CHAPTER 6

SUMMARY, CONCLUSIONS AND SCOPE FOR FUTURE WORK

6.1 Summary and Conclusions

The conceptual and algorithmic simplicity, ease of implementation and hence experimentation, high convergence characteristics and robustness of Differential Evolution algorithm have attracted, is still attracting, many researchers who are working on its performance improvement. This consequently is resulting in multitude of DE variants. The means by which the research community is attempting to enhance DE’s performance efficacy is manifold viz. new as well as problem specific mutation strategies; control parameters adaptations; suitable modifications to the algorithmic structure; hybridization with other heuristic methods and local search algorithms; theoretical insight about the working of DE; parallelization of DE etc. to cite but a few. Despite the active research in DE, an extensive empirical analysis of the basic (classical) DE variants to provide insight about their efficacy to the practitioners has not been take up well. Little research effort has been devoted to understand and compare the efficacy of existing classical (barebones) DE variants to solve problems with different features.

Consequently, this thesis has carried out an extensive empirical comparative analysis of the performance of fourteen classical DE variants on a well known fourteen unconstrained global optimization benchmark problem suite which is grouped by their modality and decomposability. This analysis has been undertaken to identify the competitive DE variants which perform reasonably well on a range of problems with different characteristics. The analysis identified DE/rand/1/bin, DE/rand/2/bin, DE/best/2/bin and DE/rand-to-best/1/bin as the most competitive variants.

Observing the fact that from the point of view of population updating DE is still static, dynamic population updating has been proposed for DE/rand/1/bin and DE/best/1/bin variants, in the literature. Interestingly very little research effort has been devoted towards the Dynamic Differential Evolution (DDE). However, since this thesis considered this small modification to DE as an effective improvement, this dynamicity has been extended to the 12 remaining variants and an extensive empirical analysis has been carried out. Invariably, the DDE variants outperformed their DE counterparts. Interestingly the most competitive variants are
DDE/rand/1/bin, DDE/rand/2/bin, DDE/best/2/bin and DDE/rand-to-best/1/bin (the same DE variants identified earlier). In fact the thesis also extended multiple trial vector generation strategy to all the 14 DE and 14 DDE variants. However the mtvDE (multiple trial vector DE) and mtvDDE (multiple trial vector DDE) variants are found to demand more number of function evaluations with no promising performance. Consequently, this thesis intended to focus only on DE and DDE algorithms.

Having identified four competitive DE variants and their DDE counterparts, the thesis made an attempt to analyze the behavior of only the identified competitive DE variants for the sake of easier analysis. The analysis has been attempted to gather insight for possible improvement and/or effective design of the variants to make them more robust and efficient. The behavioral analysis of the four identified DE variants has been attempted through deriving analytical relationship as well as empirical observation of the evolution of population variance as against the number of generations.

In fact a theoretical relationship between the expected population variance after mutation-crossover and the initial population variance has been derived for DE/rand/1/bin in the literature. This thesis directly extended the theoretical measure of population diversity, derived for DE/rand/1/bin to the three other identified variants viz. DE/rand/2/bin, DE/best/2/bin and DE/rand-to-best/1/bin. The derived theoretical measures were tested with success using a simple experimental setup. The simulation results showed that the ‘pure’ rand variants’ (DE/rand/1/bin and DE/rand/2/bin) runs display gradual loss or gradual increase of population variance revealing a slow convergence pattern. The best variant’s (DE/best/2/bin) runs display a sharp loss or sharp increase in population variance revealing a greedy behavior. However, in case of DE/rand-to-best/1/bin both decrease and increase in population variance is found to be much more balanced as against those of rand and best variants. This behavior of DE/rand-to-best/1/bin is by virtue of the contribution of both random and best candidate solutions in the population.

The population variance behavior of DE/rand-to-best/1/bin suggested/hinted that mixing of perturbation schemes may contribute to the exploration-exploitation balance resulting in robust optimization characteristics. Based on this the thesis proposed that the mixing of effective/competitive DE variants with diverse characteristics – in such a way that they are allowed to evolve independently but are made to suitably exchange their search information
amongst others – will co-operatively enhance the efficacy of the system as a whole with robust optimization characteristics as compared to when they operate separately.

The thesis proposed to mix the identified competitive DE variants with diverse characteristics, each in an island, in a distributed framework. This mixing of heterogeneous DE variants in islands based distributed framework resulted in a class of algorithms called distributed mixed variants Differential Evolution (dmvDE). The dmvDE constitutes various proportions and combinations of DE variants as subpopulations. The dmvDE is different from the typical distributed Differential Evolution (dDE) in that the islands are populated by different DE variants not the same variants (as in dDE). For the sake of easier analysis, dmvDE configuration is restricted to only 4 islands and different combinations of two of four DE variants in each island thus empirically analyzing only 19 dmvDE variants. To serve as a frame of reference as well as comparison against dmvDE, 14 dDE (distributed Differential Evolution) variants and 14 dDDE (distributed Dynamic Differential Evolution) variants were implemented and analyzed with the 14 test functions.

The simulation results obtained by dmvDEs displayed a relatively marginal competitive performance advantage, if not considerable improvement, over dDE/DE variants. However to elucidate the robust optimization characteristics of dmvDEs, the best of the 19 variants has been benchmarked on a new 13 benchmark functions suite (of 500 and 1000 dimensions each) against five state-of-the-art distributed Differential Evolution algorithms. The dmvDE displayed a superior performance over the five algorithms displaying robust optimization characteristics as this performance has been achieved with a new set of test functions. The dmvDE was not only promising in 30 dimensions, but also scaled well to 500 and 1000 dimensions displaying scalability.

With the intention of analyzing the dynamics of mixing between variants, the thesis instead of just mixing DE variants in each island attempted one more level down in mixing i.e. mixing DE and DDE variants in a distributed framework thus resulting in yet another class of algorithms called distributed mixed variants (Dynamic) Differential Evolution (dmvD³E). The dmvD³E is different from dmvDE in that the islands are populated by both DEs and DDEs. With the similar experimental assumptions as that of dmvDE, dmvD³E outperformed the five state-of-the-art algorithms as well as the dmvDE thus reiterating the efficacy of mixing DE variants.
6.2 Scope for Future Work

The competitive performance of \textit{dmvDE} as well as \textit{dmvD^2E} may largely be attributed to the co-operative evolution of constituent \textit{DE/DDE} variants with each of them evolving independently in each island but also exchanging information amongst themselves to co-operatively enhance the efficacy of \textit{dmvDE/ dmvd^2E} as a whole. However a better insight about the dynamics of mixing calls for a very extensive systematic empirical analysis. It is worth admitting that the simulation studies employed only 19 \textit{dmvDEs} and 4 \textit{dmvD^2E} by virtue of the constraints on island size as well as limited combinations. Despite the fact that relaxing these constraints would amount to huge number of possibilities of mixing, a systematic analysis of such possibilities is a must for better understanding of dynamics of mixing.

Interestingly, the mixing has been attempted for 4 competitive variants. The mixing between \textit{DE} variants of different performance efficacies can be carried out. This would reveal the implications of efficacy differences over mixing. Also, careful mixing of \textit{DE} variants to match the optimization problem characteristics leading to a \textit{recipe} of variants’ mixing for a given problem is another potential direction for research.

Apart from the selection of constituent \textit{DE/DDE} variants for \textit{dmvDE/dmvD^2E}, the empirical setting of parameters like migration strategy, proportions of migrants, topology etc. also implicates \textit{dmvDE/dmvD^2Es} performance. Careful empirical analyses of above parameters provide another direction for research. In fact the resulting insight would provide answers to interesting questions like

(1) If the number of constituent \textit{DE/DDE} variants are increased further in case of \textit{dmvDE/dmvD^2E}, which migration strategy would suit better?

(2) For a distributed computing environment with tens of islands, what are the design considerations as far as the distributed framework parameters are concerned?

To cite but a few examples. Since the efficacy of \textit{dmvDE} and \textit{dmvD^2E} – outside the artificial benchmark arena – in solving real world optimization problems would serve as a best critic and would stand as a testimony to the success of the idea, it would form a definite subset of the future work.