for DMU_k is given by

$$\left.\begin{array}{lll} & Max\ \beta_k & (5.8a) \\ subject\ to & \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j y_{rj} - \beta_k\, y_{rk} \geq y_{rk} & (5.8b) \\ & \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j x_{ij} + \beta_k\, x_{ik} \leq x_{ik} & (5.8c) \\ & \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j = 1 & (5.8d) \\ & \lambda_j \geq 0,\ j = 1,2,\cdots,n\,;\ j \neq k & (5.8e) \\ & \beta_k\ unrestricted & \end{array}\right\} \langle \text{M } 5.8 \rangle$$

In directional distance model given in $\langle \text{M } 5.8 \rangle$, the directional vector is the corresponding inputs and outputs of the evaluating DMU. If the DMU_k is Nerlove-Luenberger super efficient, then $\beta_k < 0$ implying that the output bundle of the DMU has to be scaled down while its input bundle is scaled up in order to get an attainable input-output bundle in the modified production possibility set. Between two DMUs, one with the lower value of β_k is ranked higher in terms of super efficiency. This approach is also with limitations and they are

1. There is no feasible solution if the DMU under evaluation has any input at zero level and all other DMUs in the production set use strictly positive quantities of that input. This implies that input constraint in the production possibility set remains infeasible.

2. If Nerlove-Luenberger super efficiency score exceeds two, i.e β_k <-1, then the projected input and output involves negative values.

3. When the system involves one of the input or output that is/are negative, then this model cannot be applicable.

5.7 Ranking efficient DMUs by changing reference set technology

The complete ranking of DMUs in DEA is well established in the last decade and it has been immensely used in social and management sciences. The method discussed in Section-5.3 is applicable to discriminate the efficient units having nonnegative inputs and outputs in DEA context. These methods cannot be applied to discriminate the efficient units involving negative inputs and outputs. Here, efforts have been made to rank the efficient DMUs by modifying improved directional distance formulation of DEA model $\langle M\ 3.2\rangle$ as developed in Chapter-3. Modification is based on the changing reference technology as introduced in Gholam Reza Jahanshaloo *et al.* (2007). This method overcomes the limitation encountered by Ray (2008). The model re-evaluates the efficiency of each inefficient DMU when efficient DMU is deleted from the reference set. The modified improved directional distance formulation of DEA model is given by

$$
\left.
\begin{array}{lll}
Min\ \rho_{a,b} = t - \sum_{i=1}^{m} W_i B_{io}^- & & (5.9a) \\
subject\ to \quad t + \sum_{r=1}^{s} Z_r B_{ro}^+ = 1 & & (5.9b) \\
\sum_{\substack{j=1 \\ j\neq b}}^{n} \Lambda_j y_{rj} - B_r^+ R_{ro}^+ \geq t y_{ro} & & (5.9c) \\
\sum_{\substack{j=1 \\ j\neq b}}^{n} \Lambda_j x_{ij} + B_{io}^- R_{io}^- \leq t x_{io} & & (5.9d) \\
\sum_{\substack{j=1 \\ j\neq b}}^{n} \Lambda_j = t & & (5.9e) \\
\sum_{i=1}^{m} W_i = 1, \qquad \sum_{r=1}^{s} Z_r = 1 & & (5.9f) \\
\Lambda_j \geq 0,\ j = 1,2,\cdots,n;\ j \neq b\ i = 1,2,...m;\ r = 1,2,...k & & \\
B_{ro}^+,\ B_{io}^- \text{ are unrestricted} & &
\end{array}
\right\} \quad \langle M\ 5.9\rangle
$$

Where $j=\{1,2,\ldots,n\}$ is the set of all DMUs, $a \in N$ is the set of inefficient DMUs *and* $b \in E$ is the set of efficient DMUs. $\rho_{a,b}$ is the efficiency of the DMU$_a$, when efficient DMU$_b$ is excluded from the efficient frontier. The main idea proposed in this model is to rank efficient DMUs by excluding efficient DMUs from the reference set one by one and calculate the efficiency of inefficient DMUs (set {J-B}). Further, average these efficiencies for each of the efficient DMUs and rank according to averaged efficiencies. The strong efficient DMU is one that when it is excluded from the reference set then the average distance between the efficient frontier and the inefficient DMUs will be minimal.

After calculating the re-evaluated efficiency measure $\rho_{a,b}$ for all inefficient DMUs, the average efficiency of inefficient DMU when efficient DMU excluded from the reference set is denoted by

$$E_b = \frac{\sum_{a \in N} \rho_{a,b}}{\tilde{n}} \qquad (E\ 5.5)$$

Where, E_b is the average efficiency of all re-evaluated inefficient DMUs when efficient DMU$_b$ is excluded from the reference set, b is an efficient DMU and $\tilde{n}$ is the number of inefficient DMUs.

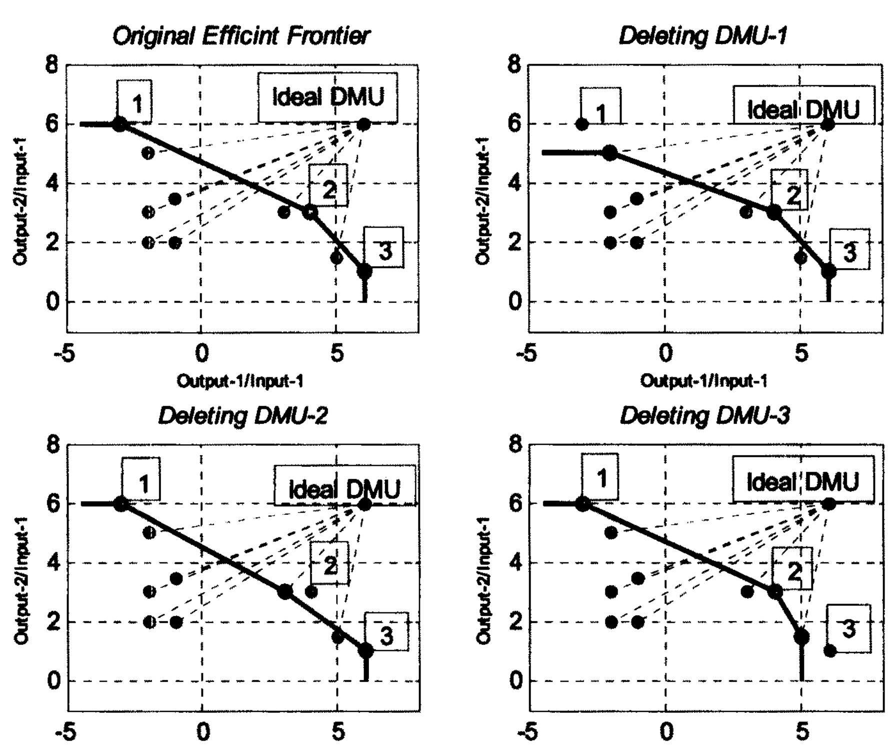

Figure-5.2: Changing the reference set technology.

Illustration of changing the reference set is given in the Figure-5.2. Analyzing the figure we can see that among the efficient DMUs (DMU_1, DMU_2, DMU_3) one that makes the original efficient frontier farthest from the inefficient DMUs in the direction of Ideal DMU is the most efficient. Hence, we can rank accordingly based on their efficiency when efficient DMU is deleted from the reference set.

5.7.1 Properties:

τ_a^* is the efficiency of the model $\langle M\ 3.2\rangle$ as defined in the Section-3.2 of Chapter-3.

1. $\rho_{a,b}^* \geq \tau_a^*$ for each DMU_b, $b \in E$, set of efficient DMUs and $a \in N$, set of in-efficient DMUs.

Proof: The feasible space of $\langle M\ 3.2\rangle$ is the subset of the feasible space of $\langle M\ 5.9\rangle$. Hence the optimal solution of $\langle M\ 5.9\rangle$ is greater than or equal to the optimal solution of the $\langle M\ 3.2\rangle$ $\quad \rho_{a,b}^* \geq \tau_a^*$

2. If $a = b$ and DMU_a is efficient, then $\rho_{a,b}^* \geq 1$.

Proof: Since DMU_a is efficient $\tau_a^* = 1$ from the property (1) $\rho_{a,b}^* \geq \tau_a^*$

$$\Rightarrow \rho_{a,b}^* \geq 1$$

3. If $a \neq b$ and DMU_a is inefficient then $0 \leq \rho_{a,b}^* \leq 1$.

Proof: $\tau_a^* = \dfrac{y_{ra}^* - y_{ra}}{R_{ra}^+}$, where $R_{ro}^+ = \underset{j}{Max}\{y_{rj}\} - y_{ra},\ r = 1,2,\cdots,s$

$\underset{j}{Max}\{y_{rj}\}$ is falls outside the production possibility set, at an optimal solution $\underset{j}{Max}\{y_{rj}\} > y_{ra}^*$, Hence $\rho_{a,b}^* \geq 0 \Rightarrow 0 \leq \rho_{a,b}^* \leq 1$

4. If DMU_a is inefficient and DMU_b does not belong to reference set of DMU_a, then $\rho_{a,b}^* = \tau_a^*$.

Proof: DMU_b does not belong to the reference set of DMU_a $\Rightarrow \Lambda_b^* = 0$ for optimal solution of $\langle M\ 3.2\rangle$. Each optimal solution with $\Lambda_b^* = 0$ is the feasible solution for $\langle M\ 5.9\rangle$. Therefore $\rho_{a,b}^* \leq \tau_a^*$, by property (1), $\rho_{a,b}^* = \tau_a^*$.

5. If DMU_a is efficient in $\langle M\ 3.2\rangle$ and $a \neq b$, then $\rho_{a,b}^* = 1$ for each DMU_b.

Proof: DMU_a is efficient in $\langle M\ 3.2\rangle$ then $\tau_a^* = 1$

by property (1) $\rho_{a,b}^* \geq \tau_a^*$

by property (3) $0 \leq \rho_{a,b}^* \leq 1$

$\therefore\ \rho_{a,b}^* = 1$

7. **If $\rho_{a,b}^* < 1$, then DMU_a is inefficient.**

Proof: By property (1) $\rho_{a,b}^* \geq \tau_a^* \quad \Rightarrow \tau_a^* < 1$

Hence DMU_a is inefficient.

5.7.2 Numerical example:

This section illustrates methodology developed in the Section-5.7 through simple numerical examples. Data for changing reference technology is shown in the Figure-5.2 for model $\langle M\ 5.9\rangle$ is shown in Table-5.7. This contains one unit input and two outputs. Results of the changing reference technology for $\langle M\ 5.9\rangle$ are given in Table-5.8. A program has been developed to find the inefficiency scores for all the DMUs based on model $\langle M\ 5.9\rangle$ using the *MATLAB 7.7* and part of the program is given in the Appendix, Program-6.

DMU	Input-1	Output-1	Output-2
1	1	-3	6
2	1	4	3
3	1	6	1
4	1	-1	2
5	1	-2	3
6	1	-1	3.5
7	1	-2	2
8	1	3	3
9	1	5	1.5
10	1	-2	5

Table-5.7: Illustrative data.

DMU	Efficiency	1	2	3
4	0.6747	0.6747	0.7089	0.6747
5	0.7	0.7273	0.7059	0.7
6	0.7527	0.8	0.7692	0.7527
7	0.6667	0.6667	0.6957	0.6667
8	0.8571	0.8571	1	0.8571
9	0.8	0.8	0.8889	1
10	0.7778	1	0.8	0.7778
E_b		**0.7894**	**0.7955**	**0.7756**

Table-5.8: Re-evaluated efficiency score of inefficient DMUs when efficient DMU is excluded from the reference set.

Among the data given in Table-5.7, DMU_1, DMU_2 and DMU_3 are efficient DMUs. In Table-5.8, column-1 is the inefficient DMUs, column-2 is the efficiency score of the inefficient DMUs using $\langle M\ 3.2\rangle$. Columns 3,4,5 are the re-evaluated efficiency scores of the inefficient DMUs when efficient DMU_1, DMU_2, DMU_3 is excluded from the reference set respectively through the model $\langle M\ 5.9\rangle$.Taking the average of the re-evaluated efficiency through $(E\ 5.5)$, one can rank the efficient DMUs. Here DMU_2 influences more on the efficiency of the inefficient DMUs. When DMU_2 is deleted from the reference set, the efficiency of the inefficient DMUs is more compared to remaining two efficient DMUs.

In order to illustrate the performance of proposed model for ranking the efficient DMUs, we have taken the real time data pertaining to the pollutant processing system analyzed in Sharp *et al.* (2007) shown in the Table-5.9. Data contains 13 DMUs which has one positive input (cost), one negative input (effluent), one positive output (saleable output) and two negative outputs (methane and CO_2). Also efficiency of the DMUs based on $\langle M\ 3.2\rangle$ is given in the Table-5.9.

DMU	Cost (I1)	Effluent (I2)	Saleable (O1)	CO2 (O2)	Methane (O3)	Efficiency
1	1.03	-0.05	0.56	-0.09	-0.44	0.4492
2	1.75	-0.17	0.74	-0.24	-0.31	0.5093
3	1.44	-0.56	1.37	-0.35	-0.21	1
4	10.8	-0.22	5.61	-0.98	-3.79	0.4667
5	1.3	-0.07	0.49	-1.08	-0.34	0.5374
6	1.98	-0.1	1.61	-0.44	-0.34	0.6354
7	0.97	-0.17	0.82	-0.08	-0.43	1
8	9.82	-2.32	5.61	-1.42	-1.94	1

9	1.59	0	0.52	0	-0.37	0.6563
10	5.96	-0.15	2.14	-0.52	-0.18	0.5359
11	1.29	-0.11	0.57	0	-0.24	1
12	2.38	-0.25	0.57	-0.67	-0.43	0.4064
13	10.3	-0.16	9.56	-0.58	0	1

Table-5.9 : Data on pollutant processing system.

Rankings of efficient DMUs based on changing reference set technology for $\langle M\ 5.9\rangle$ is given in the Table-5.10.

DMU	3	7	8	11	13
1	0.4492	1	0.4492	0.4492	0.4492
2	0.6327	0.5466	0.5093	0.5958	0.5093
4	0.4759	0.4667	0.4843	0.4667	1
5	0.5374	0.5942	0.5374	0.5665	0.5374
6	0.7824	0.6354	0.6354	0.6354	0.7985
9	0.6563	0.6563	0.6563	1	0.6563
10	0.66	0.5359	0.5359	0.5359	1
12	0.5369	0.4353	0.4064	0.4064	0.4064
E_b	**0.5914**	**0.6088**	**0.5268**	**0.5820**	**0.6696**

Table-5.10: Re-evaluated efficiency score of inefficient DMUs when efficient DMU is excluded from the reference set.

E_b is the average re-evaluated efficiency of inefficient DMUs when respective efficient DMUs in the columns of Table-5.10 is deleted from the reference set. DMU_{13} is the most efficient followed by DMU_7, DMU_3, DMU_{11} and DMU_8 is least efficient among the efficient DMUs. In this way one can rank the efficient DMUs using changing reference set technology in directional distance formulation of DEA.

5.8 Conclusions:

The inputs and outputs of the virtual DMU are not affected by extreme values and are also based on all the inputs and outputs of all the DMUs. The virtual DMU takes care of extreme weight variation of input and output variables. Hence the current method avoids favouring extreme DMUs in higher order of ranking. The method discussed in this chapter ranks the higher maverick indexed DMUs in the lower order of the efficient DMUs. Maverick DMUs show the greatest relative increment in the efficiency when shifting from peer appraisal to self appraisal by taking the advantage of favourable weights.

From the numerical study, it may be concluded that SE method favours extreme DMUs in higher order of ranking by taking very high weights to favourable variables and least weights to unfavourable variables. A key feature of the procedure developed in this chapter when compared to its counterparts in earlier approaches in ranking the efficient DMUs, is based on a single virtual inefficient DMU. The proposed approach is simpler compared to other methods. This method gives good results both in CRS and VRS model for ranking the efficient DMUs. Basic idea in the proposed approach idea is to compare the efficient units by evaluating the efficiency of virtual DMU with the linear combination of all the efficient units by excluding the efficient units one by one. The proposed approach of ranking the DMUs is simpler and robust compared to CE, SE and JJLA methods.

In the presence of negative inputs/outputs, existing methods of ranking efficient DMUs cannot be used for directional distance approach of DEA formulation. The changing reference set technology of ranking efficient DMUs is applied in this chapter. This method overcomes the limitations encountered by Ray (2008). The developed approach is more appropriate to rank the efficient DMUs where negative inputs and/or outputs are involved in the system. Although there are many ranking methods in DEA context (see Adler *et al.* (2002)), no ranking method is developed till now for ranking efficient DMUs where negative inputs and outputs occur. This method discriminates the efficient DMUs effectively.