CHAPTER VI

IDENTIFICATION OF A ZERO REFERENCE LEVEL

To get the absolute topography of the different isobaric surfaces, an isobaric surface which coincides with a level surface should be selected as a zero reference level. Since no water motion is possible at this level, it is called a level of no motion. Proper selection of a zero reference level is important for the presentation of an oceanic circulation pattern. This chapter presents a method for the identification of a zero reference level.

6.1. Determination of an appropriate reference surface

For the determination of the absolute magnitudes of computed currents, the reference surface used for the computation of the relative currents should either be a level of no motion or the distribution of the current velocity along the reference surface should be known. The latter method is not widely used for the following reasons:

1) Current measurement at sea is very difficult and subject to uncertainties.

2) The measured current is the total current which may include the effects of winds, tides, internal waves, etc. in addition to the geostrophic component of the current. Hence, for the computation of absolute currents, identification of a 'level of no motion' is usually attempted.

Literature cites several methods for the identification of a level of no motion. Some earlier investigators assumed a level of no motion as deep as possible reasoning that the velocity decreases with depth and that at great depths the isobaric surfaces are nearly horizontal. This method has the disadvantage that the error in the computed current may make the picture of the circulation pattern completely unreliable since the error in the computed currents increases with increase in the depth of reference surface. Other methods developed by different authors were critically examined by Fomin (1964) and may be summarised as follows:

a) Dietrich's method

The idea that the intermediate oxygen minimum layer in the ocean corresponds to a layer with minimum horizontal water motion was first advanced by Jacobsen (1916) and was further developed and applied in practice by Wust (1935) and Dietrich (1936). Several authors have refuted this idea (Rossby, 1936; Iselin, 1936; Wattenburg, 1938; Seiwell, 1937) and have argued that an oxygen minimum layer need not necessarily be a layer of minimum current velocity. Also Sverdrup (1938) showed that an oxygen minimum layer cannot have any general dynamic significance.

b) Parr's method

Parr (1938) advanced the idea that ~~since~~ the thickness of a layer bounded by two isopycnic surfaces (surfaces of equal density) cannot remain constant in the region of a current and should vary perpendicular to the direction of current and where the thickness of such a layer is constant, the horizontal water motion is zero or it is a layer of no motion. He also suggested a graphical method to identify such a layer. Fomin (1964) showed that the above is a necessary but not a sufficient condition for the existance of a layer of no motion because the vertical variation of current velocity depends not only on the slope of the isopycnic surfaces but also on the vertical density gradient. A layer bounded by two isopycnic surfaces in a region where there is a strong vertical density gradient will be least distorted compared to the overlaying and underlaying layers in the presence of a strong geostrophic current.

c) <u>Hidaka's method I</u>

The current velocity field in the sea is in constant interaction with the field of any physical or chemical property of sea water and there is mutual adjustments between these two fields. This fact was used by Hidaka (1949) to develop a method for the determination of the level of no motion in the sea from the salinity distribution. He argued that the surface where the second derivative of the vertical salinity distribution, $\frac{\partial^2 S}{\partial z^2}$, is equal to zero should be a level of no motion. Fomin (1964) showed that Hidaka's method provides the identification of a surface that has very definite structural features of the salinity field, but that such features are not uniquely related to the current velocity field.

d) <u>Hidaka's method II</u>

Hidaka (1940a,1940b,1950) suggested another method, for the identification of the level of no motion, based on continuity considerations for the stationary distribution of certain physical and chemical properties of sea water. He considered a volume of water in the form of a tetrahedral prism that extends from sea surface to the bottom. Assuming continuity of water volume, Hidaka obtained a set of equations, the solution of which lead to the identification of the level of no motion. Defant (1941a), however, has raised objection on the practical applicability of this method on two grounds:

i) The assumption on the continuity of water volume is not strictly true theoretically because, continuity condition requires constant mass and not constant volume.

ii) The set of equations obtained by Hidaka is practically inconsistent and cannot be solved with the existing accuracy of measurements at sea.

e) Sverdrup's method

Sverdrup *et al*. (1942) suggested a method for the identification of the level of no motion which uses the known fact that, in the steady state, the total water transport through any cross section of an oceanic area, which extends from one shore to the other, should be zero. He considered a horizontal reference surface and argued that the reference surface will be in the layer of no motion when the water transport through the section above the reference surface is equal to the water transport below the reference surface. Fomin (1964)

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points out that this method is unsuitable on three counts:

i) The currents that compensate each other need not necessarily be in the vertical plane. They can be in the horizontal plane as well.

ii) The accuracy of water density determination is insufficient for a successful computation of water transport, particularly in the deep layers.

iii) For a successful identification of the level of no motion, the pure drift component of the current should also be considered which will not only complicate the computation but also introduce additional errors caused by the uncertainty of the wind field that corresponds to the stationary case.

f) Defant's method

Defant (1941b), while comparing the differences in the relative dynamic depth anomalies of given isobaric surfaces between adjacent oceanographic stations in the Atlantic ocean, found that at certain levels these differences were practically constant over a large depth interval. Such constant relative pressure differences between adjacent stations can be interpreted in two ways:

i) The whole layer of deep water has a constant velocity.

ii) The whole layer is uniformly at rest.

The first interpretation should be considered as unreasonable since the constant velocity in most cases proves to be considerably larger than the surface velocity, a result which is against the present oceanographic experience. Hence, Defant concluded that the second interpretation is valid and the layer may be considered as a layer of no motion.

Eventhough Defant's method is the most justified and widely used one for the identification of the layer of no motion, it is not without objections. Wust (1951) put forward two arguments against the method:

i) It is doubtful whether we can find a level of no motion in a layer of constant relative pressure differences when the current computed has the same direction above and below the layer.

ii) If current is computed in the immediate vicinity of the bottom relative to a level of no motion identified in a layer of constant relative pressure differences, it is frequently found that the magnitude of the computed current exceeds the magnitude of the current at the surface and in a direction opposite to the direction of the surface current. This is an unreasonable result according to the present knowledge of the oceans.

Fomin (1964) pointed out that in a vertical section of the differences in the relative dynamic depth anomalies of isobaric surfaces between adjacent stations, selection of the depth of level of no motion is not always unique because, in some cases, there may be several layers with constant relative pressure differences and selection of any one of them is arbitrary. Also, for some pairs of stations, there may not be a layer with constant relative pressure difference at all. He also pointed out that in a region where the current velocity is low, Defant's method is liable to fail because the relative dynamic depth differences between adjacent stations in such regions will be of the same order of magnitude as the computational error itself. The method is liable to fail in a weakly stratified water body for the same reason. It will give good results only in strongly stratified bodies of water and in regions where the current velocity is high.

Sastry and D'Souza (1971) have reported that application of Defant's method in the Arabian sea region did not yield a zero level which could be accepted with any reasonable degree of certainty.

g) Mamaev's method

Mamaev (1955) suggested a method which is very similar to Defant's method. His method is based on the fact that in a layer where the differences in relative dynamic depth anomaly between adjacent stations is constant, the specific volume anomalies should be equal. Hence a vertical distribution of the differences in specific volume anomalies at two neighbouring stations will show a zero value whenever the differences in the relative dynamic depth anomalies between these two stations are constant. This method has the advantage that the minimum or zero value of the differences in specific volume anomaly is easily found compared to a layer where the differences in dynamic depth anomalies are constant. Since this method is similar to Defant's method in principle, this method also is subject to the objections raised in the context of the latter.

6.2. Identification of a level where the current velocity is negligibly small.

The foregoing discussion on the methods used for the identification of the level of no motion in the sea brings out the fact that there is no fool-proof method for the same. In a complicated mass field, as in the real ocean, it is unreasonable to think either that the level of no motion coincides with an isobaric surface over large areas or that it coincides with real surfaces in the ocean such as isothermal surface, isohaline surface, etc. In oceanic areas where the current continues from the surface right to the bottom without changing direction, a level of no motion will not exist at all. A surface where there is no water motion should be considered as having a very complex topography, which sometimes crop out at the surface where there are two opposing currents in the horizontal plane. The identification of such a surface in an oceanic area is beyond the means of present day dynamic methods in oceanography. Hence it is advisable to try to identify a level where the water motion is negligibly small and use this level for the computation of currents so that the computed currents will not be very much different from the absolute currents. A method for the identification of such a level has been described by Fomin (1964) by the application of the density model of Shtokman (1950,1951) to the oceanic mass field.

The component of the geostrophic current velocity, V_x, in the northern hemisphere, perpendicular to a vertical cross section is obtained by equating the pressure gradient force with the Coriolis force, assuming that the motion is non-accelerated and frictionless.

i.e., $$f V_x \rho = \frac{\partial p}{\partial y} \qquad (6.1)$$

where $f = 2\,\omega \sin\phi$ is the Coriolis parameter, ρ is the density of water and p is the sea pressure. The right hand side of equation (6.1) may be written in terms of density ρ by the use of hydrostatic equation, namely,

$$dp = g\,\rho\,dz \qquad (6.2)$$

where g is the acceleration due to gravity and z is the depth from the sea surface. Integrating Equation (6.2) we get

$$\int_0^z dp = p = g \int_0^z \rho\,dz \qquad (6.3)$$

Differentiating equation (6.3) with respect to y

$$\frac{\partial p}{\partial y} = g \int_0^z \frac{\partial \rho}{\partial y}\,dz \qquad (6.4)$$

Substituting equation (6.4) in equation (6.1)

$$f V_x \bar{\rho} = g \int_0^z \frac{\partial \rho}{\partial y}\,dz$$

$$\therefore \quad V_x = \frac{g}{f\bar{\rho}} \int_0^z \frac{\partial \rho}{\partial y}\, dz \qquad (6.5)$$

where $\bar{\rho}$ is the average density of the water column.

Shtokman's density model, when applied to the oceanic mass field, assumes that the density at the reference surface, which is the lower boundary of the baroclinic layer, is constant and has a value equal to $\rho(o)$ and that the deviation of density, $\rho(x,y,z)$, at any point from this constant value is given as the product of two functions, one of which depends only on the vertical coordinate, and the other only on the horizontal coordinate.

$$\text{i.e.,} \quad \rho(o) - \rho(x,y,z) = \delta(z)\ f(x,y) \qquad (6.6)$$

where f(x,y) is known as the function of influence. Hence, along a vertical, f(x,y) should be a constant.

$$\text{i.e.,} \quad \rho(o) - \rho(z) = K\,\delta(z) \qquad (6.7)$$

The assumption that the function of influence is constant implies that, in an oceanic region where the vertical distribution curves of density are similar in appearance, the function $\delta(z)$ will be different only by a constant multiplier. This will be true in oceanic areas where the T-S curves are similar and similarity of T-S curves is retained over very large oceanic areas.

Now let us locate the origin of the coordinate system on the assumed reference surface with the z axis directed vertically upwards. Substituting equation (6.6) in equation (6.5) we get

$$V_x(z) = -\frac{g}{f\bar{\rho}} \frac{\partial f(x,y)}{\partial y} K \int_0^z \delta(z)\, dz \qquad (6.8)$$

Assuming z = H at the sea surface, the geostrophic current at the sea surface is

$$V_x(H) = \frac{-g}{f\bar{\rho}} \frac{\partial f(x,y)}{\partial y} K \int_0^H \delta(z)\, dz \qquad (6.9)$$

Solving for $\frac{\partial f(x,y)}{\partial y}$ from equation (6.9) and substituting in equation (6.8) we obtain

$$V_x(z) = V_x(H) \frac{\int_0^z \delta(z)\, dz}{\int_0^H \delta(z)\, dz} = V_x(H)\, \phi(z) \qquad (6.10)$$

where $\phi(z)$ is known as the stratification function. Equation (6.10) describes the vertical distribution of geostrophic current velocity and may be used to compute any subsurface current, once the absolute value of the geostrophic component of the sea surface current is known by measurement. Equation (6.10) shows that the strarification function will be the same where the function $\delta(z)$ at different points is different only by a constant multiplier.

Integration of equation (6.10) between the limits z = 0 to z = H yields

$$\int_0^H V_x(z)\,dz = \int_0^H V_x(H)\,\phi(z)\,dz$$

$$= V_x(H)\int_0^H \phi(z)\,dz$$

$$= \frac{V_x(H)}{F(H)} \qquad (6.11)$$

where the function F(H) is defined by

$$F(H) = \frac{1}{\int_0^H \phi(z)\,dz}$$

$$= \frac{\int_0^H \delta(z)\,dz}{\int_0^H \int_0^z \delta(z)\,(dz)^2} \qquad (6.12)$$

If the geostrophic current velocity component in equation (6.11) is considered as average current between two stations at a distance L, then the time rate of water transport between the verticals at the two stations between the sea surface and the assumed reference surface is given by

$$L\int_0^H V_x(z)\,dz = \frac{L\,V_x(H)}{F(H)} \qquad (6.13)$$

It is easy to see that since $L\,V_x(H)$ is a constant, the time rate of water transport varies inversely with the function F(H). It will be illustrated later in this section that function F(H) decreases in magnitude as the value of H increases, and becomes constant below a particular value of depth of the assumed reference surface. Hence the time rate of water transport remains constant irrespective of the reference surface selected, once it is below the particular reference surface where the value of the function F(H) becomes constant. This means that the layer of water below this reference surface is a layer of no motion. Hence the reference surface where the value of the function F(H) becomes constant is a level of no motion. The function F(H) will be the same in a region where the function $\delta(z)$ is different only by a constant multiplier.

In fact, Shtokman's density model, when applied to the mass field of the ocean, does not ensure that the reference level thus identified is actually a level where the geostrophic current velocity is zero. The model only tells that if a current exists at the selected reference level, it will continue unabated to the bottom, because the constancy of the function F(H) in a layer indicates zero vertical density gradient, a condition which cannot change the magnitude of current in the vertical. The result that reference surface identified by the application of Shtokman's model to the oceanic mass field is a level of no motion is due to a tactical assumption that the velocity of geostrophic current at the sea bottom is zero. Since we generally expect only negligibly small current at the bottom of the sea where depths are large, the reference surface selected by the application of Shtokman's density model may be considered as a surface where the geostrophic current velocity is negligibly small and hence may be considered as a zero surface for the purpose of computing absolute currents.

Shtokman's density model cannot be applied in an oceanic region where the current changes in direction with depth and where it first increases and then decreases with depth. This is because of the assumption that the function of influence is constant along the vertical and so the velocity of geostrophic current decreases with depth without change in direction. Hence the boundary between two oppositely directed currents can only be vertical.

This method is applied to the Arabian sea region for the identification of a zero reference level where the geostrophic current velocity is negligibly small. The data used are obtained from the Cruise No.31-0197 of the U.S. Research Vessel 'Atlantis' in the Arabian sea during August-September 1963. Fig.3 gives the location of the stations used in the present study.

We have already seen that Shtokman's density model can be applied only in an oceanic region where the magnitude of current decreases with depth and where the direction of current does not change with depth. This cannot be generally expected in the surface layers, particularly in a tropical oceanic region like the Arabian sea. So, for the purpose of computing function F(H) using Shtokman's density model, an oceanic region below about 1500 m is selected where generally the current velocity is expected only to decrease with depth with no change in direction. A region below 1400 m is taken for the purpose so that z = H now represents the 1400 d bar surface.

The method of computing function F(H) for Station No.82 is shown in Table VII assuming that the lower boundary of the current is 4000 m. For the purpose of this computation it is convenient to write the equation for function F(H) in finite difference form as

Fig. 3: Map showing location of stations.

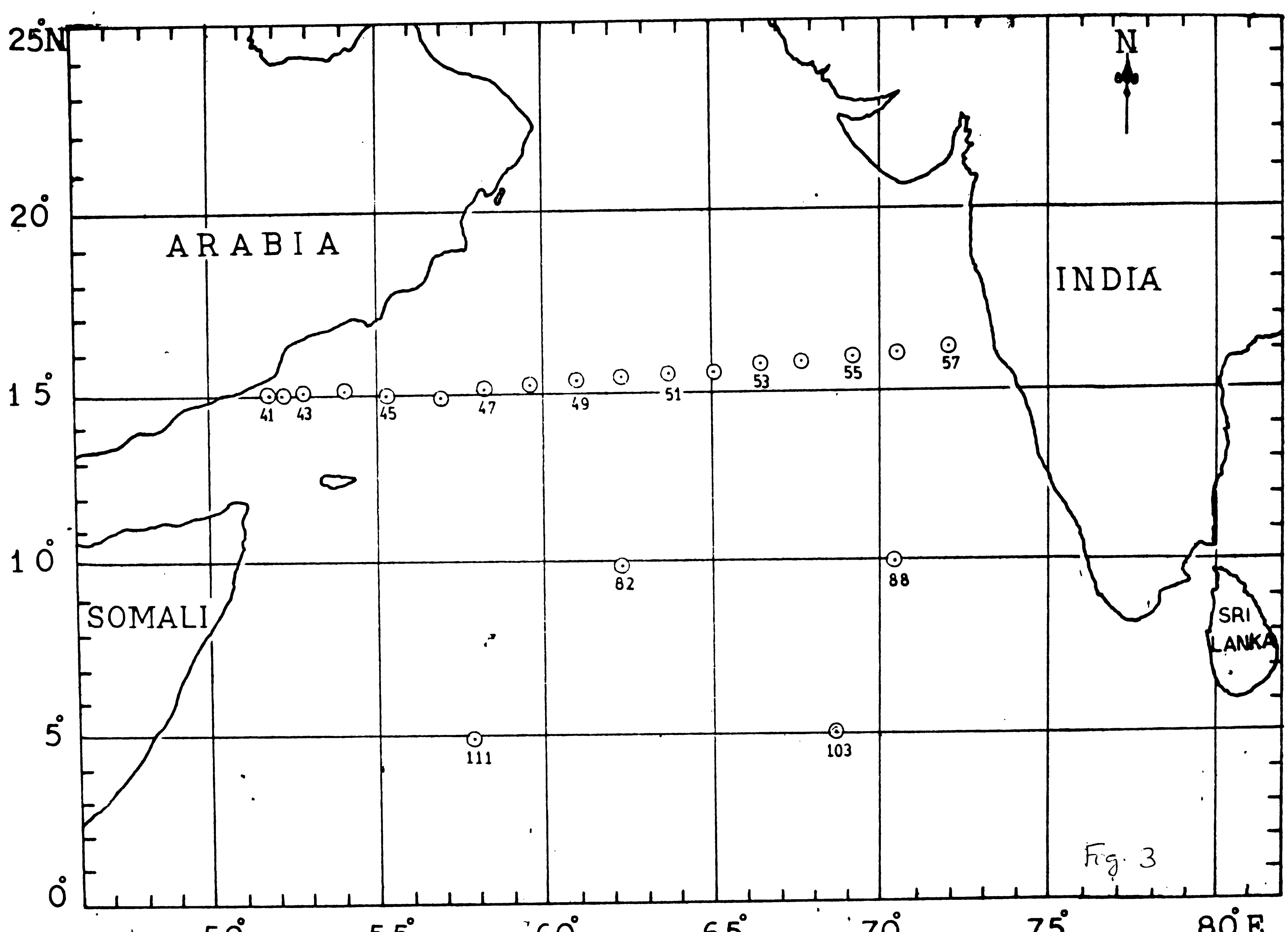
25°N
20°
15°
10°
5°
0°
50°
55°
60°
65°
70°
75°
80°E
ARABIA
INDIA
SOMALI
SRI LANKA
N
41
43
45
47
49
51
53
55
57
82
88
111
103
Fig. 3

Table VII

Computation of function F(H)

H–z (m)	σ_t	$k\delta(z)$	$K\overline{\delta(z)}$	$K\overline{\delta(z)}\Delta z$	$I = K\sum_{0}^{z}\overline{\delta(z)}\Delta z$	$\bar{I}$	$\bar{I}\Delta z$
1	2	3	4	5	6	7	8
1400	27.65	0.16			64.0		
			0.14	14.0		57.0	5700
1500	27.69	0.12			50.0		
			0.09	22.5		38.8	9700
1750	27.75	0.06			27.5		
			0.05	12.5		21.3	5325
2000	27.78	0.03			15.0		
			0.02	10.0		10.0	5000
2500	27.80	0.01			5.0		
			0.01	5.0		2.5	1250
3000	27.81	0.00			0.0		
			0.00	0.0		0.0	0.0
4000	27.81	0.00			0.0		

$$\sum_{0}^{H} \bar{I}\Delta z = 26975$$

$$F(H) = 2.37 \times 10^{-3}$$

$$= 2.37 \times 10^{-5}\ \text{cm}^{-1}$$

$$F(H) = \frac{\int_0^H \delta(z)\, dz}{\int_0^H \int_0^z \delta(z)\, (dz)^2}$$

$$= \frac{\sum_0^H \overline{\delta(z)}\, \Delta z}{\overline{\sum_0^H \left(\sum_0^z \overline{\delta(z)}\, \Delta z\right) \Delta z}} \qquad (6.14)$$

In Table VII, the first column gives the depth of the different isobaric surfaces from the sea surface. The vertical distribution of σ_t is given in the second column. The third column shows the values of $[K\delta(z)]$, that is, values of $[\sigma_t(o) - \sigma_t(z)]$. The $[K\delta(z)]$ values averaged by layers are given in the fourth column. In the fifth column are given the products of the average values of $[K\delta(z)]$ and the corresponding depth intervals. The sixth column gives the summation $\sum_0^z \overline{\delta(z)}\Delta z$ where z can have values in the interval $0 \leq z \leq H$. Column 7 contains values of the sixth column averaged by layers. The last column shows the average values of the seventh column multiplied by the corresponding depth intervals. Now the function F(H) is obtained by dividing the number in the first row of the sixth column, which represents $K \sum_0^H \overline{\delta(z)}\,\Delta z$, by the sum of the values of the last column, which represents $K \sum_{0}^{H} (\sum_{0}^{z} \overline{\delta(z)\Delta z})\Delta z$. To convert the result to the cgs system, we must multiply the same by 10^{-2}. The same computations are repeated for the same station assuming the lower boundary of the current at 3000 m, 2500 m, 2000 m and 1750 m. Similarly function F(H) for different depths of the lower boundary of current are computed for the stations 48,88,103 and 111. The stations selected are well-spread over the whole of the Arabian sea region so that a zero surface identified with this method may be applied to the whole of this oceanic region. The computed values of function F(H) are tabulated in Table VIII. The first column shows the assumed depths of the lower boundary of the current from the sea surface and the other columns show the computed values of function F(H) for the different depths and for the selected stations. Fig.4 is a graphical representation of function F(H) for the stations 48,88 and 103. Both Table VIII and Fig.4 show that the function F(H) depends little on position and agree rather well in magnitude. This means that in the region selected the water is comparatively uniform in the horizontal plane. The T-S characteristics of the water masses of the Indian ocean published by Sastry (1971) and Sastry and D'Souza (1972) confirm this fact by showing that the T-S curves of the water masses of the Arabian sea region coincide

Table VIII

Values of $F(H) \times 10^{-5}$ cm^{-1} for different depths of assumed reference surfaces

H-z (m)	Station Numbers				
	48	82	88	103	111
1750	7.07	7.38	7.72	7.38	7.72
2000	4.80	4.92	4.69	4.92	4.69
2500	2.87	3.39	2.83	2.92	2.83
3000	2.25	2.37	2.24	2.27	2.18
4000	2.25	2.37	2.24	2.27	1.53

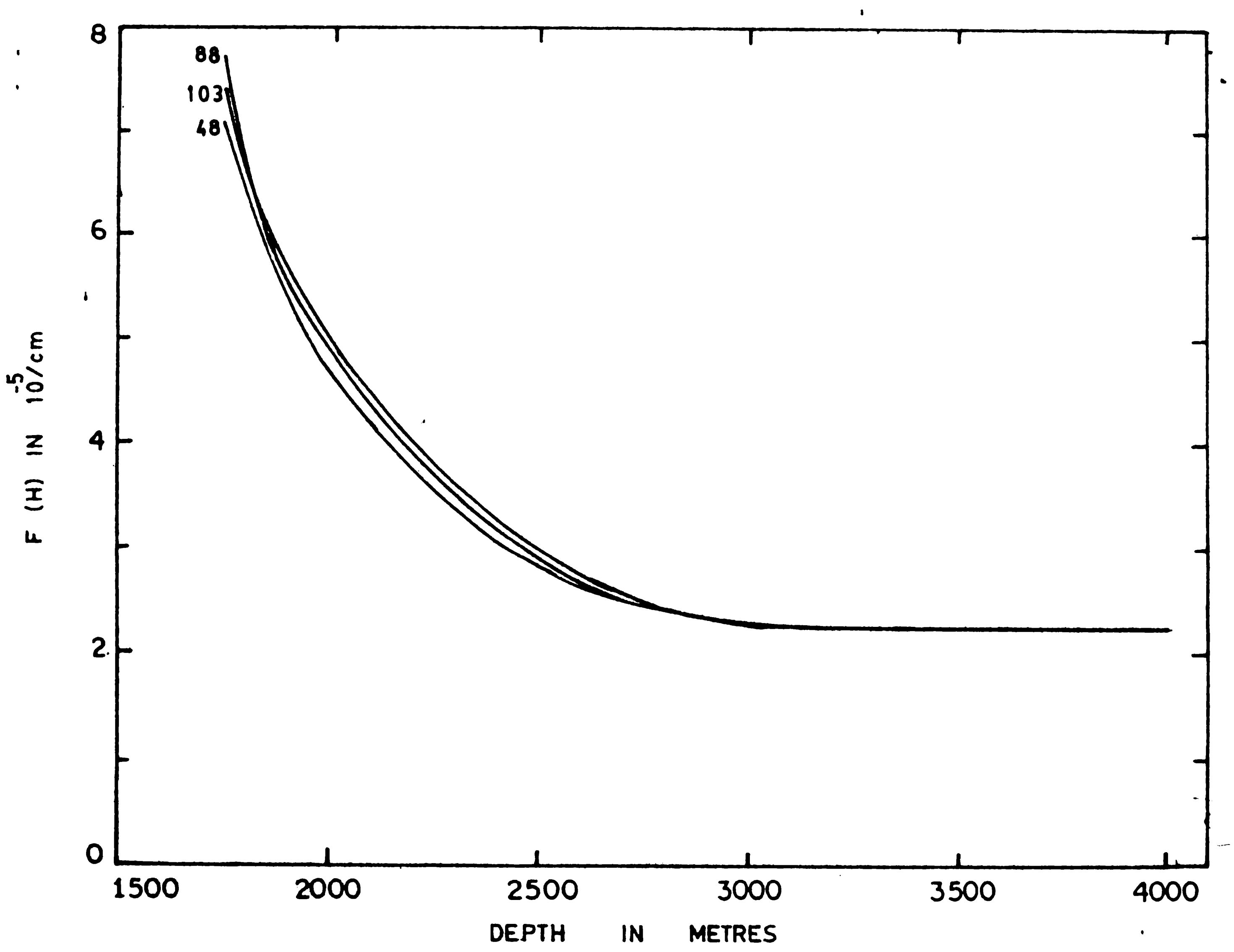

Fig. 4: Curves of function F(H) at three stations.

below 40 cl/ton isosteric surface which represents an oceanic region below 1500 m. The table and the figure also show that the function F(H) changes rapidly in the comparatively shallow regions and except for station 111 (Table VIII), the function F(H) is independent of depth when the depth is greater than 3000 m. At station 111 the vertical density gradient is not zero below 3000 m but is very weak compared to the upper layers. Hence, we may take the 3000 d bar surface as a zero surface in the Arabian sea with negligibly small geostrophic current.