CHAPTER-1

1. INTRODUCTION

1.1 Inventory Management

Almost all industrial operations are subject to the problem of inventory management. Inventories are nothing but materials stored, waiting for processing, or experiencing processing. For example, observation of almost any company balance sheet reveals that a significant portion of its assets comprises inventories of raw materials, components and subassemblies within the production process, and finished goods. Raw material inventories provide a stable source of input required for production. In-process inventories minimize the impacts of the variability of the production rates in a plant and also protect against failures in the processes. Final goods inventories were used for better customer service. The important marketing considerations are variety and easy availability of the product. Other kinds of inventories, viz., spare parts inventories for maintenance and excess capacity built into facilities to take advantage of the economies of scale of construction. A large inventory requires less replenishment and may reduce ordering costs because of economies of scale. The existence of inventory reflects a temporary gap between two activities viz., supply and demand. Inventories also incur costs for the care of the stored material and are subject to spoilage and obsolescence.

Inventory theory is a fertile area for cost reduction. Even in manufacturing or service sectors the inventory theory is important as large amount of the total assets is tied up as inventories. Love (1979) defined inventory as a quantity of goods or material in the control of an enterprise and held for a time in a relatively idle or unproductive state, awaiting its intended use. A proper and scientific inventory management system is necessary to find stability between two extremes of high inventory and shortages. Hence efficient management of inventories is not only necessary but also essential for an industry or business to stay competitive in the global business environment.

Because of their practical and economic importance, the subject of inventory management is a major concern in many situations. Inventory theory usually answer questions of when and how
much raw material should be ordered, when a production order should be released to the plant, what level of safety stock should be maintained at a retail outlet, or how in-process inventory is to be maintained in a production process. These questions are amenable to quantitative analysis with the help of inventory theory.

Formulation of an inventory model is very important to address the above questions. Two important aspects of the problem have been considered while formulating an inventory model. One of them being the perishability of items and the other one is the variation in the demand rate. Many inventory models assume that the demand rate to be independent of inventory levels which may not be true for certain types of consumer goods, consumables, food grains etc. It also assumes that the stock in storage is non-perishable which again may not true in practice in case of products like fruits, milk, photographic films etc.

1.2 Focus of the Thesis

1.2.1 Inventory Models with Quadratic Demand

In last few decades several researchers have extensively studied various aspects of inventory models considering two kinds of time–varying demand so far, viz., (a) continuous-time and (b) discrete-time. Most of the continuous–time inventory models have been developed considering either linearly increasing /decreasing demand \[ D(t) = a + bt, \ a > 0, \ b \neq 0 \] or exponentially increasing /decreasing demand \[ D(t) = Ae^{at}, \ A > 0, \alpha \neq 0 \]. It is well known that the demand rate for physical goods may also be depending on stock and price. For a comprehensive review of literature on inventory models with time/price/stock varying linear or exponential demand rate, one may refer the work of Goyal and Giri (2001).

Several mathematical modelers argued that, in realistic terms, the demand need not follow either linear or exponential trend. So, it may be reasonable to assume that the demand rate, in certain commodities, due to seasonal variations follow quadratic function of time \[ D(t) = a + bt + ct^2; \ a \geq 0, b \neq 0, c \neq 0 \] or a quadratic function of price \[ D(p) = a + bp + cp^2; \ a \geq 0, b \neq 0, c \neq 0 \] or a quadratic function of stock level \[ D(I) = a + bI + ci^2; \ a \geq 0, b \neq 0, c \neq 0 \]. Here \( a \) denotes the initial rate of demand, \( b \) is the rate with which the demand rate increases (decreases) and \( c \) is the acceleration in the demand rate. These functional
forms of time/price/stock level dependent quadratic demands explain the accelerated/retarded growth/decline in the demand patterns which may arise due to seasonal demand rate (Khanra and Chaudhuri, 2003). Here we have \( \dot{D}(t) = 2ct + b \) and \( \ddot{D}(t) = 2c \). When \( \dot{D}(t) = 0 \), we get
\[
t = -\frac{b}{2c}.
\]
Now \( t \) is positive if ‘\( c \)’ and ‘\( b \)’ are of opposite signs. Thus \( D(t) \) is (i) maximum at
\[
t = -\frac{b}{2c}
\]
for \( b > 0 \) and \( c < 0 \) and (ii) minimum at \( t = -\frac{b}{2c} \) for \( b < 0 \) and \( c > 0 \). In the first case demand gradually go up to a maximum and then gradually decreases while in other case the demand gradually falls to a minimum and then increases gradually. Normally we can come across the first case in real market where as the second one is very rare. Depending on the signs of ‘\( c \)’ and ‘\( b \)’, following are the different types of realistic demand curves:

(i) \( c > 0 \) and \( b > 0 \) gives accelerated growth in demand model  
(ii) \( c > 0 \) and \( b < 0 \) gives retarded growth in demand model  
(iii) \( c < 0 \) and \( b > 0 \), we have retarded decline in demand model  
(iv) \( c < 0 \) and \( b < 0 \), we have accelerated decline in demand model

The above four types of demand curves are given below:

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<th>Accelerated Growth Demand Curve</th>
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Optimal replenishment policy for linear (increasing/decreasing) demand is established by Donaldson (1977). He considered the EOQ problems over a finite horizon for linear (trended) demand, and derived an analytical solution to obtain the optimal number of replenishments and its duration. However, his procedure requires iterative calculations which may be quite tedious.

1.2.2 Inventory Models with Inflation

Classical inventory models assume that costs related to inventory are independent of time and constant over the period under review. The general tendency in inflationary environment is to purchase large amount of items in order to reduce the total system cost, which may be economically justified in some situations. However in the inventory systems, ordering large quantities would not be economical for perishable items because of material spoilage/wastage due to deterioration. Thus it is important to study the inventory management system under constant inflation rate. Under inflation the assumption of constant unit price is not valid. The tendency in inflationary environment is to buy more in order to reduce the total system cost, which may be true in certain situations but it is not true when consumption rate of items is dependent on initial stock level since buying more quantity under inflationary environment leads to more consumption resulting in higher total system cost.

1.2.3 Inventory Models with Trade Credit Policy

Trade credit policy is generally one of the accepted principles of financial analysis. A permissible credit period is usually allowed to a retailer to pay back the dues without paying any interest to the supplier. The retailer can pay the supplier either at the end of the credit period or later incurring interest charges on the unpaid balance for the overdue period. The retailer is expected to settle the account at a time before the end of the inventory cycle time because the payable interest rate is generally higher than the earned interest rate. The unit price seller charged by the seller and the length of the credit period offered by the seller to the buyer both influence the final demand for the product.

In developing mathematical models in inventory control it is assumed that payment will be made to the supplier for the goods immediately after receiving the consignment. However, in practice, it is found that the supplier allows a certain fixed period to settle the account. During this fixed period no interest is charged by the supplier, but beyond this period interest is charged under the
terms and conditions agreed upon and, moreover, interest can be earned on the revenue received during the credit period.

1.2.4 Inventory Models with Controllable Deterioration Rate

The nature of the inventory models varies depending upon the items under consideration. Generally goods can be classified into two categories viz., perishable and non-perishable items. Perishable items become obsolete and cannot be used after certain time period. The study of deteriorating items in inventory system has gained the attention of many researchers in this area of research. The study the inventory of deteriorating items was opened up by Within (1957). In his study, he considered the deterioration of fashion goods at the end of prescribed storage period. Ghare and Schrader (1963) extended the classical EOQ formula with exponential decay of inventory due to deterioration and gave a mathematical model of inventory of deteriorating items. The literature is replete with inventory models for deteriorating items on the basis of demand variations and various other conditions or constraints.

The rate of deterioration is faster in some products which causes loss to the retailer. The life span of such products can be increased using some preservatives. This rate of deterioration of items can be controlled using some preservation technology which reduces the deterioration rate thereby the retailer may increase the profit. The deterioration rate of items in the literature is viewed as an exogenous variable which is not controllable. In practice, the deterioration rate of products can be controlled and reduced through various efforts such as procedural changes and specialized equipment acquisition. The consideration of preservation technology is important due to rapid social changes and the fact that preservation technology can reduce the deterioration rate significantly. By the efforts of investing in preservation technology we can reduce the deterioration rate.

1.2.5 Weibull Distribution

The Weibull distribution is one of the most widely used life-time distribution. The main reason for considering the Weibull distribution is that it has been tested experimentally to provide a good fit for different types of characteristics for lifetime goods. The distribution is amenable and also has the advantage of having a closed form of cumulative distribution function.
While developing inventory models it is assumed that the lifetime of the commodity is random and follows three parameters Weibull distribution.

The formula for the probability density function of the general Weibull distribution is

\[ f(x) = \frac{\gamma}{\alpha} \left( \frac{x - \mu}{\alpha} \right)^{\gamma - 1} \exp \left( - \left( \frac{x - \mu}{\alpha} \right)^\gamma \right) \quad x \geq \mu; \ \gamma, \ \alpha > 0 \]

where \( \gamma \) is the shape parameter, \( \mu \) is the location parameter and \( \alpha \) is the scale parameter. The case where \( \mu = 0 \) and \( \alpha = 1 \) is called the standard Weibull distribution. The case where \( \mu = 0 \) is called the 2-parameter Weibull distribution. The equation for the standard Weibull distribution reduces to

\[ f(x) = \gamma x^{\gamma - 1} \exp \left( - x^\gamma \right) \quad x \geq 0; \ \gamma > 0 \]

The shape of the curve depends on the values of all the three parameters \( \alpha, \gamma, \mu \) and are shown in the following figure:
Exponential decay is a special case of Weibull distribution when $\gamma = 1$ and $\mu = 0$. It also includes various deterioration rates for different values of the parameters $\alpha$, $\gamma$ and $\mu$.

1.3 Literature Review

In this work, though the demand rate is assumed to be a quadratic function of time/price/stock, review of literature is classified into two categories viz., (i) linear (or exponential) demand rate and (ii) quadratic demand rate. The focus is on inventory models with regard to financial parameters viz., inflation, trade credit policy, controllable deterioration etc.

1.3.1 Perishable Inventory Models with Linear or Exponential Demand Rate

Halley and Higgins (1973) tried to investigate the relationship between inventory policy and trade credit policy in the context of the basic lot-size model. It is showed that, in general, optimality requires order quantity and payment time decisions determined simultaneously. The conditions under which the standard solutions become optimal are also developed.

Buzacott (1975) has shown that with inflation the choice of the inventory carrying charge used in the EOQ formula depends on the company's pricing policy. If prices change independently of replenishment order timing the inventory charge should be low and independent of the inflation rate. However, when no "double ticketing" is permitted and the company uses a constant percentage mark up the carrying charge is high and depends on the inflation rate and the mark-up. Only if the company is allowed a fixed monetary margin is the classical result for carrying charge valid.

Donaldson (1977) examined the classical no-shortage inventory policy for the case of a linear trend in demand. Using methods of calculus a computationally simple procedure for determining the optimal times for replenishment of inventory is established. Table extracts for the functions required are included.

Mishra (1979) presented a note on optimal inventory management under inflation. This paper develops a discounted-cost model that is similar to the classical economic order quantity model but includes inflation rates as parameters of the inventory system. A numerical problem is solved to illustrate the effects.
Bradshaw and Erol (1980) derived nearly time-optimal control policies for a class of linear time-invariant models of production-inventory systems which comprise a cascade of basic production-inventory subsystems with bounded input. The sub-optimal control policies are obtained by constructing the sampled-data model of the production-inventory system where the sampling period is chosen to give the maximum control input for the given shipping rate. The theory is illustrated by the presentation of the results of simulation studies which show the transient behavior of two production-inventory subsystems in cascade.

Chapman et al (1984) derived an economic order quantity model which explicitly considers possible credit periods allowed by suppliers. It is shown that the model is very sensitive to the length of the credit period, and to the relationship between the credit period and inventory level. It is also shown to be more sensitive to estimates of demand for inventory items and less sensitive to order costs than the basic economic order quantity model. A practical example illustrates this sensitivity, shows how inventory costs may be considerably reduced by taking the existence of a credit period into account, and demonstrates the implications for inventory and credit policies.

Davis and Gaither (1985) attempted to develop optimal order quantities for firms that are offered a one-time opportunity to delay payment for an order of a commodity. Optimal order quantities are also developed for extended payment privileges that occur at a reorder point of between reorder points. Six suppliers' extended payment scenarios are evaluated. A simulation analysis is conducted to determine the sensitivity of derived models to changes in the various input parameters. The simulation with realistic parameter values reveals that the additional discounted order quantity is insensitive to large changes in the ordering cost and unit price; sensitive to changes in the carrying cost and return rate of funds, but without significantly affecting the total cost; and extremely sensitive to the annual demand. Simple analytic decision rules are provided to guide firms that are offered such extended payment privileges.

Goyal (1985) developed mathematical models for obtaining the economic order quantity for an item for which the supplier permits a fixed delay in settling the amount owed to him. An example has been solved to illustrate the method.

Dallenbach (1986) discussed the issues related to inventory Control and Trade Credit. It is then showed that if trade credit surplus is taken into account, the optimal replenishment quantities decrease, rather than increase, as argued in some papers.

Shah et al (1988) studied economic ordering quantity when delay in payments of orders and Shortages are Permitted. Sensitivity of the model is discussed with numerical examples.

Murdeshwar (1988) derived an analytical procedure to obtain the optimal number of replenishments, the replenishment points and the time points at which the inventory reduces to zero. Numerical examples are also given to illustrate the results.

Carlson and Rousseau (1989) studied EOQ under date-terms supplier credit, making explicit the separate effects on inventory policy of the two components of carrying cost-namely, financing cost and other variable holding costs. It is shown that the calculation of EOQ is quite complicated when a distinction between these types of holding costs is made. Rather, optimal order quantity must be determined by search over a well-defined range of order quantities which encompasses the classical EOQ. The conclusion currently contained in the literature that the optimal order quantity under date terms is always given by an integer multiple of monthly demands no longer applies. In particular, a unique feature of date-terms credit is the possible existence of multiple EOQs.

Mandal and Phaujdar (1989a) discussed some EOQ models under permissible delay in payments.

Mandal and Phaujdar (1989b) developed an order-level inventory model for deteriorating items with uniform rate of production and stock-dependent demand. Shortages are allowed, and excess demand is backlogged. Results are illustrated with numerical examples.

Hamid (1989) presented a heuristic model for determining the ordering schedule when an inventoried item is subject to deterioration and demand changes linearly over time. It is noted that while the optimizing model developed by researchers fixes the ordering interval and varies
the ordering size, the heuristic permits variation in both replenishment-cycle length and the size of the order. Further it is noted that the heuristic produces a better solution than optimizing models in the study presented here.

Datta and Pal (1990) considered a stock level dependent demand rate and analyzed an infinite time horizon deterministic inventory system without shortage, which has a level-dependent demand rate up to a certain stock level and a constant demand for the rest of the cycle.

Haiping and Wang (1990) presented a (Ti, Si) inventory policy model for deteriorating items is presented in this paper. The model is developed based on the assumptions that (1) demand rate is deterministic and linearly changes with time, (2) deterioration rate is constant, (3) planning horizon is finite and known, and (4) replenishment periods are not equal. The model is solved using a dynamic programming method. The result is applicable to the case where the demand is either increasing or decreasing. A numerical example is provided to illustrate the model and solution procedure.

Padmanabhan and Vrat (1990) studied an EOQ model for items with stock dependent consumption rate and exponential decay. The model developed in this paper helps to determine optimum ordering quantity for stock dependent consumption rate items under inflationary environment with infinite replenishment rate without permitting shortages. The results are illustrated with a numerical example.

Goh (1994) apparently provides the only existing inventory model in which the demand is stock dependent and the holding cost is time dependent. Actually, Goh (1994) considers two types of holding cost variation: (a) a nonlinear function of storage time and (b) a nonlinear function of storage level.

Aggarwal and Jaggi (1995) developed an inventory system to obtain the optimum order quantity of deteriorating items under a permissible delay in payments. A numerical example is also given and discussed its sensitivity.

Padmanabhan and Vrat (1995) presented EOQ models for perishable items under stock dependent selling rate. It is assumed that the selling rate is assumed to be a function of current inventory level and rate of deterioration is taken to be constant. Under instantaneous
replenishment with zero lead time, the model incorporates aspects such as complete, partial, and no backlogging. EOQ is determined for maximizing the total profit in each of the situations. The models developed are illustrated through numerical examples and sensitivity analysis is reported.

Burwell et al (1997) developed an economic lot size model for price-dependent demand under quantity and freight discounts. An algorithm is developed to determine the optimal lot size and selling price for a class of demand functions, including constant price-elasticity and linear demand. A numerical example is provided to illustrate the model and a computer program is developed to implement the model derived in the paper.

Ray and Chaudhuri (1997) take the time value of money into account in analyzing an inventory system with stock-dependent demand rate and shortages. Two types of inflation rates are considered: internal (company) inflation, and external (general economy) inflation.

Chung (1998) discussed the economic quantity under conditions of permissible delay in payments. First, it is shown that the total annual variable cost function is convex. Second, with convexity, a theorem is developed to determine the economic order quantity. The theorem also reveals that the economic order quantity under conditions of permissible delay in payments is generally higher than the economic order quantity given by the classical economic order quantity model. Numerical examples are given to illustrate the theorem.

Jamal et al (2000) developed an inventory system for wholesaler–retailer problem as a cost minimization problem to determine the optimal payment time under various system parameters. The model is solved through an iterative search procedure and the overall findings indicate that the retailer has always an option to pay after the permissible credit period depending on interest rates, unit purchase and selling price, and the deterioration rate of the products.

Shao et al. (2000) determined the optimum quality target for a manufacturing process where several grades of customer specifications may be sold. Since rejected goods could be stored and sold later to another customer, variable holding costs are considered in the model.

Goyal and Giri (2001) reviewed the advances of deteriorating inventory literature since the early 1990s. The models available in the relevant literature have been suitably classified by the shelf-life characteristic of the inventoried goods. They have further been sub-classified on the basis of
demand variations and various other conditions or constraints. The motivations, extensions and generalizations of various models in each sub-class have been discussed in brief to bring out pertinent information regarding model developments in the last decade.

Beltran and Krass (2002) analyzed the dynamic lot sizing problem with positive or negative demands and allowed disposal of excess inventory. Assuming deterministic time varying demands and concave holding costs, an efficient dynamic programming algorithm is developed for this finite time horizon problem.

Chung and Huang (2003) tried to extend Goyal (1985) to the case that the units are replenished at a finite rate. It is observed that the work of Goyal (1985) will be a special case of this paper when the replenishment rate approaches to infinite.

Abad and Jaggi (2003) considered a seller–buyer channel in which the end demand is price sensitive and the seller may offer trade credit to the buyer. This paper provides procedures for determining the seller's and the buyer's policies under non-cooperative as well as cooperative relationships. In the non-cooperative case, they determined for the seller the optimal unit price and the length of the credit period. Further, for the cooperative structure, they have given a procedure for characterizing Pareto efficient solutions.

Shinn and Hwang (2003) studied the problem of determining the retailer's optimal price and lot size simultaneously when the supplier permits delay in payments for an order of a product whose demand rate is represented by a constant price elasticity function. It is assumed that inventory is depleted not only by customer's demand but also by deterioration. Investigation of the properties of an optimal solution allows us to develop an algorithm whose validity is illustrated using an example problem.

Mondal et al (2003) discussed an instantaneous replenishment inventory model of ameliorating items for prescribed time period. It is assumed that the rate of amelioration follows the Weibull distribution and the demand rate is a function of selling price of an item. Shortages are not allowed. The salvage values of deteriorated units due to death of such items and the cost for feeding the life stocks are taken into account. The model is numerically illustrated with the demand rate dependent linearly on the selling price of the item. Finally, the effect of
amelioration as well as the deterioration on an inventory policy has been studied. The sensitivity of the average profit with respect to the amelioration parameters has also been presented with the help of Analysis of Variance method.

You (2005) investigated the problem of jointly determining the order size and optimal prices for a perishable inventory system under the condition that demand is time and price dependent. It is assumed that a decision-maker has the opportunity to adjust prices before the end of the sales season to influence demand and to improve revenues. A mathematical model is developed to find the optimal number of prices, the optimal prices and the order quantity. Analytical results showed that a stationary solution to the Kuhn–Tucker necessary conditions can be found and it is shown to be the optimal solution. The analytical results lead us to derive a solution procedure for determining the optimal order size and prices.

Chung et al (2005) discussed the problem of determining the economic order quantity under conditions of permissible delay in payments. The delay in payments depends on the quantity ordered. When the order quantity is less than the quantity at which the delay in payments is permitted, the payment for the item must be made immediately. Otherwise, the fixed trade credit period is permitted. The minimization of the total variable cost per unit of time is taken as the objective function. An algorithm to determine the economic order quantity is developed. The results obtained in this paper generalize some already published results.

Wu et al (2006) considered the problem of determining the optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand. In this model, shortages are allowed and the backlogging rate is variable and dependent on the waiting time for the next replenishment. The necessary and sufficient conditions of the existence and uniqueness of the optimal solution are shown. As the special cases, the results for the models with instantaneous or non-instantaneous deterioration rate and with or without shortages are derived. Sensitivity analysis of the optimal solution with respect to major parameters is carried out. Finally, four numerical examples are presented to demonstrate the developed model and the solution procedure.

Huang (2007) investigated the optimal retailer’s replenishment decisions under two levels of trade credit policy within the economic production quantity (EPQ) framework. It is assumed that
the supplier would offer the retailer a delay period and the retailer also adopts the trade credit policy to stimulate his/her customer demand to develop the retailer’s replenishment model under the replenishment rate is finite. It is also assumed that the retailer’s trade credit period offered by supplier M is not shorter than the customer’s trade credit period offered by retailer N. The author modeled the retailer’s inventory system as a cost minimization problem to determine the retailer’s optimal replenishment decisions. Then three theorems are developed to efficiently determine the optimal replenishment decisions for the retailer. Also some previously published results of other authors are deduced as special cases. Finally, numerical examples are given to illustrate the theorems obtained in this paper.

Teng et al (2007) extended Goyal’s model to develop an Economic Order Quantity (EOQ) model in which the supplier offers the retailer the permissible delay period M, and the retailer in turn provides the trade credit period N (with N ≤ M) to his/her customers. In addition, we assume that (1) the retailer's selling price per unit is necessarily higher than its unit cost, and (2) the interest rate charged by a supplier or a bank is not necessarily higher than the retailer's investment return rate. We then establish an appropriate EOQ model with trade credit financing, and provide an easy-to-use closed-form solution to the problem. Furthermore, it is found that a well-established buyer may order a lower quantity and take the benefit of the permissible delay more frequently, which contradicts to the result by the previous researchers. Sensitivity analyses are performed to illustrate the theoretical and managerial results.

Teng and Goyal (2007) extended the work of Huang (2003). It is also proposed a generalized formulation to the problem and established the proper theoretical results to obtain the optimal solution. The sensitivity of the proposed model is discussed with a real-world inventory problem and its optimal solution.

Ajanta Roy (2008) developed a deterministic inventory model when the deterioration rate is time proportional. It is assumed that the demand rate is a function of selling price and holding cost is time dependent. The model is first solved allowing shortage in inventory. The case of without shortage is also discussed. The results are illustrated with the help of numerical example. The sensitivity of the solution with the changes of the values of the parameters associated with the model is discussed.
Valliathal and Uthayakumar (2009) studied an EOQ model for perishable items under stock and time-dependent selling rate with shortages. The authors studied the effects time dependent demand on the total profit and time factors. It is proved that the optimal replenishment solution not only exists but is also unique. Numerical examples are given to illustrate the application of developed model.

Yen-Wen Wang et al (2010) developed inventory models for deteriorating items with variable selling price under stock-dependent demand. A theoretical analysis of the existence and uniqueness of the optimal solutions without shortages and with complete backlogging is presented. Numerical examples of the parameters are also presented to illustrate these two models. Finally, we compare the optimal solutions without shortages to those with complete backlogging.

Valliathal and Uthayakumar (2011) discussed a new study of an EOQ model for deteriorating items with shortages under inflation and time discounting. The authors discussed the effects of inflation and time discounting on an EOQ model for deteriorating items under stock-dependent demand and time-dependent partial backlogging. The inventory model is studied under the replenishment policy starting with no shortages. Numerical examples are presented to determine the developed model and the solution procedure. Sensitivity analysis of the optimal solution with respect to major parameters is presented.

Singh and Malik (2011) developed an inventory model with stock-dependent demand with two storages capacity for non-instantaneous deteriorating items. In this model, shortages are allowed and the backlogging rate is variable and dependent on the waiting time for the next replenishment. The necessary and sufficient conditions of the existence and uniqueness of the optimal solution are obtained. Further a numerical example is presented to demonstrate the developed model.

Kuo-Lung Hou et al. (2011) presented an inventory model for deteriorating items with stock-dependent selling rate and partial backlogging under inflation. It is assumed that shortages are allowed and the unsatisfied demand is partially backlogged at the exponential rate with respect to the waiting time. The optimal order quantity and the total present value of profits are obtained. A
numerical example and sensitivity analysis are presented to illustrate the proposed model and particular cases of the model are also discussed.

Patra and Ratha (2012) discussed an inventory replenishment policy for deteriorating items under inflation in a stock dependent consumption market with shortage. The effects of inflation and time value of money are incorporated into the model. It is assumed that the goods in the inventory are deteriorating over time at a constant rate. The inventory policy is discussed over a finite time horizon with several reorder points. The results are discussed with a numerical example and a sensitivity analysis of the optimal solution with respect to the parameters of the system is carried out.

Maisuriya and Bhatawala (2013) discussed an EOQ model with stock dependent demand rate and variable time. It is used instantaneous case of replenishment and finite case of replenishment for different functional relationship to get optimum value of stock quantity and total cost per unit time. EOQ models are used to get an optimal order quantity level.

Rekha et al. (2013) developed an optimal inventory model for time dependent decaying items with stock dependent demand rate and shortages. It is assumed that the rate of deterioration is a linear function of time and demand rate is stock dependent in linear form. Shortages are allowed and shortages are completely backlogged. A numerical example is given to illustrate the model developed. The model is solved analytically by maximizing the total profit.

Mishra (2013) developed an inventory model for instantaneous deteriorating items with the consideration of the facts that the deterioration rate can be controlled by using the preservation technology (PT). It is assumed that the demand rate and holding cost both are linear function of time, deterioration rate is constant and backlogging rate is variable. Further it is assumed that the length of the next replenishment, shortages are allowed and partially backlogged and obtained an analytical solution which optimizes the total cost of the proposed inventory model.

Raman and Reena (2014) developed an inventory model for Weibull deteriorating items with variable selling price and stock dependent demand. It is assumed that the holding cost is linear function of time and shortages are not allowed. Numerical example is presented and sensitivity analysis is also carried out for parameters.
Raman and Reena (2014) studied deteriorating items inventory model with stock dependent demand under shortages and variable selling price. The model is solved assuming the holding cost is linear function of time and shortages are allowed and completely backlogged. A numerical example is given and sensitivity analysis is also carried out for all the parameters.

Pal and Chandra (2014) studied a periodic review inventory model with stock dependent demand, permissible delay in payment and price discount on backorders. Permissible delay in payments is allowed in the model. A numerical example is given to illustrate the model.

Yashveer et al. (2014) studied an inflation induced stock-dependent demand inventory model with permissible delay in payment. This inventory model is developed for non-instantaneous deteriorating items. The purpose of this paper is to obtain the optimal policies for maximizing the total profits. Numerical examples are provided to demonstrate the developed model and also to provide the solution algorithm.

Tripathi and Mishra (2014) developed an inventory model with inventory-dependent demand for deteriorating items in a single warehouse system. Where shortages are allowed and it is completely backlogged. The planning horizon is finite. It is shown that there exists a unique optimal cycle time to minimize the total inventory cost per cycle. Truncated Taylor’s series expansion is used for finding closed form optimal cycle time, optimal time to finish positive inventory and optimal total inventory cost per cycle. A numerical example is given to validate the proposed model. The sensitivity analysis of the solution with the changes of the values of parameters associated with the model has also been discussed.

Nita Shah et al (2014) studied optimal integrated inventory policy for stock-dependent demand when trade credit is linked to order quantity. The authors analyzed integrated inventory policy for vendor-buyer when demand is stock-dependent and trade credit is linked to order quantity. The joint total profit is maximized to determine buyer’s order quantity and the number of shipments from the vendor to the buyer during one cycle. Numerical examples and sensitivity analysis are given to find critical inventory parameters. Managerial insights are also obtained.
1.3.2 Perishable Inventory Models with Quadratic Demand Rate

Bhandari and Sharma (2000) considered a single-period inventory problem where demand is taken to be quadratic in nature. The model contains usual ingredients of a newsboy problem: sale price, purchase cost, salvage value and shortage cost. In addition, there is a cost per unit of marketing effort, and the demand distribution is stochastically increasing in that effort. It is assumed that the distribution of demand can be shifted up by an increase in sales effort. The paper deals with the simultaneous determination of optimal order quantity and sales effort. The optimum profit associated with optimum marketing effort is also determined. The sensitivity analysis of the results is provided, to consider variation in optimal order quantity and sales effort with the changes in unit sales price, unit purchase cost, unit salvage value, unit shortage cost and cost per unit of marketing effort.

Kharna and Chaudhuri (2003) discussed an order-level inventory problem with the demand rate being represented by a continuous, quadratic function of time. It is assumed that a constant fraction of the on-hand inventory deteriorates per unit of time. The solution of the model is discussed both for infinite and finite time-horizon. A numerical example is taken up to illustrate the solution procedure and sensitivity analysis is also carried out.

Bhandari and Sharma (2004) have studied a single period inventory problem with quadratic demand distribution under the influence of marketing policies.

Sana and Chaudhary (2004) developed a stock-review inventory model for perishable items with uniform replenishment rate and stock-dependent demand. The deterioration function per unit time is a quadratic function of time. The associative cost function under some constraints is optimized due to the limitation of storage capacity.

Ghosh and Chaudhuri (2004) developed an inventory model for a deteriorating item having an instantaneous supply, a quadratic time-varying demand and shortages in inventory. A two-parameter Weibull distribution is taken to represent the time to deterioration. The model is solved analytically to obtain the optimal solution of the problem. It is then illustrated with the help of a numerical example. The sensitivity of the optimal solution towards changes in the values of different system parameters is also studied.
Kalam et al (2010) discussed a production lot-size inventory model for Weibull deteriorating item with quadratic demand, quadratic production and shortages. Shortages of cycle are allowed in the inventory system. The solution of the model is discussed for finite time horizon. The numerical example is taken up to illustrate the solution procedure and sensitivity analysis of the model.

Patra et al (2010) studied an order level EOQ model for deteriorating items in a single warehouse system with price depended demand in non-linear (quadratic) form. The sensitivity of the model is discussed with an illustration.

Begum et al (2010) developed order level inventory models for deteriorating items with quadratic demand. It is assumed that the finite production rate is proportional to the demand rate and the deterioration is time proportional. Also it is assumed that the unit production cost is inversely proportional to the demand rate. The model is solved to minimize the total average cost. Numerical examples are used to illustrate the two developed models. Sensitivity analysis of the optimal solution with respect to major parameters is carried out.

Venkateswarlu and Mohan (2011) developed inventory models for deteriorating items with time dependent quadratic demand rate. The salvage value is included for deteriorated items in the model. The results are explained with suitable examples. The sensitivity of these models is discussed to study the effect of salvage value.

Venkateswarlu and Mohan (2013) also studied a deterministic inventory model for deteriorating items when the demand rate is assumed to be a function of price which is quadratic in nature and the deterioration rate is proportional to time. The model is solved when shortages occur in inventory. Later, the case of salvage is discussed. The sensitivity of the model is discussed with a numerical example.

Singh and Pattnayak (2013) were motivated by Khanra, Ghosh and Chaudhuri’s (2011) paper extending their model to allow for a variable rate of deterioration when delay in payment is permissible. The time varying demand rate is taken to be a quadratic function of time. For settling the account, the model is developed under two circumstances: case-1: The credit period is less than or equal to the cycle time and case-2: the credit period is greater than the cycle time.
Nita H. Shah et. al., (2014) developed the supplier gives its customers a credit period to settle the account which attracts more customers and boosts market demand. Yet the present of credit period leads to default risk for the supplier. The default risk associated with sales revenue is taken into consideration in objective of profit maximization. Inventory system deals with quadratic demand which is function of permissible trade credit. Also products in the inventory are deteriorating at constant rate. The necessary condition to obtain the seller’s optimal decision about setting the permissible credit period and purchase quantity is discussed.

Venkateswarlu and Mohan (2014) studied an inventory model for deteriorating products with demand rate is quadratic function of time. The salvage value is used for deteriorated items in the system and the deterioration is considered as constant. Suitable numerical example and sensitivity analysis is also discussed.

Umakanta Mishra (2015) studied an order level inventory system with time dependent Weibull deterioration and quadratic demand rate where holding costs as a linear function of time. The proposed model considered here to allows for shortages, and the demand is partially backlogged. The model is solved analytically by minimizing the total inventory cost.

1.4 Motivations

The purpose of the study is to give a new dimension to the inventory literature on time-varying demand patterns. Researchers have extensively discussed various types of inventory models with linear trend (positive or negative) in demand. The main limitation in linear time-varying demand rate is that it implies a uniform change in the demand rate per unit time. This rarely happens in the case of any commodity in the market. In recent years, some models have been developed with a demand rate that changes exponentially with time. Demands for spare parts of new aero-planes, computer chips of advanced computer machines, etc. increase very rapidly while the demands for spares of the obsolete aero-planes, computers etc. decrease very rapidly with time. Some modelers suggest that this type of rapid change in demand can be represented by an exponential function of time. But it is felt that an exponential rate of change in demand is extraordinarily high and the demand fluctuation of any commodity in the real market cannot be so high. A realistic approach is to think of accelerated growth (or decline) in the demand rate in the situations cited above and it can be best represented by a quadratic function of time.
1.5 Objectives

The main objective of this study is to get more insights in inventory management models considering different financial parameters viz., inflation, trade credit policy, controllable deterioration etc. when demand follows a quadratic function for perishable items. In view of tremendous amount of literature on inventory models over last six decades, a brief review of some of the research articles relating to this work is presented in the following section.

1.6 Our Contribution

From literature review, it is observed that not much research has gone into inventory models for perishable items with quadratic demand rate. Thus an attempt is made to understand inventory systems with perishable items which follow quadratic demand rate. The organization of the thesis is as follows:

In Chapter – 2, an attempt is made to develop an inventory model for perishable items with the assumption that the demand is quadratic function of time under inflation. It is assumed that the rate of deterioration is constant. Under instantaneous replenishment with zero lead-time, EOQ is determined for optimizing the system total cost. The sensitivity analysis is carried out with a numerical example.

Chapter – 3 deals with inventory models for perishable items when the demand rate is price dependent quadratic demand. It is assumed that the deterioration rate is directly proportional to time. The total profit is calculated when shortages are not allowed. The effect of inflation is observed on the models. The sensitivity analysis is done with a numerical example at the end of this chapter.

In Chapter – 4, inventory models are developed with stock dependent quadratic demand for perishable items. It is assumed that the deterioration rate is constant. The effect of inflation on the system total profit is studied for items having inventory level dependent quadratic demand. At last, the sensitivity of the models is presented and analyzed.

Chapter – 5 is devoted to developing deterministic EOQ models for deteriorating items with time dependent quadratic demand rate. It is assumed that the deterioration rate is constant and the supplier offers the retailer a credit period to settle the account of the procurement units. To solve
the model it is further assumed that shortages are not allowed and the replenishment rate is instantaneous. To minimize the retailers total inventory cost, salvage value is also taken to see its effect on the total cost. A numerical example is given to study the effect of allowable credit period and the total cost of the retailer. Sensitivity is presented to test the effectiveness of the models.

In Chapter – 6, an inventory system is considered with trended demand which is assumed to be a function of price and time dependent quadratic demand. For minimizing the total cost of the inventory system, it is assumed that the deterioration rate is constant and the supplier offers the retailer a credit period to settle the account of the procurement units. To solve the model it is further assumed that shortages are not allowed. Salvage value is also considered and observed its effect on the total cost. A numerical example is given to test the strength of the model. Critical parameters are identified by studying the sensitivity of the system.

Chapter - 7 presents a deterministic inventory model for deteriorating items when the deterioration rate is controllable using preservation technology and the demand rate is a quadratic function of time. The optimal cost of the system is calculated allowing shortages. Further it is assumed that the holding cost is a linear function of time. A numerical example is given and the robustness of the model is tested through sensitive analysis.

Chapter – 8 deals with production lot size inventory models for deteriorating items with time dependent quadratic demand rate. It is assumed that the deterioration rate follows Weibull distribution. It is further assumed that the holding cost is a linear function of time. Inventory models are developed without considering shortages. The salvage value is considered while calculating the optimal policies that maximize the revenue of the system. Numerical example is given and discussed the sensitivity of these models.