CHAPTER 3
FUZZY LOGIC-BASED IMAGE RESTORATION

3.1 INTRODUCTION TO FUZZY LOGIC (FL)

In the previous Chapters, introduction and the literature survey in the area of image restoration based on Fuzzy Logic, Neural Networks (NN) and Fuzzy Neural Networks (FNN) and work done so far have been discussed. The concept of fuzzy logic, membership functions, fuzzy image processing and proposed fuzzy similarity algorithm for image restoration are discussed in this Chapter.

Fuzzy logic provides an inference morphology that enables approximate human reasoning capabilities to be applied to knowledge-based systems. It represents qualitative knowledge and acts as a body of concepts and techniques for dealing with imprecision, approximate reasoning and computing with words. Fuzzy logic provides a remarkably simple way to draw definite conclusions from vague, ambiguous or imprecise information. Unlike classical logic which requires a deep understanding of a system, exact equations, and precise numeric values. Fuzzy logic incorporates an alternative way of thinking, which allows modeling complex systems using a higher level of abstraction originating from our knowledge and experience. Fuzzy logic allows expressing this knowledge with subjective concepts such as very hot, bright red, and a long time which are mapped into exact numeric ranges.
Fuzzy logic has been gaining increasing acceptance during the past few years. There are over two thousand commercially available products using fuzzy Logic, ranging from washing machines to high speed trains. Nearly every application can potentially realize some of the benefits of Fuzzy logic, such as performance, simplicity, lower cost, and productivity. Fuzzy techniques represent and process knowledge in the form of fuzzy if-then rules. Fuzzy processing is desirable in computer vision because of the uncertainties that exist in many aspects of image processing. A fuzzy logic system essentially defines a nonlinear mapping of the input data vector into a scalar output using fuzzy rules. The mapping process involves input/output membership functions, fuzzy logic operators, fuzzy if–then rules, and defuzzification. The fuzzy logic system maps crisp/fuzzy inputs into crisp/fuzzy outputs depending on the nature of application. The fuzzy logic system contains four components namely the fuzzifier, inference engine, knowledge base, and defuzzifier as shown in figure 3.1. The rule base contains linguistic rules that are provided by experts. It is also possible to extract rules from numeric data.
Fuzzy logic is an excellent mathematical tool to identify uncertainty in systems. It was founded by Dr. Lofti A Zadeh (University of California, at Berkley). Fuzzy logic is a problem-solving control system methodology that lends itself to implement systems ranging from simple, small, embedded micro-controllers to large, networked, multi-channel PC or workstation-based data acquisition and control systems. It can be implemented in hardware, software or a combination of both. Fuzzy logic provides a simple way to arrive at a definite conclusion based upon vague, ambiguous, imprecise, noisy, or missing input information. Fuzzy logic's approach to control problems mimics how a person would make decisions, but much faster.
3.2 FEATURES OF FUZZY LOGIC

Fuzzy logic offers several unique features that make it a particularly good choice for many control problems.

1. It is robust since it does not require precise, noise-free inputs and can be programmed to fail safely if a feedback sensor quits or is destroyed. The output control is a smooth control function despite a wide range of input variations.

2. Since the Fuzzy logic controller processes user-defined rules governing the target control system, it can be modified and tweak easily to improve or drastically alter system performance. Sensors can easily be incorporated into the system simply by generating appropriate governing rules.

3. In rule-based operation, any reasonable number of inputs can be processed (1-8 or more) and numerous outputs (1-4 or more) generated, although defining the rule base quickly becomes complex, if too many inputs and outputs are chosen for a single implementation since rules defining their interrelations must also be defined. It would be better to break the control system into smaller chunks and use several smaller Fuzzy logic controllers distributed on the system, each with more limited responsibilities.

4. Fuzzy logic can control nonlinear systems that would be difficult or impossible to model mathematically. This opens the door for control systems that would normally be deemed unfeasible for automation.
3.3 FUZZY MEMBERSHIP FUNCTIONS

Fuzzy logic starts with the concept of a fuzzy set. A fuzzy set is any set that allows its members to have different grades of membership (membership function) in the interval [0, 1].

The simplest membership functions are formed using straight lines. Of these, the simplest is the triangular membership function, and it has the function name \textit{trimf}. It is nothing more than a collection of three points forming a triangle. The trapezoidal membership function, \textit{trapmf}, has a flat top and really is just a truncated triangle curve. These straight line membership functions have the advantage of simplicity.

The generalized bell membership function is specified by three parameters and has the function name \textit{gbellmf}. The bell membership function has one or more parameters than the Gaussian membership function, so that it can approach a non-fuzzy set, if the free parameter is tuned. Because of their smoothness and concise notation, Gaussian and bell membership functions are popular methods for specifying the fuzzy sets. Both of these curves have the advantage of being smooth and non-zero at all points.

Although the Gaussian membership functions and bell membership functions achieve smoothness, they are unable to specify asymmetric membership functions, which are important in certain applications. Next we define the sigmoidal membership function, which is either open left or right. Asymmetric and closed (i.e. not open to the left or right) membership
functions can be synthesized using two sigmoidal functions, in addition to the basic \textit{sigmf}. We also have the difference between two sigmoidal functions \textit{dsigmf}, and the product of two sigmoidal functions \textit{psigmf}.

Polynomial based curves account for several of the membership functions in the toolbox. Three related membership functions are the Z, S, and Pi curves, all named because of their shape. The function \textit{zmf} is the asymmetrical polynomial curve open to the left, \textit{smf} is the mirror-image function that opens to the right, and \textit{pimf} is zero on both extremes with a rise in the middle.

\textbf{Summary of Membership Functions}

- Fuzzy sets describe vague concepts (fast runner, hot weather, and weekend days).
- A fuzzy set admits the possibility of partial membership in it. (Friday is a sort of a weekend day, the weather is rather hot).
- The degree an object belongs to a fuzzy set is denoted by a membership value between 0 and 1. (Friday is a weekend day to the degree 0.8).
- A membership function associated with a given fuzzy set maps an input value to its appropriate membership value.

\textbf{Fuzzification}

Fuzzification establishes the fact base of the fuzzy system. First, it identifies the input and output of the system. Fuzzification then defines
appropriate IF-THEN rules and uses raw data to derive a membership function. At this point, one is ready to apply fuzzy logic to the system.

Defuzzification

Defuzzification – output is single crisp value. Defuzz \( (x, mf, \text{type}) \) returns a defuzzified value of a membership function \( mf \) positioned at \( x \) associated variable value type, using one of the several defuzzification strategies, according to the argument \( \text{type} \).

The variable \( \text{type} \) can be one of the following.

- centroid: centroid of area method
- bisector: bisector of area method
- mom: mean of maximum method
- som: smallest of maximum method
- lom: largest of maximum method

If the \( \text{type} \) is not one of the above, it is assumed to be a user-defined function. \( x \) and \( mf \) are passed to this function to generate the defuzzified output.

Fuzzy Rule (Fuzzy implication)

If-then structure

If \( X \) is \( x_1 \) then \( Z \) is \( z_1 \), where \( X \) and \( Z \) are linguistic variables and \( x_1 \) and \( z_1 \) are terms. For example if motor is \textbf{hot} then alarm is \textbf{high}.

Fuzzy Rule sets and Fuzzy Inference

If \( X \) is \( x_1 \) then \( Z \) is \( z_1 \), if \( Y \) is \( y_1 \) then \( Z \) is \( z_2 \), if \( X \) is \( x_2 \) then \( Z \) is \( z_3 \), the rule set
is evaluated using the min-max interpolation. The result $Z$ is represented as a membership function.

### 3.4 FUZZY RULES

Human beings make decisions based on rules. Although we may not be aware of it, all the decisions we make are based on if-then statements. If the weather is fine, then we may decide to go out. If the forecast says the weather will be bad today, but fine tomorrow, then we make a decision not to go today, and postpone it till tomorrow. Rules associate ideas and relate one event to another. Fuzzy machines, which always tend to mimic the behavior of man, however the decision and the means of choosing that decision are replaced by fuzzy sets, and the rules are replaced by fuzzy rules. Fuzzy rules also operate using a series of if-then statements. For instance, if $X$ then $A$, if $Y$ then $B$, where $A$ and $B$ are all sets of $X$ and $Y$. Fuzzy rules define fuzzy patches, which is the key idea in fuzzy logic. A machine is made smarter using a concept designed by Bart Kosko called the Fuzzy Approximation Theorem (FAT). This theorem generally states that a finite number of patches can cover a curve as seen in figure 3.2. If the patches are large, then the rules are sloppy. If the patches are small then the rules are fine.

![Figure 3.2 Fuzzy Patches](image)

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In a fuzzy system this simply means that all our rules can be seen as patches and the input and output of the machine can be associated together using these patches.

**Fuzzy Control**

Fuzzy control, which directly uses fuzzy rules, is the most important application in fuzzy theory. Using a procedure originated by Ebrahim Mamdani in the late 70's, three steps are taken to create a fuzzy controlled machine.

1) Fuzzification
2) Rule evaluation
3) Defuzzification

### 3.5 ADVANTAGES OF FUZZY LOGIC

- Mimic human in decision making to handle vague concepts
- Ability to deal with imprecise or imperfect information
- Improved knowledge representation and uncertainty reasoning
- Modeling of complex, non-linear problems
- Natural language processing/programming capability

### 3.6 FUZZY IMAGE PROCESSING

Fuzzy image processing is a collection of different fuzzy approaches to image processing. Nevertheless, the following definition can be regarded as an attempt to determine the boundaries: Fuzzy image processing is the collection of all approaches that understand, represent and process the
images, their segments and features as fuzzy sets. The representation and processing depend on the selected fuzzy technique and on the problem to be solved.

**Steps in fuzzy image processing**

Fuzzy image processing has three main stages: image fuzzification, modification of membership values, and, if necessary, image defuzzification as shown in figure 3.3.

![Figure 3.3 The General Structure of Fuzzy Image Processing](image)

The coding of image data (fuzzification) and decoding of the results (defuzzification) are steps that make it possible to process images with fuzzy techniques. The main power of fuzzy image processing is in the middle step (modification of membership values). After the image data are transformed from gray-level plane to the membership plane (fuzzification), the appropriate fuzzy technique modifies the membership values. This can be a fuzzy clustering, a fuzzy rule-based approach or a fuzzy integration approach.
3.7 IMPORTANCE OF FUZZY IMAGE PROCESSING

There are many reasons to use fuzzy techniques in image processing.

1. Fuzzy techniques are powerful tools for knowledge representation and processing.

2. Fuzzy techniques can manage the vagueness and ambiguity efficiently.

In many image processing applications, we have to use expert knowledge to overcome the difficulties (e.g. object recognition, scene analysis). Fuzzy set theory and fuzzy logic offer us powerful tools to represent and process human knowledge in the form of fuzzy if-then rules. On the other side, many difficulties in image processing arise because the data/tasks/results are uncertain. This uncertainty, however, is not always due to the randomness but to the ambiguity and vagueness.

For filtering images, median filter is particularly effective, when the noise pattern consists of strong spike like components and the characteristic to be preserved as edge sharpness. When the noise rate is more than 30%, the median filter cannot do well. The Adaptive Fuzzy Median Filter (AFMF) has been introduced to deal with this problem. Since the input space of fuzzy systems must be divided into fuzzy regions, it is very difficult to apply fuzzy systems to problems in which the number of input variables is large. Moreover, another difficulty arises in knowledge acquisition through interviewing experts. The application of fuzzy techniques in image processing is a promising research field [45]. Fuzzy techniques have already been
applied in several domains of image processing (e.g., filtering, interpolation [95] morphology [62, 63]) and have numerous practical applications (e.g., in industrial and medical image processing [11, 73]). Several fuzzy filters for noise reduction have been developed, e.g., the well-known FIRE-filter in [74, 78, 79], the weighted fuzzy mean filter in [52, 54] and the iterative fuzzy control-based filter in [28]. Most fuzzy techniques in image noise reduction mainly deal with fat-tailed noise like impulse noise. These fuzzy filters are able to outperform rank-order filter schemes (such as the median filter). Nevertheless, most fuzzy techniques are not specifically designed for Gaussian noise or do not produce convincing results when applied to handle this type of noise.

3.8 NEED FOR IMAGE RESTORATION IN MEDICAL FIELD

Medicine in general is very demanding in practice. A minute error can cause a catastrophic effect. Fortunately, advancement in radiology and nuclear medicine has given rise to many imaging facilities at diagnostic stage. For instance, Magnetic Resonance Imaging (MRI) has long been used as standard method to find out the properties of matter at molecular level. These diagnostic images can help physicians to plan future treatment of patients.

It is the physic of the imaging technique; however, some considerations have to be given. If it is not the cost of producing the images a primary concern, the quality of the images is another. Poor quality images cause diagnosis a very hard task. Hence it is necessary to use image restoration algorithm to enhance the quality of the images so that the restored
image can be studied easily and hence allow disease pattern to be detected as early as possible.

The existence of unwanted data or noise in medical images cannot be avoided due to the nature of the system. This results in poor quality images. Various restoration techniques are employed to remove this noise. The method chosen depends on what kind of imaging technique is used. Here, we will take a closer look at several examples.

First, we might want to know how these unwanted signals can end up in our images. In MRI, patient movement such as the respiratory motion during the scanning process causes a blurring effect and also a ghost-like image to appear called artifact. In addition, geometrical and intensity distortion can arise from changes in the density of human body. In ultrasound images however, artifacts contamination arises from changes in velocity of the sound beam while passing through the tissues of different densities. This change in velocity refracts the beam and sends inaccurate signal to the receiver. There are also other reasons such as random production of photons in X-Ray images and background noises, but the above are the common problems. Filters are typically designed for and tested on a certain type of noise. Two of the most common noise types in the literature are Gaussian and salt-and-pepper noise. Even if a filter is claimed to be designed for dealing with impulse noise, it may be tested only on salt-and-pepper and in fact it can perform worse, when the amplitude of the noise impulses is random. Filters can be designed according to a number of criteria, such as an
objective error measure (MSE, SNR), visual appearance of the image, processing time, hardware implementability, tenability and robustness against changes in the noise distribution.

3.9 INTRODUCTION TO THE PROPOSED FILTER

Nowadays, image restoration is also needed in surgical procedure and other areas of medical treatment. Surgeons are now using many optical devices and computer systems which rely heavily on the accuracy of the image produced to perform surgery. This gives rise to the need of fast and cost-effective restoration technique.

In the restoration process, it can be said that basic algorithm for reconstruction of the image is employed. This includes using various filters such as non-linear filters and non-linear edge filters. Though ideally filtering is processes of restoration, some authors think that restoration is a different stage. A linear process called Wiener filter is commonly used for image restoration. However, there are also several generic techniques introduced to tackle specific task depending on the objective of restoration. While considering medical images, Fuzzy logic works well when compared to the existing methods in quality and reality.

3.10 PROPOSED FUZZY SIMILARITY-BASED FILTER

Reduction of noise in images is one of the most basic image processing operations. In recent years, fuzzy logic-based filters have shown to provide efficient image filtering. The fuzzy filters can be
1. Fuzzification of weighted mean filters,

2. Fuzzy combinations of the outputs of several classical sub filters

or

3. "Pure" fuzzy filters, in which no classical filter operators are used, but the filter output is directly determined by a set of rules.

A new fuzzy-based filter is designed to reduce mixed noise (Gaussian and impulse) and is able to operate in real-time at frame rates higher than normal video rates. The filter is based on the evaluation of fuzzy similarities between pixels within a local processing window. Eighteen 4-pixel templates are represented in the following figure 3.4. The outputs of a set of sub filters are then combined to provide the final filter output.

![Figure 3.4 Different 4-pixel Templates](image)
3.10.1 Noise reduction based on Fuzzy Similarity

Consider a small neighborhood of a center pixel $x_0$, in a noise-free image. If we assume that the image details are no smaller than the processing window, the brightness will be more or less constant in edge-free regions. This indicates that the intensities of the pixels inside this window are similar, except when the window contains an edge. As noise is added to the image, the image pixels change their original intensities and become less similar to each other. High similarities between the pixels in the window imply the presence of short-tailed noise, whereas low similarities indicate the existence of long-tailed noise. Different sub-filters are then activated to certain degrees, depending upon the similarities found in the window. When the similarities are low, a filter for long-tailed noise is activated. Here, we use a simple median filter, since it has been widely used for the suppressing of long-tailed and impulse noise while preventing blurring of edges. Hence, the basic idea in the Fuzzy-Based (FB) filter approach is to investigate whether the pixels in the local processing window are similar to each other. In order to check the similarity in different parts of the window, the pixels are arranged into a set of 4-pixel templates.

3.10.2 Fuzzy Similarity

The similarity between two pixels are allowed to take values between 0 and 1, hence it is called fuzzy similarity. In this work, it is evaluated as a 1-D triangular membership function taking the intensity difference as input variable in the following equation 3.1.
\[ \mu_{x=x_j} = \begin{cases} 
1 - \frac{|x_i - x_j|}{\alpha}, & |x_i - x_j| < \alpha \\
0, & \text{elsewhere} 
\end{cases} \quad (3.1) \]

Generally, the fuzzy similarity can be modeled as a 2-D fuzzy relation surface. This allows for a membership function that varies with the two input parameters. In order to get the degree of similarity between the central pixel and a template, the similarities between the central pixel and each pixel in the template are calculated. Then, they are combined using an aggregation operator.

### 3.10.3 Rules

The FB filter approach is based on the evaluation of a set of rules. The original rules are the following:

**R\(_1\):** IF the central window pixel and all templates have similar intensities

THEN the window has nearly uniform intensity distribution

AND do not change the central pixel;

**R\(_2\):** IF the window does not have uniform intensity distribution

THEN the max similarity templates best portray the local image properties

AND change the central pixel according to the intensity of the max similarity templates;

The idea of rule R\(_2\) is that if we can find pixels that are similar to the central pixel, these pixels are more likely to portray the local image characteristics. The activation degree rule R\(_1\) is never calculated explicitly, but it is automatically fulfilled in the defuzzification step. However, although
similar to a central pixel, pixels can be still noisy themselves. Hence, we introduce a couple of rules that take into account the similarity between pixels within the templates.

\( R_{3,j} \): IF template \( j \) is not homogenous

AND the central pixel is not similar to template \( j \)

THEN template \( j \) does not contribute to the noise cancellation;

\( R_{4,j} \): IF template \( j \) is homogenous

AND the central window pixel is not similar to template \( j \)

THEN template \( j \) portrays the local image characteristics

AND change the central pixel according to a function of the pixels in template \( j \);

\( R_{5,j} \): IF template \( j \) is homogenous

AND the central window pixel is similar to template \( j \)

THEN the central pixel and template \( j \) both portray the local image characteristics

AND change the central pixel according to a function of the pixels in template and itself;

Since there is no relevant consequent part associated to rule \( R_{3,j} \), it is not used in the filtering, but merely serves as clarifying the idea behind the filter design. We also define an additional rule, associated with high dissimilarity within the window.

\( R_{dis} \): IF no other rule is activated
THEN change the central pixel according to a default method/function \( f_E \) of the pixels in the templates and itself.

The firing strength of this rule is defined as the degree to which none of the other rules is activated:

\[
\mu_{\text{dis}} = 1 - \max (\mu_i) \quad (3.2)
\]

The rules can be written compactly as

- **R\(_{2,j}\)**: IF sim\(_j\) is high
  
  THEN \( \Delta x_{2,j} = g(X_j) \);

- **R\(_{4,j}\)**: IF hom\(_j\) is high
  AND sim\(_j\) is low
  THEN \( \Delta x_{4,j} = h() \);

- **R\(_{6,j}\)**: IF hom\(_j\) is high
  AND sim\(_j\) is high
  THEN \( \Delta x_{4,j} = h(x_0, X_j) \);

In this work, \( g(x) = \text{median}(x) \), \( h(x) = \text{mean}(x) \), and \( f_E \) is the median of 4-neighborhood. However, the functions can be chosen according to a priori knowledge about the noise type. For example, replacing the function \( g(X_j) \) with the central pixel value, results show that the performance for impulse noise increases, as the ability to reduce Gaussian noise decreases. In general, it can be any function/method capable of reducing long-tailed noise. However, due to implementation issues, more complicated functions/methods cannot be implemented. The minimum operator was used for the “AND” operation in the rules.
The update of the intensity of the central pixel $\Delta x$ is generally obtained through centroid defuzzification

$$
\Delta x = \frac{\sum_{i=2, 4, 5}^{n} \sum_{j=1}^{n} \mu_{i,j} \Delta x_{i,j} + \mu_{\text{dis}} \Delta x_{\text{dis}}}{\sum_{i=2, 4, 5}^{n} \sum_{j=1}^{n} \mu_{i,j} + \mu_{\text{dis}}} 
$$

(3.3)

where $\mu_{i,j}$ is the firing strength of rule $R_{i,j}$.

3.11 EXPERIMENTAL RESULTS

The above fuzzy similarity algorithm is applied to different degraded images. The algorithm removes the noises such as Gaussian and salt and pepper present in the degraded images and restores the images. In all the cases the parameter $a = 64$ and the minimum operator was used for aggregation. Some of the tested degraded images and restored images are shown in figures 3.5(a) to 3.5(h).

Figure 3.5(a) Degraded input image
3.12 SUMMARY

The use of fuzzy concepts, membership functions and the proposed fuzzy similarity algorithm are discussed in this chapter. The fuzzy-based filter approach was found useful for reduction of noise in images with a number of different noise types and noise levels. This filter is fairly robust against the changes in noise distribution. From the results, the filtering quality seems to be comparable to other existing filters. However, the performance of FB filter
is less efficient for high levels of salt and pepper noise because template consisting of noisy pixels is still homogeneous. Subsequently, the intensity change implied by the corresponding rule will be assigned a large weight in the defuzzification implying an inaccurate central pixel update. The neural network-based image restoration is discussed in chapter 4.