The present thesis entitled “Some Aspects of Records and Generalized Order Statistics” is being submitted for the award of the degree of Doctor of Philosophy (Statistics). It is based on five chapters and deals mainly with exact expressions and some recurrence relations for single and product moments and moment generating functions of various ordered random variables from specific continuous distributions. We have also exploited the conditional expectation of functions and recurrence relations for single moments of ordered random variables to characterize these distributions.

Several models of ordered random variables, including order statistics and record values, are contained in the model of generalized order statistics introduced by Kamps (1995). The concept of generalized order statistics provides a large class of models with many interesting and useful properties for both description and analysis of practical problems. Applications are multifarious in a variety of disciplines and in particular in reliability.

The model of generalized order statistics can be easily applicable in practice problems except when $F(x)$ is so called inverse distribution function (inverse exponential, inverse Weibull and inverse Pareto, etc). For this, the concept of lower generalized order statistics was given by Pawlas and Szynal (2001). Later Burkschat et al. (2003) introduced it as dual generalized order statistics so as to enable a common approach to descending ordered random variables like reversed order statistics and lower record values.

The introductory Chapter I contains a brief and comprehensive account of the historical developments of different models of ordered random
Abstract

variables such as order statistics, upper and lower record values, $k$–th records, generalized order statistics and lower (dual) generalized order statistics and deals with some basic concepts and results associated with them. Some continuous distributions, which are used in subsequent chapters, have also been discussed.

Chapter II contains exact expressions and some recurrence relations satisfied by single and product moments of upper and lower record values drawn from Gompertz distribution given by

$$F(x) = 1 - \exp\left(-\frac{\beta}{\alpha}(e^{\alpha x} - 1)\right), \quad x \geq 0, \alpha > 0, \beta > 0,$$

and generalized inverse Weibull distribution

$$F(x) = e^{-\theta(\alpha x)^\beta}, \quad x > 0, \alpha > 0, \beta > 0, \theta > 0.$$  

Further, conditional expectation of functions and recurrence relations for single moments of upper and lower record values have been utilized to characterize these distributions.

Chapter III deals with the recurrence relations for single and product moments of generalized order statistics from doubly truncated linear exponential distribution

$$F(x) = 1 - e^{-(\lambda x + \theta \alpha^2 / 2)}, \quad x > 0, \lambda > 0, \theta > 0.$$  

These relations are deduced for moments of record values and order statistics. Further, the recurrence relation of single moments of generalized order statistics has been applied to characterize the linear exponential distribution. Exact expression for single moments has also been obtained.

Chapter IV discusses the explicit expressions for single moments and some recurrence relations for single and product moments of lower
Abstract

generalized order statistics from generalized inverse Weibull distribution. The relations for order statistics and lower records are deduced from the relations derived. Further, the distribution has been characterized on using conditional expectation of function of lower generalized order statistics and a recurrence relation for single moments. Using the established explicit expressions we calculate the first four moments and variances of order statistics and lower records for $n = 1(1)5$.

Chapter V embodies the explicit expressions and recurrence relations satisfied by single and joint moment generating functions of generalized order statistics and lower generalized order statistics drawn from Gompertz distribution and generalized exponential distribution

$$F(x) = (1 - e^{-x})^\alpha, \quad x > 0, \quad \alpha > 0.$$  

The results for order statistics, upper and lower record values are deduced as special cases. Further, conditional expectation of functions of generalized and lower generalized order statistics have been utilized to obtain the characterizations of the Gompertz and generalized exponential distributions.

A comprehensive bibliography has also been given at the end, which has been referred during our research.

References

