1.1 INTRODUCTION TO STATISTICAL QUALITY CONTROL

The term quality now-a-days occupies an important place in many fields of research. In the beginning, this was applied to an industrial output, but in recent times it is playing a very important role in almost all fields like Management, Biology, Medicine and Research. Basically the term quality, with reference to consumer products means, ‘General excellence’ more precisely it means excellence in relation to certain thing that a consumer wants in a particular product. There are a good number of reasons, why a product may have unsatisfactory quality in this popular sense. There may be certain characteristics desired by a customer, for instance, performance, durability, appearance, colour, strength, safety, fool-proof and fail-safe and so on, which were not designed into the product because the manufacturer did not intend to do so. Thus, the quality of a product can be defined as follows:
“The totality of features and characteristics of a product of service that bear on its ability to satisfy the stated or implied needs of the user of the product”

The term ‘Quality’ is mostly used by Industrialists. Quality means ‘fit for use’. Quality is the most important consumer - decision factor in selecting products and services. In modern times we have professional societies, Governmental regulatory body which aim at assuring the quality of products sold to consumers. Every customer has a ‘right’ to get good quality product, which is consumed by ‘Consumer Protection Act, 1986’. Quality is defined based on the viewpoint that products and services must meet the requirements of the consumers. Quality control is a process employed to ensure a certain level of quality in a product or service. Essentially, quality control involves the examination of a product, service, or process for certain minimum levels of quality. The basic goal of quality control is to ensure that the products, services or processes are provided to meet the specific requirements and are dependable satisfactory and fiscally sound.

It is clear to say that when manufacturing began and competition accompanied manufacturing, consumers would compare and choose the products based on qualities such as performance, reliability, durability, serviceability, aesthetics (visual appeal of the product), features, company reputation and standards required by the consumers. It is worthwhile to note that quality is inversely proportional to variability in quality characteristics. Thus, if quality of the product varies from certain respective characteristics then quality of that product will down-grade. As a consequence of this, the company’s reputation will come down. For example, if you make regular business trips using a particular airline and the flight does not arrive on time, the prospective customer will choose an alternative mode of travel and hence, the airline company loses its reputation and business as well. Therefore the main objective of the manufacturing company is to enhance the company’s reputation, maintain low cost, improve quality of the product, reduction of the variability and ensure that whether the
product satisfies consumer’s needs or not. Most organizations find it difficult to provide good quality products (identical from product to product and meet customer expectations).

On the other hand, Statistical quality control is comparatively new than Science of Statistics. The earlier applications were made in Astronomy and Physics and in the Biological and Social Sciences. It was not until the 1920’s that statistical theory began to be applied effectively to quality control.

In the year 1924, the concept of quality control in manufacturing was first advised by Walter. A. Shewhart [49] of the Bell telephone Laboratories. At the end of 20th century Dodge and Roming [12] both developed acceptance sampling plans as an alternative to 100 % inspection in their Bell Telephone Laboratories. However, the value of Statistical quality control was not widely recognized by industry. Later, in the Second World War the Statistical quality control methods were attracted the attention of many researchers and had gained importance. “Economic Control of Quality of Manufactured Product was published by Walter. A. Shewhart [46] in the year 1931.

In 1946, the American Society for quality promotes the use of quality improvement techniques. In 1950’s, designed experiment for product and process improvement was first introduced in the United States. The initial applications were in the chemical industry. Later the applications spread outside the chemical industry. In late 1970’s and early 1980’s, many western companies discovered designed experiments for producing quality products that compete with Japanese produced products. In 2000’s ISO 9000:2000 standard is issued. Supply-chain management and supplier quality became even more critical factors in business success. Quality improvement activities expanded, beyond the traditional industrial setting into many other areas including financial services, health care, insurance and utilities.
Academic engineering curriculum, mythological development in engineering and industrial organisation has been extended by this discovery. Statistical method for quality and overall business improvement in the United States has been grown since 1980. This has motivated us to develop acceptance sampling plans based on Continuous Acceptance Sampling Plans-Cumulative Sum (CASP-CUSUM) control charts.

Though it may be of low quality, the customer prefers the product with less cost. Hence, quality of a product is to be judged on the basis terms of customer satisfaction.

The term ‘quality’ uses by all entrepreneurs, industrialists and manufacturers, because of severe competition among various firms for salability of their products, quality of the product has become the defining factor. A sustainable process has to be put in place to bring out a high quality product that suits customer-requirements. This means that the process must be capable of operating with little variability around the target. It is in this context that one has to study Statistics process control. This Statistics Process Control (SQC) has developed numerous methods applicable in different situations and can be broadly classified as follows:

1. Acceptance sampling plans,

2. Sequential sampling plans,

3. Reliability,

4. Rectifying instructions and

5. Control Chart.
Huge quantities of finished products are submitted for inspection by every industry and manufacturing unit. The purpose, as well as the aim of accepting sampling plans is to accept or reject these finished products. These types of techniques are mainly used in experiments where destruction may be involved. For example, manufacturing of crackers, electric bulbs and batteries and so on. In these cases, it is not feasible to adopt 100% inspection because the test procedures are destructive in nature. These plans may not have a direct impact on control of a product’s quality.

Cumulative Sum (CUSUM) to maintain current control was introducing by British Statistician E.S Page [39] in 1954. CUSUM charts are basically used to identity a sudden and persistent change in the process average. We can also locate the time of change more sharply with the help of CUSUM charts. The basic difference between Shewart control charts and CUSUM charts is that, in the former, a decision is taken on the basis of a sample taken at the time ‘t’ whereas in the latter the decision is taken on the basis of all the sample taken up to the time ‘t’ which justifies the term cumulative in the title.

Further Acceptance Sampling Plans are introduced mainly to accept or reject the lots of finished products. Such type of techniques are properly used where testing involves destruction, for instance, in the manufacturing of Crackers, Bullets, Batteries, Bulbs and so on, it is impossible to go for 100% inspection. Even though these techniques of Acceptance Sampling Plans do not have direct impact on controlling the quality of the product but they have a lot of indirect effects to improve the same. For instance, a particular product is continuously rejected for lack of quality; automatically the producer strives to improve the quality of the product, otherwise, the consumer will opt for another better quality product.
In this middle of the century H.F. Dodge [11] introduced the concept of continuous sampling plans in contrast to lot-by-lot acceptance sampling plans. Continuous sampling plans are popularly used in those processes where lots are not clearly defined. For instance, the production of cars coming off an assembly line, Soft drink bottles from a continuous glass ribbon machine, Welded Leads emanating from a welding operation and so on. In these situations the lot size ‘N’ is equal to 1 since units are produced item by item. It is a well proved fact that continuous acceptance sampling plans can fruitfully be used in the place of lot-by-lot acceptance sampling plans.

In addition, the formation of lots eliminate majority of problem in the formation of lots. For instance, under conditions of continuous production, lotting requires either the items be allowed to accumulative at given points of production or the “lot” be artificially marked as given segment of the items in the production line. The first procedure which creates bunk of items at various production points requires extra space, increases inventories and case of explosive material produces additional safety. In the second procedure, a lot of special difficulties arise in demarcating the lots. To avoid these difficulties continuous sampling plans came into existence. At First these procedures were applied to non-destructive inspection on a go-no-go basis of a continuous flow of individual units of product offered to the inspector order of production.

In persuasion, continuous sampling is an acceptance rectification inspector procedure which involves alternative periods of 100% and sampling inspection. To start with, the product is sampled off the line in pre-determined ratio ‘f’ until a product is found. The finding of one or more defects, depending up on the plan is used, is the signal for 100% inspection to begin. Sampled is resumed when a certain number of successive defect-free units ‘i’ have been inspected and thus the cycle is repeated.
1.2 CONTINUOUS ACCEPTANCE SAMPLING PLANS

1.2.1 INTRODUCTION

Lot acceptance sampling plans (LASP) are used during production to test units submitted for evaluation against certain hypotheses. Based on company quality, LASP’s provide a check. Most LASP samples of a product are carried out in lots. In standard sampling a hypotheses of the sample makes up the criteria by which the process is judged. These units are then accepted or rejected on the basis of the set forth hypothesis. If a process has tested adequately, the lot or unit is accepted and passed on to the retailer or customer. If, however, the quality control is not sufficient, sampling will prevent unacceptable product from leaving the manufacturer. Accepting or rejecting a lot or unit is synonymous with not rejecting or rejecting the null hypothesis in the hypothesis test. Because grouping into lots is not always advantageous, continuous sampling as outlined below, takes a slightly different approach to quality control in manufacturing.

1.2.2 CONTINUOUS SAMPLING

Continuous sampling planning organised in 1943 by Dodge [09] known as CSP-1, continuous sampling is used where product flow is continuous and not easily grouped in lots. Two parameters exist for continuous sampling. One is the frequency (f) and the second is the clearing number (i). The frequency (f) is defined by a number such as 1/20, 1/30, or 1/X. The clearing number (i) is a number such as 30 or 60. A company checks all of its product until 100% of i number of units are inspected and found to be defect free. After 100% of i number of units are found to be defect free, 100% inspection is ceased, and one out of every X number of units is checked. The sampling continues until a defect is found. After finding a defect the cycle repeats itself until 100% of i number of units has been found to be defect free. At this point the sample 1/X will begin again.
1.2.3 USING CONTINUOUS SAMPLING

Continuous sampling plan is simple and used in three ways: 1. Inspect all ‘i’ data. 2. If no defects are found, randomly sample fraction ‘f’ of data and check again for defects. 3. Whenever a defect is found, correct the flaw and repeat step1. There are two main parameters to consider when executing a continuous sample. All other relevant measurements for continuous sample planning can be derived from these two parameters.

AOQ Average outgoing quality for long –run CSP-1 plan.

\[ AOQ = (p)(P_a)(\frac{N-n}{N}) \]

AFI Average fraction inspected for long-run CSP- 1 plan

\[ AFI = \frac{f}{f + q'(1−f)} \]

f  Sampling frequency for short-run CSP –1 plan
i  Clearance number for short-run CSP – 1 plan
p  Incoming quality level
\( p_a \) Probability of accepting incoming unit
\( q' = 1-p \)
n Lot size

When a series defects are gathered, a good continuous sampling plan will inspect 100% the rejected units and replace and defective parts or products. In this case all defects are made whole and accepted. AOQ refers to the long term defect level of this sampling plan and 100% inspection of rejected units. The average fraction inspected is simply the average of the fractions 1/X sampled. Results can then be further analyzed by creating an Average Outgoing Quality Curve.
1.3 MOTIVATION

The main objective of any researcher in the field of Statistical Quality Control is to suggest methods to produce a qualitative product. With this motivation many Statistical quality control techniques emerged from various researchers to tackle the problem arising in different types of production. Beattie, D.W [5] in his paper suggested a continuous acceptance sampling procedure when the items are being manufactured continuously and the decision rule is required to decide whether to continue or to stop the production of that product. Evan, W.D; and Kemp, K.W. [15] suggested the method of using CUSUM charts in constructing continuous acceptance sampling plans. They demonstrated the use of such plans in the following situation where ‘Thermal Batteries’ are produced at the rate of 50 per day. The test to access the quality of Battery is destructive test in which a principal output is rise time. It is desired to discriminate between two qualities, where quality is measured by the proportion of rise times above a specification limit of 1-5 seconds, and to accept or reject that day’s production by means of a small sample taken daily. The complexity of the test production schedules and inspection man power require that the sampling range remains constant. They further assumed that the rise-time of a Thermal Battery follows a Normal distribution.

Schneider, H [46] studied the performance of standard sampling plans when the variable under study is distributed as truncated Normal distribution. He also showed how variable sampling plan can be designed when the truncation points and the Normal population, standard deviation are known.

Kakoty, S and Chakraborty, A.B. [24] described an acceptance sampling plan for doubly truncated normal variate based on CUSUM chart. They further investigated the effect of truncation points on ARLs and Type-C OC curves of the plan.
Recently the optimization methods have occupied prominent place in modern research of various disciplines. These optimization methods can also fruitfully be applied to optimize the parameters of various sampling plans.

Very recently Narayana Murthy, B.R et.al. [37, 38] described optimization of CASP- CUSUM Schemes for truncated Log-Logistic and Rayleigh distributions. They further investigated the effect of truncation points on ARLs and Type-C OC curves of the plan.

Motivated thus, by the above facts, an attempt is made in this thesis to develop new continuous acceptance sampling plans when the variable under consideration follows:

1. Truncated Burr distribution

2. Truncated Dagum distribution

for each distribution we applied two methods of Integrations. Further, exploiting the application potentialities of computers, a number of programs were developed for each situation discussed above to optimize different parameters of these sampling plans.

Thus, the chief object of this dissertation is four fold namely:

1. to suggest new sampling plans when the variable under consideration follows any one of the above mentioned distributions.

2. to truncate the variable under study and to examine the effect of various truncation points on the parameters of the scheme.
3. to compare the optimization results relating to above mentioned distributions with different integration methods, and

4. to demonstrate the utility of computers in optimizing the parameters of various schemes under consideration.

To compare many kinds of methods of optimization techniques used to increase the probability of accepting a lot is the main aim of this dissertation. Akhtar,P.Md.and Sarma,K.L.A.P [1] introduced various optimization methods in his Ph.D Thesis in 1993. These methods are applicable in case of Accepting Sampling Plans. They basically used “Gauss-Chebyshev Polynomial” to obtain solutions and to evaluate various integrals. These integrals are across the calculation to derive \( P(A) \), which signifies the probability of accepting a lot. The method of optimization is compared with another method in this dissertation. Lobatto method of integration is used in the latter method. After that, the results are compared with those obtained by Akhtar,P.Md and Sarma,K.L.A.P [1]. To evaluate any specified integral, a number of methods are available. The methods are to be chosen carefully, keeping in view, the nature as well as the type of integral. Various integral values are obtained, based on selected nodes and weights in different methods of integration. However, the choice of nodes and weights must be in such a manner that there will be minimum error in the predicted results. In Gaussian Integration Method, these nodes and methods are determined by making formulae exact polynomials of degree up to ‘n’ some of the established Gaussian integration methods are:

1. Gauss – Legendre Integration Method (GLIM)
2. Lobatto Integration Method (LIM)
3. Radau Integration Method (RIM)
4. Gauss Laguerre Integration Method (GLAIM) and
5. Gauss-Hermite Integration Method (GHIM)
The aim of each one of these methods is to secure more and more accurate value of a given integral. This is the case where the integral has a specific form. M. K. Jain, SRK Iyengar and R.K Jain [23], not only discussed these methods elaborated, but also formulated various tabulations of nodes and weights for different values of ‘n’. The method of optimization followed i.e. Gauss-Chebyshev method of integration is compared to Lobatto integration method and this is explained in Chapter-II and Chapter-III respectively relating to Burr distribution and in Chapter-IV and Chapter-V respectively relating to Dagum distribution.

In the past research, there were many truncated probability distributions used other than Burr distribution and Dagum distribution to obtained truncated optimal point at which the probability of acceptance of the lot was maximum and also determined acceptance zone L(0), rejection zone L’(0). Therefore in thesis, we obtained P(A), L(0) and L’(0) by using Gauss-Chebyshev Integration Method, Lobatto Integration Method.

In this connection, Continuous Acceptance Sampling Plans (CASP) are used for analysis of probability of various parameters, computer programmes have been developed. They involve a number of integrals, which in turn involve a large number of integrations to evolve. Critically comparative conclusions are drawn suitably, on calculating the probabilities.

1.4 A BRIEF LITERATURE REVIEW

The objective of this dissertation is to suggest various continuous acceptance sampling plans when the variable under consideration follows serval truncated statistical laws which are rarely used in the field of SQC. Hence, this section is completely concentrated to review the relevant literature regarding continuous acceptance sampling plans and its related topics.
1.4.1 CONTINUOUS ACCEPTANCE SAMPLING PLANS (CASP)

H.F. Dodge and Perry, R.L [11] of Bell Telephone Laboratories developed CASP in contrast to lot-by-lot Acceptance Sampling Plans (ASP), in case of continuous production. When lots are not created as a natural result of production process, it is difficult to demarcate lots. These plans are found to be very useful. It is already mentioned that lotting requires accumulating items at a given point of production or the lots being placed at different points along the production line. We find development of special sampling inspection schemes for continuous production, as a result of lacunae in the existing methods. A continuous flow of individual units of products are submitted to the inspector on go-no- go basis for a non-distractive inspection. This is in the order of production, as suggested H.F. Dodge [09], naming it CSP-1, which is explained as follows:

1.4.2 OPERATION SCHEME OF CSP -1

This is to be started with 100% inspection of products produced continuously. This 100% inspection is discontinued, after ‘i’ consecutive units of products are found to be free from defects, then fractions (f) of the units are inspected. From the flow of products, the individual sample units are to be selected, one at a time, at random. Then, 100% inspection is to be reverted, followed by the completion of a cycle as soon as a sample unit is found to be defective. The quantity ‘i’ is called “Clearance Interval” and ‘f’ is called “the sampling frequency”. There is correction or replacement of these units with good ones. “The sampling frequency” and “The clearing interval” are selected on practical consideration.
The CSP-1 is presented in Fig.1.4.1

![Diagram](image)

**Figure 1.4.1**

In reality, the defects of a product can be classified as major and minor. In case of each category of these defects, CSP-1 can be applied. To be more precise, let there will be ‘$m_1$’ major and ‘$m_2$’ minor defects, but the same ‘f’ can be used in both cases with any obstacle, under this joint scheme, however, it will be a sample inspection for minor defects, where as it will be a 100% inspection for major defects. The reverse is also true in many cases. When both of them are under sampling inspection, same units of sample are used both for major defects and minor defects. Under sampling inspection for minor defects, discovery of a unit of product among non-sampled units, a long side 100% inspection for major defects, cannot be taken as a cause for shifting to 100% inspection of minor defects. The same applies to major defects under sampling inspection, when minor defects are fewer than 100% inspection.
In course of time, in the first version of Dodge Continuous Sampling Plan -1 [CSP-1] some variations were found. Following are two concepts on which variations are mainly based.

1) It is a common practice that single isolated defective item invites 100% inspection in CSP-1. However, in situations concerned with minor defects, there may be a violation of this. Hence, there is necessity for modification of CSP-1. H.F. Dodge and M.N. Torrey [11] suggested modified versions known as CSP-2 and CSP-3. Until two defective samples units are found within the range of k-sample units, 100% inspection cannot be restored. The possibility is that ‘K’ will be taken as equal to ‘i’. Though it is true that CSP-3 follows CSP-2, it gives additional protection against spotty production. In case a defective unit is found in sampling inspection, the next four units should also be inspected. A 100% inspection procedure is to be adopted in any of the four defects. CSP-2 can be continued, in case no defect is found. The inter-relationships among CSP-1, CSP-2 and CSP-3 are explained clearly in Fig.1.4.2

![Diagram showing the inter-relationships among CSP-1, CSP-2 and CSP-3](image-url)
2) Further, in CSP-1 G.J. Lieberman and H. Solomon [30] proposed sampling plans, based several levels, known as Multi Level Sampling Plans, in order to avoid these abrupt changes. The beginning of operation procedure of these schemes is on the lines of CSP-1. These plans start with 100% inspection and go to fraction ‘f’ as soon as a run of ‘i’ non defective units has been found. In case, a run of ‘i’ consecutive sample units are found at the rate \( i/f \), free of defects, then the method of sampling at the rate of \( i/f^2 \) is followed. This is repeated in other units, found free of defects. However, if a defective unit is found under sampling inspection, there has to be immediate return to the next lower level of sampling. The effect is that reduces inspection load, where the production is good and it enhances the load, when the production is bad.

Multilevel plans similar to the Lieberman – Solomon plans were suggested by J.A. Greenwood [20]. The exception is that 100% inspection is called for all units of the product represented by the sample unit, which is proved to be defective. A schematic representation of multi-level plan is given in Fig. 1.4.3.

![Figure 1.4.3](image_url)
A. Wald and J. Wolfowitz [54] suggested Block Continuous Plans, which stand for other continuous sampling plans. The plans earlier discussed assume a study flow of production, without segregating the product into lots or segments. But by sampling at random, a fraction of ‘f’ units from successive segments of a given size, any of the two procedures can be implemented. The plans divide the sequence of production into successive blocks, with a prescribed sample being taken from each block. M.A. Girshick [18] has given a modified approach to avoid the need of segmenting production. This modification also gives the same result.

1.4.3 CUMULATIVE RESULT PLANS (CRP)

Cumulative Result Plans are processed for all plans discussed above. This is a part of rule for any sample inspected. The continuous sampling plans belong to the class of cumulative result plans. The other members of the class are “Skip-lot Plan”, “Chain Sampling Plan” and “Cone-Dodge Cumulative Result Plan.” Certain assumptions are to be met about the nature of inspection in case of cumulative result plans Dodge [09] proposed following assumptions while introducing chain sampling plans.

1. The lot should be one of a continuous series of supply.

2. Lots should normally be expected to be of the same quality.

3. The consumer must have confidence in the supplier, in that no advantage would be taken of a good record to slip-in a substandard lot.

4. The consumer should have no reason to believe that the lot to be inspected in poorer than any of the immediately preceding lots.
1.4.4 SKIP-LOT SAMPLING PLANS (SLSP)

Individual units produced in a serial order from a continuous flow of supply can be subjected to method of Continuous Sampling Plans. The principle of these plans can also apply to individual lots received in a steady stream from a trusted supplier. Lots may also be skipped like units, so the plan is named as Skip-lot Sampling Plan, abbreviation being SKSP. Though it may appear surprising, protection per sampled unit can be enhanced by skip-lotting. H.F. Dodge and Perry, R.L [10] introduced the first version of Skip-lot Plan – (SKSP-1) for inspection of raw material purchased from a common source. The skip-lot procedure is schematically represented in Fig.1.4.4.

![Figure 1.4.4](image)

The purpose of introducing SKSP-1 was to use in situations of a simple and absolute go- no-go decision. On the other hand, in case of continuous sampling approach, when a standard sampling plan is applied to each lot, skipping lots can be of great utility. The risk of a producer or a consumer should be considered in case of using sampling plans. Dodge and Perry [10], in their development of SKSP-2 have factored these risks. Some discrete items are sampled by means of a standard reference sampling plan. The above plans are to be applied in such cases.
1.4.5 Chain Sampling Plans (Ch.SP)

At the same time, reduction of sample size, maintaining or extending protection are used to instance of use of cumulative results in achieving a reduction of sample. These plans introduced by H.F. Dodge [09] had the purpose of overcoming the lack of discrimination in C =0 sampling plans. It is to “chain” together the most recent inspections, the chaining is done in such a way as to build up the shoulder of the operating characteristic curve of C = 0 plans. This is particularly used in situations where small samples are demanded due to economic or physical problem of obtaining a sample. The sampling scheme is usually known as ChSP-1. The operational procedure of ChSP-1 is schematically presented in Fig.1.3.5

![Diagram of Chain Sampling Plans (Ch.SP)](image)

**Figure 1.4.5**
1.4.6 Continuous Acceptance Sampling Plans Based on Cumulative Sum (CASP-CUSUM) Control Charts

E.S. Page [38], a British Statistician, first introduced CUSUM Control Charts. The basic difference between Shewart Control Chart and CUSUM Control has already been mentioned. Also, CUSUM Charts are mainly used to maintain current control of a process. Being equally effective and less expensive than Shewart Chart, it is the advantage of CUSUM Chart.

W. D. Ewan [15] and K.W. Kemp [15] introduced the concept of CUSUM technique. This was to overcome various kinds of problems arising in day-to-day quality control. In a situation of production being continuous, successive values of variables are compared with reference value and it forms the basic feature of CUSUM technique. If accumulation reaches or exceeds a pre-determined decision interval, it is an indication that a change has occurred in the mean level of a variable.

A small persistent change occurring in the process of production can be rapidly identified by CUSUM chart compared to Shewart Chart [49] and D.W. Marquardt [34] has demonstrated this in the U.S.A. industry by maintaining tight control over the process. There are fundamentally two criticisms in application of CUSUM procedure, as pointed out by Willam H. Woodall [55]. They are as follows:

1. CUSUM may be too sensitive to small shifts in the process means in some applications. According to I.N. Gibra [17], when some slack in the process is permissible, CUSUM chart should be used with greater care, if at all used. W.H. Woodall [55] recommends that in-control in-difference and out-of-control regions of shifts be defined, in order to overcome this problem.

2. The second criticism of the CUSUM procedure is that it does not detect large shifts in the mean as fast as Shewart Chart does. J. M. Lucas and R.B. Crosier [32] answer this criticism by combining Shewart control limit with the CUSUM procedure.
Researchers have suggested may other methods to overcome the above two criticisms. Remarkable research work is going on in this direction. CUSUM charts with various sampling intervals were introduced by M.R. Reynolds et al [43]. Normally a CUSUM chart is maintained by taking samples at fixed time intervals. Plotting of CUSUM chart differences is done with sample means and target value on the chart in order of time. The process mean is to be on target as long as CUSUM statistic computed from the sample does not fall into the original region of the chart. The value of CUSUM statistic in the signal region can be taken as an indication that the process mean has changed. It also calls for investigation of possible causes of changes. M.R. Reynolds et al [43] have brought up a new chart known as Variable Sampling Intervals VSI – CUSUM Chart. When this chart falls close to the signal region, it can be taken as an indication that the mean level of the process may have shifted. When used in a short sampling interval, more information about the process can be obtained quickly. Any important change in the process mean can be detected easily and quickly. These are the two main advantages of VSI-CUSUM chart.

But, if the chart is not close to the signal region, it is an indication that the process mean is close to the chart. In this case, a longer sampling interval will be used by VSI- CUSUM chart. Different rates of sampling depend on what is being deserved from the date. VSI-CUSUM chart switches between these rates when samples are taken at a faster rate. This is, if indication is available that there may be a shift in the process mean and if the chance of a problem is less frequent. Efficiently of CUSUM chart with other charts has been compared and methods to choose parameters of the chart have been fixed.

F.F Gan [16] introduced the concept of optimization of CUSUM charts. He defined an optimal CUSUM chart as one with a fixed in control.

Average Run Length (ARL), which is the smallest for a specified shift $\Delta$ in the mean sensitivity analysis of optimal CUSUM charts for various values have been determined.
A fast and precise approximation for ARL of CUSUM control charts has been suggested by D.M. Hawkins [22]. ARL relies on the parameter ‘h’ and ‘k’ and this ARL is utilized for measuring the performance of CUSUM chart. The larger the values of the parameter, the longer are the ARL’s.

1.5 CONCEPTS USED IN THE WORK

The concepts of Operation Characteristics (OC) curve occupies an important curve and Average Run Length (ARL) are more frequently used in this thesis, hence it is more appropriate here to discuss the basic definitions, types and some important properties of these concepts which play a vital role in the theory of SQC.

1.5.1 THE OPERATING CHARACTERISTICS CURVE

The Operation Characteristics (OC) curve occupies an important place in acceptance sampling plan while judging the best efficiency of various sampling plans. In order to select the best sampling plan it is desirable to compare the performance of various sampling plans under considerations over a range of possible quality levels of submitted product. An excellent picture of this performance is given by the OC curve. For any given fraction defective ‘p’ in a submitted lot OC curve shows the probability of $P_A$ that such lot will be accepted by the given sampling plan. In other words the OC curve shows the long-run percentage of submitted lots that would be accepted if a great many lots of any stated quality were submitted for inspection.

1.5.2 CONSTRUCTION OF OC CURVE

The method of construction of OC curve is mainly based on calculation of the probability of lot acceptance denoted by $P_A$ for various assumed values of lot’s quality
denoted ‘p’. ‘p’ is usually represented on the X-axis and \( P_A \) on the Y-axis. A number of points \((p, P_A)\) will be obtained and plotted on the graph. These points are joined by a smooth curve which is the OC curve of the plan, calculation of \( P_A \) changes from plan to plan.

### 1.5.3 CLASSIFICATION OF OC CURVES

Basically two types of OC curves come across in the literature. They are Type-A and Type-B OC curves. The Type-A OC curve gives the probability of acceptance an isolated lot whereas Type-B OC curve gives the probability of accepting of an average quality ‘p’. In brief the basic difference between Type-A and Type-B OC curve is explained as follows:

The Type-A OC curve describes how a consumer is likely to view the Operating Characteristics of a sampling plan. When he buys isolated lots of material or thing about the quality of individual lots rather than the average quality of a stream of lots. The Type-B OC curve describes how a consumer is likely to view the Operating Characteristics of a sampling plan, when he is buying a study stream of material from a given supplier.

It is important note that Type-A OC curve depends on the size of the lot under consideration and as the lot size is large Type-A OC curve approaches Type-B OC curve. In other words Type-B OC curve can be taken as a good approximation to Type-A OC curve when the lots are large, say, more than 10 times of the sample. Usually Type-A OC curve for large lots will be denoted by Type-A and for small lots, it is denoted by Type-A OC curves.

### 1.5.4 AVERAGE RUN LENGTH (ARL)

Just like the OC curve of an acceptance sampling plan brings out the salient characteristics of the plan, Average Run Length (ARL) curve of a control scheme can also be considered as a good way of concisely picturing some of its important
characteristics. If a process average starts at $\bar{X}$ and subsequently changes by an amount $\Delta$ if this change persistent until detected, the number of sample points that will on the average have to be plotted before the given control scheme associated with the specified change in the process. In this case of Shewhart control chart it is easy to find out the ARL because with the help of an OC curve we can find the probability $P_A$ of failing to pick-up a specified change in a single sample after the change has occurred, Hence $1 - P_A$ gives the probability of identifying the change, whereas in CUSUM charts determining ARL is not an easy job. For continuous variables direct derivation of ARL usually requires the assistance of computer either to solve the integral equations that arise in the analytical formulation of the problem or through simulation.

Basically there are two types of ARL curves. They are (i) ARL curve for one sided CUSUM scheme and (ii) ARL for two sided CUSUM scheme

i) ARL FOR ONE SIDED CUSUM SCHEME

Kemp, K.W has constructed a nomograph for finding approximate value of the ARL’s of some one-sided CUSUM schemes when the variable plotted is normal distributed. The nomograph constructed by him is given in Fig 1.5.1. The method of using this nomograph is explained as follow:

Let $m_a$ denoted an acceptable quality level and $m_r (> m_a)$ be barely tolerable, and worse quality being rejectable level. Let ‘k’ denote the reference value such that $m_a < k < m_r$ and let the process standard deviation $\sigma$ is known. Then for a one sided CUSUM scheme with a specified reference value
Figure 1.5.1

‘k’, decision interval ‘h’ and sample size ‘n’ we can find ARL’s at $m_a$ and $m_r$ are as follows. In the Fig 1.5.1 note the point on the $\frac{h \sqrt{n}}{\sigma}$ line yielded by the given values for ‘h’ and ‘n’, denote this point as ‘A’. To find the ARL at $m_a$, find the point on the line $|k-m| \frac{\sqrt{n}}{\sigma}$ given by setting $m= m_a$ denoted this point by ‘B’. Join these points ‘A’ and ‘B’ through a line which cuts line $L_a$. This point on $L_a$ is the ARL at $m_a$. To find the ARL at $m_r$, find the point on the $|k-m| \frac{\sqrt{n}}{\sigma}$ by substituting $m= m_r$ denoted this point by ‘C’ now join ‘A’ and ‘C’ through a line such that it cuts $L_r$. This point on $L_r$ is the ARL at $m_r$. 

The same procedure can be used to find the ARL’s for other values of m, using L line, when $m < k$ and L line, when $m > k$. A similar procedure can be used to sketch the ARL curve for a negative one sided CUSUM chart when the barely tolerable quality level $m(<m)$ and the decision interval is ‘–h’.

(ii) ARL FOR TWO SIDED CUSUM SCHEME

The ARL for two sided CUSUM schemes can readily be described with the help of the ARL’s for two one sided schemes. K.W. Kemp has shown that $L_{1(m)}$ is the ARL at ‘m’ of a one sided schemes with reference value $k_1(>m)$ and decision interval ‘h’, and if $L_{2(m)}$ is the ARL at ‘m’ of a one sided scheme with reference value $k_2(<m)$ and decision interval ‘–h’, $L_{d(m)}$ is the ARL at m, then

$$\frac{1}{L_{d(m)}} = \frac{1}{L_{1(m)}} + \frac{1}{L_{2(m)}}$$

............... (1.5.1)

Various methods have been used to evaluate ARL’s for CUSUM schemes. Among these S.Vardeman and et.al obtained solution for ARL’s of CUSUM schemes when observations were exponentially distributed. Such type of results have immense value when the observations arise naturally in the context of monitoring the occurrence of rare events. Lucas,J.M applies the CUSUM idea to data on the occurrence of industrial accidents. The monitoring of congenital malformations discussed by Chen,R and of background radiation discussed by Marshall, A.G are other areas when exponential distribution can fruitfully be applied.

1.5.5 TYPE-C OC CURVES

The concept of Type-C OC curve was first introduction by Read, D.R. & Beattie, D.W.[42] which is the probability of acceptance of an item as a function of incoming quality. It will be clear that this probability $P_a$ is given by the proportion items accepted when the incoming quality is constant. Thus,
\[ P(A) = \text{Proportion of items accepted} \]
\[ = \frac{\text{No. of samples on the normal chart}}{\text{Total No. of samples taken}} \]

\[ P_A = \frac{L(O)}{L(O) + L'(O)} \]

\[ \text{Where } L(O) = \text{ARL on normal chart} \]
\[ \text{and } L'(O) = \text{ARL on normal chart} \]

Formula (1.5.2) assumes a constant rate of sampling on both normal and return charts. If this is not so, then denote \( g \),
\[ g = \frac{\text{fractions sampled on normal chart}}{\text{fractions sampled on return chart}} \]

And
\[ P_A = \frac{L(O)}{L(O) + gL'(O)} \]

The concept of normal chart and return chart of a continuous acceptance sampling plan is given Fig.1.5.2
In the above Fig.(1.5.2), The Norma Chart is represented by the portion of the chart in the interval \((0, h)\) and ”The Return Chart” by the portion of the chart between \(h\) and \(h’\). In the above \(h\) is called the decision interval and \(h’\) is called the return interval. Type-C OC curve plays a predominant role in the field of continuous acceptance sampling plans to determine the effectiveness of the plan. Basically there are two criteria to measure the effectiveness of the plans, the first one is based on the ARL. This criterion is used by Ewan, W.D. & Kemp, K.W[15]. They have used two ARL’s of which one ARL for the normal chart and another one for the return chart. The second criterion to determine the effectiveness of the play is through the OC curve of the plan.

By Albrecht, A. And et. al.[2] In which they used the accept of Acceptable Quality Level (AQL) and Objectable Quality Level (OQL) and used the formulae of sequential analysis introduced by Wald, A to describe the OC curve. But Wald’s formulae apply to those plans for sequential sampling from lots of fixed size. When sampling plan is done from varying lot sizes or from continuous production the method of calculating OC curve differs and this new type of OC curve is called Type-C OC curve by Read, D.R, & Beattie, D.W, which is a function of incoming quality of the product submitted for inspection.

1.6 CHAPTER SUMMARIES

The present thesis consists of six chapters. A brief summary of each chapter is given bellow:

Chapter-I: This chapter is introductory in nature where identification of the problem is introduced first, along with a brief introduction to AQC and Acceptance Sampling Plans. The present work, and describe how we motivated to develop CASP-CUSUM Schemes. Also it explains the brief literature and basic concepts needed to understand the area of the work.
Chapter-II: This chapter deals with the introduction to continuous acceptance sampling plans based on CUSUM charts and solutions are obtained for ARL and Type- C, OC curve when the variable under consideration follows a truncated Burr Distribution. A Burr Distribution can be considered as a first approximation to any random variable. Hence this chapter deals with Optimization of CASP-CUSUM Schemes based on truncated Burr distribution using Gauss-Chebyshev Method of Integration, it is assumed that the random variable under study distributed according to Burr distribution. Under this assumption it is determined that the optimal truncated point at which the probability of accepting of lot is maximum, and also determined acceptance zone L(0), rejection zone L’(0). Finally, based on the obtained results, conclusions are drawn.

Chapter-III: This chapter deals with the introduction to continuous acceptance sampling plans based on CUSUM charts and solutions are obtained for ARL and Type- C, OC curve when the variable under consideration follows a truncated Burr distribution. A Burr distribution can be considered as a first approximation to any random variable. Hence this chapter deals with Optimization of CASP-CUSUM Schemes based on truncated Burr distribution using Lobatto Method of Integration, it is assumed that the random variable under study distributed according to Burr distribution. Under this assumption it is determined that the optimal truncated point at which the probability of accepting of lot is maximum, and also determined acceptance zone L(0), rejection zone L’(0). Finally, based on the obtained results, conclusions are drawn.

Chapter-IV: This chapter deals with the introduction to continuous acceptance sampling plans based on CUSUM charts and solutions are obtained for ARL and Type- C, OC curve when the variable under consideration follows a truncated Dagum distribution. A Dagum distribution can be considered as a first approximation to any random variable. Hence this chapter deals with Optimization of CASP-CUSUM Schemes based on truncated Dagum distribution using Gauss-Chebyshev
Method of Integration, it is assumed that the random variable under study distributed according to Dagum distribution. Under this assumption it is determined that the optimal truncated point at which the probability of accepting of lot is maximum, and also determined acceptance zone $L(O)$, rejection zone $L'(O)$. Finally, based on the obtained results, conclusions are drawn.

**Chapter-V:** This chapter deals with the introduction to continuous acceptance sampling plans based on CUSUM charts and solutions are obtained for ARL and Type- C, OC curve when the variable under consideration follows a truncated Dagum distribution. A Dagum distribution can be considered as a first approximation to any random variable. Hence this chapter deals with Optimization of CASP-CUSUM Schemes based on truncated Dagum distribution using Lobatto Method of Integration, it assumed that the random variable under study distributed according to Dagum distribution. Under this assumption it is determined that the optimal truncated point at which the probability of accepting of lot is maximum, and also determined acceptance zone $L(O)$, rejection zone $L'(O)$. Finally, based on the obtained results, conclusions are drawn.

**Chapter-VI:** This Chapter deals with conclusions about the Optimization of CASP-CUSUM Schemes based on the results obtained in chapter II. III, IV, and chapter V. It also explains the further scope of the thesis.