Chapter 4

Application of Chemo-GA on Model Order Reduction Problem for Single Input and Single Output System

For a wide range of unconstrained optimization problems, the parameter recommendation has been done for CGA in the previous chapter. In this Chapter a real life Electrical Engineering problem has been solved under the same parameter setting. This unconstrained optimization problem is a Model Order Reduction (MOR) problem of linear time invariant continuous Single Input and Single Output (SISO) system.

4.1 INTRODUCTION

The present day technology, societal and environmental processes are bringing a large number of problems which are complex natured due to the involvement of larger dimension and non-linearity functions. Some earlier attempts have been made to simplify those large scale system was to decompose it into number of subsystems for efficient computational procedure and getting simplified designs. Most large scale systems are having multiplicity of measured outputs and inputs. In such cases, decentralized control structures are applied. Next, the development of large scale system modeling came into existence. Among those, Model order reduction (MOR) is such technique by which large scale complex systems can be reduced into simpler models. The basic idea behind these techniques is to produce the input-output relation in acceptable time minimizing the error between the reduced model and the original model. Now-a-days the real life problems and the large scale complex systems can be time variant, time invariant, linear, non linear, parametric or stochastic which are posing big challenges for MOR problem. The system may be Single input and Single output (SISO) or may be multiple inputs and multiple outputs (MIMO) system. The goal of the Model Order Reduction (MOR) problem includes the following points. (i) the approximation of the output of large scale system by a reduced model should be evaluated in significantly less time, (ii) the computable error bound for the reduced model should exist, and (iii) the preservation of physical properties of the original system, such as stability, minimum phase, and passivity must be followed during the MOR process.

Several approaches exist for MOR, Viz. (i) Modal Analysis approach, (ii) Aggregation Methods, (iii) Frequency domain based methods, (iv) Norm based methods, and (v) Pole replacement Techniques.

In Frequency domain based approach for MOR techniques the following are very popular approaches. (i) Pade approximation methods, (ii) Routh approximation techniques, (iii) Continued fraction method, (iv) Rational Interpolation method, (v) Modal truncation method, and (vi) Standard balanced truncation method.

Many researchers have done excellent work in this regard. To list a few, Rao et al. (1978) presents Routh-Approximant frequency domain modeling for reduced models for SISO system. A suitable truncation of the original system matrices in their canonical structure is done and the low order time domain model matrices are derived. Bai and Freund (2001) in their paper presented an algorithm for using partial pade approximants via Lanczos method for reduced order modeling. Paramar et al. (2007) presented an algorithm combining stability equation method and error minimization by GA. The numerator of the reduced polynomial are determined by minimizing the integral square error between the higher order system and lower order system using GA and the denominator of the model is obtained by Stability equation method. Deepa and Sugumaran (2011) proposed Modified PSO (MPSO) for model order reduction of linear time invariant Single input and Single output system. This method is based on the minimization of error between the higher order system and lower order system pertaining to the unit step input. The algorithm is also compared with Integral of time of squared error (ITSE) and Integral of time of Absolute error (ITAE). Bansal et al. (2011) applied Artificial Bee Colony (ABC) algorithm to solve SISO system. The results have been compared with Pade approximation, Routh approximation and other conventional methods. They designed a novel objective function for MOR problem, where minimization is carried out based on both Integral Square Error (ISE) and Impulse Response Energy (IRE). Bansal and Sharma (2012) also applied Differential Evolution (DE) and its variant Cognitive Learning in Differential Evolution (CLDE) algorithm for obtaining the reduced models. In CLDE algorithm, cognitive learning factor (CLF) is introduced in the mutation operator in Differential Evolution (DE). Further, Bansal et al. (2012) proposed Fitness Based Differential Evolution (FBDE) for solving MOR problem. Kumar and Tiwari (2012) used factor division method with Clustering technique for finding stable reduced ordered models of SISO system. Ibraheem et al. (2012) in their proposed algorithm gave more insight into the continued fraction expansion technique for SISO system. Mondal and Tripathy (2013) proposed a mixed method for SISO system where the numerator is reduced by Pade approximation and the denominator polynomial is reduced preserving the basic characteristics of higher order original system. This method tries to minimize the Integral Square error (ISE). Priya and Sunilkumar (2013) deployed approximate generalized time moments (AGTM) matching method in the first phase where the optimum selection of expansion points is carried out using Luus-Jaakola (LJ) optimization procedure. The interval parameters are derived by the optimization procedure in the second phase. Desai and Prasad (2014) have adopted a new approach to model order reduction by employing Big Bang Big Crunch (BBBC) optimization procedure with Stability Equation (SE). The numerator is obtained by Big Bang Big Crunch (BBBC) algorithm and the denominator is obtained by Stability Equation (SE).

This Chapter is organized as follows. The MOR problem definition is described in the Section 4.2. The experimental set up and the numerical examples are discussed in Section 4.3. The result discussions are given in Section 4.4 and the conclusion of the Chapter is drawn in Section 4.5.

4.2 MODEL ORDER REDUCTION PROBLEM

This Section aims at verifying the efficiency of CGA over some recent algorithms like a variant of Cognitive Learning in Differential Evolution (CLDE), named as Linearly Increasing Cognitive Learning in Differential evolution (LICLDE) (Bansal and Sharma, 2012) and Fitness Based position update process in Differential Evolution (FBDE) (Bansal et al., 2012). Three cases of Model order reduction (MOR) problems have been picked from the literature (Shamash, 1975; Pal, 1986; Lucas, 1986) and solved by CGA.

4.2.1 MOR as an optimization problem

Let's consider an n^{th} order linear time invariant dynamic single-input and single-output system (SISO) system given by

$$G(s) = \frac{N(s)}{D(s)} = \frac{\sum_{i=0}^{i=n-1} a_i s^i}{\sum_{i=0}^{n} b_i s^i} \tag{4.1}$$

Where a_i and b_i are known constants. The problem is to find out r^{th} order reduced model in the transfer function form $R(s)$, where $r < n$ represented by Eq. (4.2) such that the reduced model retains the important characteristics of the original system and approximates its step responses as closely as possible for the same type of inputs with minimum Integral square error (ISE) as well as Impulse response energy(IRE).

$$R(s) = \frac{N_r(s)}{D_r(s)} = \frac{\sum_{i=0}^{r-1} a_i' s^i}{\sum_{i=0}^{r} b_i' s^i} \tag{4.2}$$

Where a_i' and b_i' are unknown constants. Mathematically, the integral square error of step responses of the original and the reduced system can be expressed by the following error index given by Eq. (4.3)

$$J = \int_0^{\infty} \left[y(t) - y_r(t) \right]^2 \, dt \tag{4.3}$$

where $y(t)$ is the unit step response of the original system and $y_r(t)$ is the unit step response of the reduced system. The error index is the function of unknown coefficients of reduced order model so that the error index is minimized.

The Impulse response energy (IRE) for the original and the various reduced models is given by Eq. (4.4)

$$IRE = \int_0^{\infty} g(t)^2 dt \tag{4.4}$$

Where, $g(t)$ is the Impulse response of the system. In this Chapter the objective is to minimize the objective function based on both Integral Square Error (ISE) and Impulse Response Energy (IRE).

4.2.2 Proposed Method to solve MOR problem

Mukherjee and Mishra (1987) proposed a simplified method for the MOR problem of linear continuous system using matching of steady state parts and minimizing the error between the transient parts of a unit step response of the original and reduced ordered system. In that method, the dominant poles of the original system are retained.

Motivated by the above, in this Chapter, CGA (discussed in details at section 2.4) has been used to solve MOR problem of linear continuous time invariant Single Input and Single Output (SISO) system where the original system is having real and distinct Eigen values. Also in this proposed method, the steady state part of the unit step responses of the original system and reduced order models are closely matched. The minimization of the Integral Square Error (IRE) as well as Impulse Response Energy (IRE) of unit step response of the original high order transfer function and the reduced low order transfer function for transient parts is considered. In the proposed method, the reduced model is designed considering the real and distinct Eigen values.

Let's consider the original high order system transfer function is given by Eq. (4.5)

$$G(s) = \frac{b_n s^n + b_{n-1}s^{n-1} + \ldots\ldots + b_1 s + b_0}{s^m + a_{m-1}s^{m-1} + \ldots\ldots + a_1 s + a_0} \tag{4.5}$$

where m is the order of the system and $m > n$.

$$G(s) = \frac{b_n s^n + b_{n-1}s^{n-1} + \ldots\ldots + b_1 s + b_0}{(s+\lambda_1)(s+\lambda_2)(s+\lambda_3)\ldots\ldots(s+\lambda_m)} \tag{4.6}$$

Where $-\lambda_1 < -\lambda_2 < -\lambda_3 < \ldots\ldots\ldots\ldots\ldots\ldots < -\lambda_{m-1} < -\lambda_m$ are distinct real Eigen values of the system.

The unit step responses of (4.6) can be determined as

$$T(s) = \frac{G(s)}{s} = \frac{b_n s^n + b_{n-1}s^{n-1} + \ldots\ldots + b_1 s + b_0}{s(s+\lambda_1)(s+\lambda_2)(s+\lambda_3)\ldots\ldots(s+\lambda_m)}$$

$$= \frac{k_0}{s} + \frac{k_1}{s+\lambda_1} + \frac{k_2}{s+\lambda_2} + \ldots\ldots\ldots + \frac{k_{m-1}}{s+\lambda_{m-1}} + \frac{k_m}{s+\lambda_m} \tag{4.7}$$

Where k_i' s are real constants. Taking inverse Laplace transformation,

$$y(t) = k_0 + k_1 e^{-\lambda_1 t} + k_2 e^{-\lambda_2 t} + \ldots\ldots\ldots\ldots\ldots + k_m e^{-\lambda_m t} \tag{4.8}$$

Where k_0 is steady state response and the rest of the terms are transient response of the system given in equation (4.6).

Let the proposed reduced order system constructed is of 2nd order, where

$$R(s)=\frac{a_0 s+a_1}{a_2 s^2+a_3 s+a_4}=\frac{a_0 s+a_1}{(s+\mu_1)(s+\mu_2)} \tag{4.9}$$

Where, μ_1 and μ_2 are distinct and real Eigen values and $-\mu_1<-\mu_2$.

The unit step response of (4.9) can be determined as

$$T_1(s)=\frac{R(s)}{s}=\frac{k_0'}{s}+\frac{k_1'}{s+\mu_1}+\frac{k_2'}{s+\mu_2} \tag{4.10}$$

Where, $k_0', k_1', k_2', \mu_1, \mu_2$ are real constants. Let the inverse Laplace transformation

$$y_r(t)=k_0'+k_1'e^{-\mu_1 t}+k_2'e^{-\mu_2 t} \tag{4.11}$$

In order that steady state part of the responses of the original high order system (4.6) and the reduced order system (4.9) are matched exactly, the following condition should be fulfilled.

$$k_0=k_0' \tag{4.12}$$

The Integral Square Error (ISE) of the transient responses of the system of (4.6) and (4.9) is given by

$$=\int_0^\infty [y(t)-y_r(t)]^2\,dt \text{ since } k_0=k_0'$$

$$=\int_0^\infty \left\{\sum_{i=1}^{m} k_i e^{-\lambda_i t}-\sum_{j=1}^{2} k_j' e^{-\mu_j t}\right\}^2 dt \tag{4.13}$$

Where k_i' s and λ_i' s are known and k_1', k_2', μ_1, μ_2 are all unknown constants, which are randomly generated in CGA and the value of J are calculated. The reduced model $R(s)$ is obtained for 30 independent runs so that the integral square error J is minimized. In the next step that reduced model $R(s)$ is taken where IRE given by Eq. (4.4) is also minimized in Matlab7.0.

4.3 EXPERIMENTAL SETUP AND NUMERICAL EXAMPLES

In this section, 3 examples of MOR problems have been picked up from (Shamash, 1975; Pal, 1986; Lucas, 1986).

These problems (listed below) have real and distinct Eigen values.

***Example* 1:** The MOR problem proposed by (Shamash, 1975)

$$G_1(s) = \frac{N_1(s)}{D_1(s)} \tag{4.14}$$

$$N_1(s) = 18s^7 + 514s^6 + 5,982s^5 + 36,380s^4 + 122,664s^3 + 222,088s^2 + 185760s + 40,320$$

$$D_1(s) = s^8 + 36s^7 + 546s^6 + 4536s^5 + 22,449s^4 + 67,284s^3 + 118,124s^2 + 109,584s + 40,320$$

***Example* 2:** The MOR problem proposed by (Lucas, 1986)

$$G_2(s) = \frac{8169.13s^3 + 50664.97s^2 + 9984.32s + 500}{100s^4 + 10520s^3 + 52101s^2 + 10105s + 500} \tag{4.15}$$

***Example* 3:** The MOR problem proposed by (Pal, 1986)

$$G_3(s) = \frac{s+4}{s^4 + 19s^3 + 113s^2 + 245s + 150} \tag{4.16}$$

Proposed CGA aims at solving these problems with two objectives. They are (i) to minimize the Integral Square Error (ISE) and (ii) Impulse Response Energy (IRE); between original higher order system and reduced lower order system. The proposed CGA program code is designed in C++ and the experiment is carried out on a P-IV, 2.8 GHz machine with 512 MB RAM under WINXP platform. Parameter settings of CGA to solve MOR problem is defined as follows.

- The probability of Crossover $P_c = 0.9$
- The probability of Mutation $P_m = 0.01$
- The no of Chemo tactic steps taken=40
- The no of swim steps taken=4
- The minimum step size taken $\left(C_{\min} = 0.008\right)$
- The maximum step size taken $\left(C_{\max} = 0.1\right)$

Bansal et al.(2012) solved these problems recently by suing a new Fitness Based position update process in Differential Evolution (FBDE) and also Bansal and Sharma (2012) adopted new variant of Cognitive learning in Differential evolution (CLDE) named as Linearly increasing cognitive learning factor in Differential evolution (LICLDE) to solve those problems. They claimed that LICLDE and FBDE outperform many other order reduction methods. This Chapter reconsidered all these methods along with LICLDE and FBDE to compare with the proposed CGA.

At first hand a set of solutions have been collected for 30 runs where ISE is minimum given by Eq. (4.13). Then the emphasis is given in minimizing the IRE of the system given by Eq. (4.4). The solution where ISE and IRE are both minimum is considered as the global optimum. The reported solutions are plotted in form of impulse responses and step responses of the system in Mat lab 7.0.

4.3.1 Reduced Models

Reduced Model for Example 1:

The reduced model obtained for example 1 is given as

$$R_1(s) = \frac{17.32218649s + 5.372427313}{s^2 + 7.025081490s + 5.372427313} \tag{4.17}$$

The result comparison of ISE and IRE is shown in Table 4.1. The step and impulse responses are plotted in Fig.4.1 and 4.2.

Table 4.1: Comparison of the methods for example 1 (Shamash, 1975)

Method of order reduction	**Reduced Models** $R_1(s)$	**ISE**	**IRE**
Original	$G_1(s)$	______	**21.74**
CGA	$\frac{17.32218649s + 5.372427313}{s^2 + 7.025081490s + 5.372427313}$	**0.00080881929**	**21.74**
FBDE	$\frac{17.32178s + 5.3660}{s^2 + 7.0240s + 5.3660}$	0.0008089311	**21.74**
LICLDE	$\frac{17.203s + 5.3633}{s^2 + 6.9298s + 5.3633}$	0.000909249	**21.74**
DE	$\frac{20s + 5.6158}{s^2 + 9.2566s + 5.6158}$	0.03729	21.908
Pade approximation	$\frac{15.1s + 4.821}{s^2 + 5.993s + 4.821}$	1.6177	19.426
Routh approximation	$\frac{1.99s + 0.4318}{s^2 + 1.174s + 0.4318}$	1.9313	1.8705
Gutman etal.	$\frac{4[133,747,200s + 203,212,800]}{85,049.280s^2 + 552,303.360s + 812,851.200}$	8.8160	4.3426
Krishnamurthy and Sheshadri	$\frac{155,658.6152s + 40,320}{65,520s^2 + 75,600s + 40,320}$	17.5345	2.8871
Mittal et.al.	$\frac{7.0908s + 1.9906}{s^2 + 3s + 2}$	6.9159	9.7906
Mukherjee and Mishra	$\frac{7.0903s + 1.9907}{s^2 + 3s + 2}$	6.9165	9.7893
Mukherjee et.al.	$\frac{11.3909s + 4.4357}{s^2 + 4.2122s + 4.4357}$	2.1629	18.1060
Pal	$\frac{151,776.576s + 40,320}{65,520s^2 + 75,600s + 40,320}$	17.6566	2.7581
Prasad and Pal	$\frac{17.98561s + 500}{s^2 + 13.24571s + 500}$	18.4299	34.1223
Shamash	$\frac{6.7786s + 2}{s^2 + 3s + 2}$	7.3183	8.9823
Hutton and Friedland	$\frac{1.98955s + 0.43184}{s^2 + 1.17368s + 0.43184}$	18.3848	1.9868

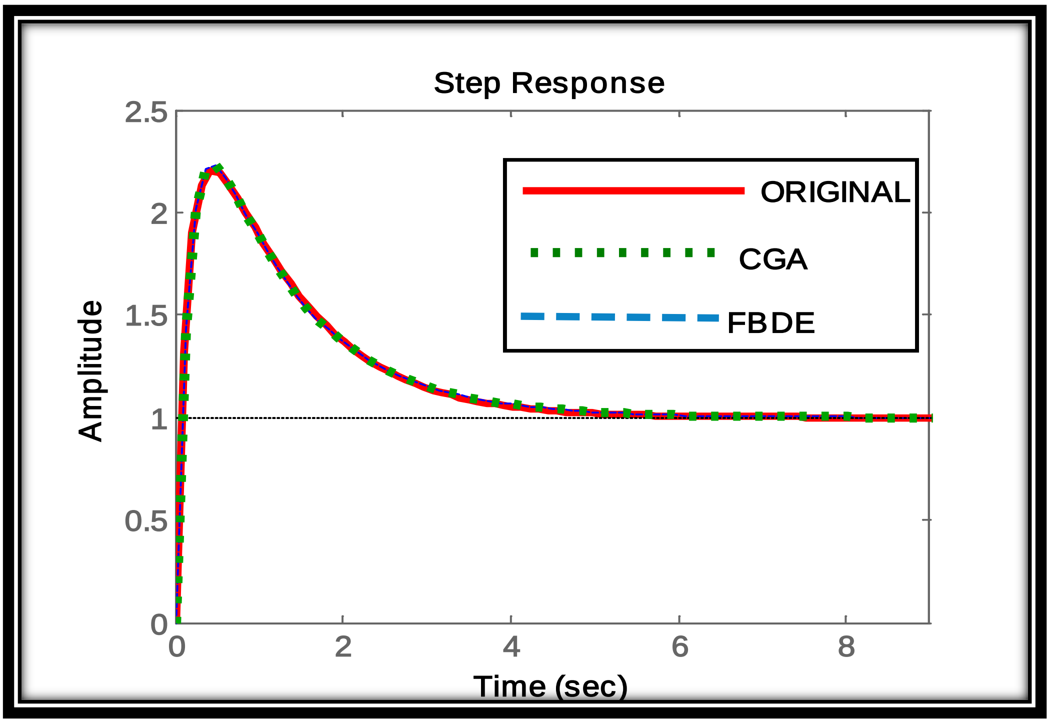

Fig. 4.1: Comparison of step responses for example 1

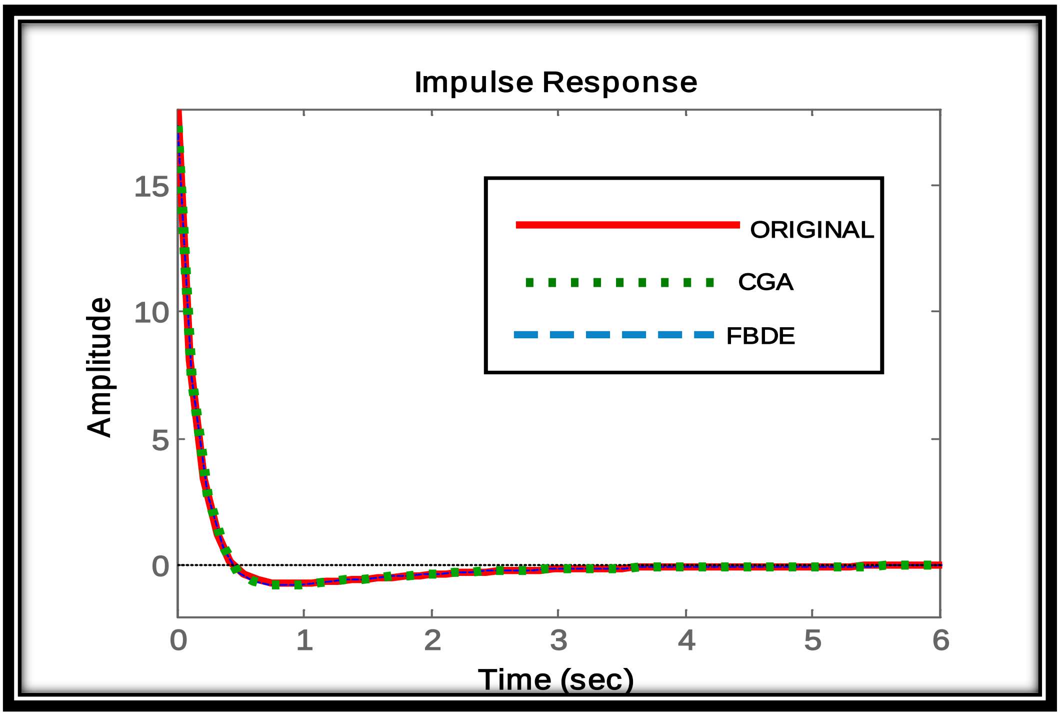

Fig. 4.2: Comparison of impulse responses for example 1

Reduced Model for Example 2:

$$R_2(s) = \frac{78.03187648s + 201.4534442}{s^2 + 92.318454s + 201.4534442} \qquad (4.18)$$

The result comparison of ISE and IRE is shown in Table 4.2. The step and impulse responses are plotted in Fig.4.3 and4.4 respectively.

Table 4.2: Comparison of the methods for example 1 (Lucas, 1986)

order reduction	**Reduced Models** $R_2(s)$	**ISE**	**IRE**
Original	$G_2(s)$	———	**34.069**
CGA	$\frac{78.03187648s + 201.4534442}{s^2 + 92.318454s + 201.4534442}$.	**0.001538260.**	**34.06923470**
FBDE	$\frac{85.33529245s + 462.3004006}{s^2 + 113.6582937s + 462.3004006}$	0.0017826566	**34.06884**
LICLDE	$\frac{101.3218182s + 867.893179}{s^2 + 169.4059231s + 867.893179}$	0.0036228741	**34.069918**
DE	$\frac{220.8190s + 35011.744}{s^2 + 1229.450s + 35011.744}$	0.004437568	**34.069218**
Singh	$\frac{93.7562s + 1}{s^2 + 100.10s + 10}$	0.008964	43.957
Pade approximation	$\frac{23.18s + 2.36}{s^2 + 23.75s + 2.36}$	0.0046005	11.362
Routh approximation	$\frac{0.1936s + 0.009694}{s^2 + 0.1959s + 0.009694}$	2.3808	0.12041
Gutman etal.	$\frac{0.19163s + 0.00959}{s^2 + 0.19395s + 0.00959}$	2.4056	0.11939
Chen et al.	$\frac{0.38201s + 0.05758}{s^2 + 0.58185s + 0.05758}$	1.2934	0.17488
Marshall	$\frac{83.3333s + 499.9998}{s^2 + 105s + 500}$	0.00193	35.450

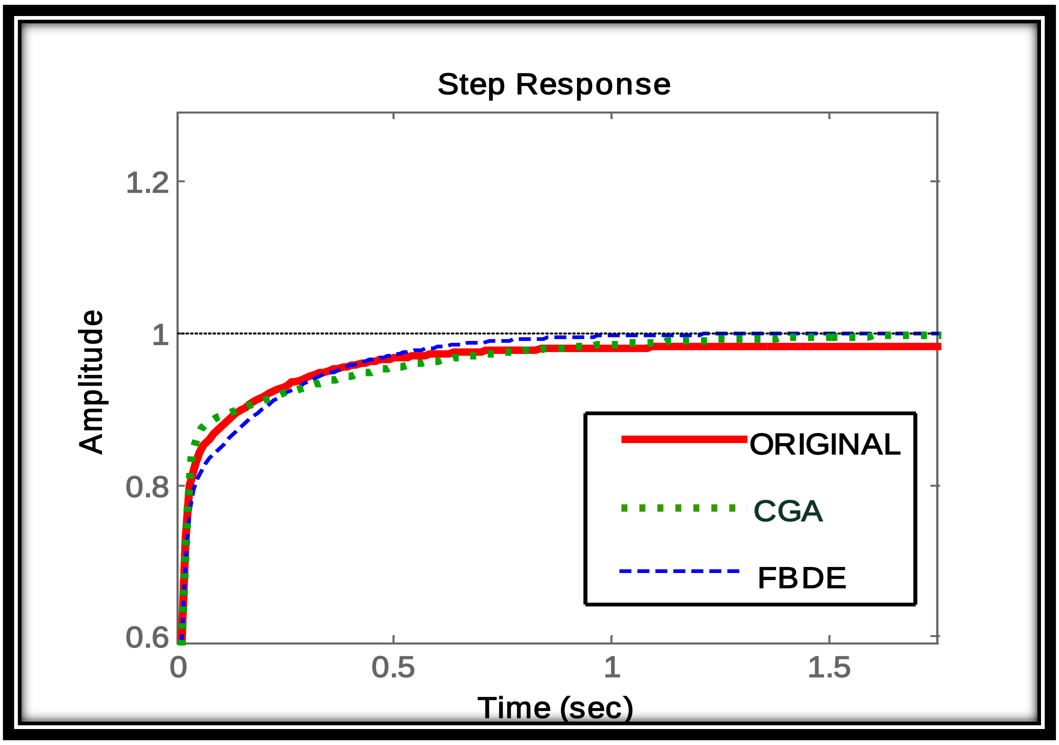

Fig. 4.3: Comparison of step responses for example 2

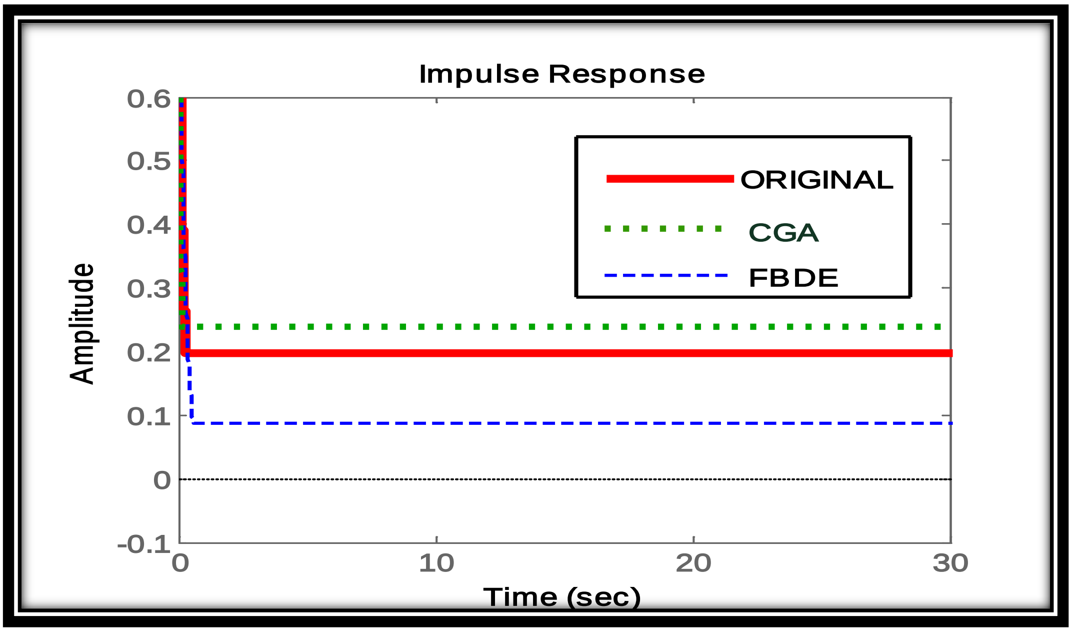

Fig. 4.4: Comparison of impulse responses for example 2

Reduced Model for Example 3:

The reduced model obtained for example 3 is given as

$$R_3(s) = \frac{-0.004821915s + 0.081246761}{s^2 + 4.0266s + 3.042949850} \tag{4.19}$$

The result comparison of ISE and IRE is shown in Table 4.3. The step and impulse responses are plotted in Fig.4.5 and 4.6 respectively

Table 4.3: Comparison of the methods for example 3 (Pal, 1986)

Method of order reduction	Reduced Model $R_3(s)$	ISE	IRE
Original	$G_3(s)$	________	**0.00026938**
CGA	$\frac{-0.004821915s + 0.081246761}{s^2 + 4.0266s + 3.042949850}$	**5.7038e-009**	**0.000271441**
LICLDE (Bansal and Sharma)	$\frac{-0.0195s + 0.2884}{s^2 + 14.9813s + 10.82}$	4.3168e-006	**0.00027**
DE	$\frac{0.0296s + 0.2175}{s^2 + 12.3952s + 8.156}$	1.451930426e-05	**0.00027**
Singh	$\frac{-404.596s + 405.48}{150s^2 + 2487s + 15205.5}$	2.856e-03	0.0002476
Pade approximation	$\frac{-0.005017s + 0.08247}{s^2 + 4.09s + 3.093}$	∞	**0.00027192**
Routh approximation	$\frac{0.009865s + 0.03946}{s^2 + 2.417s + 1.48}$	∞	0.00023777

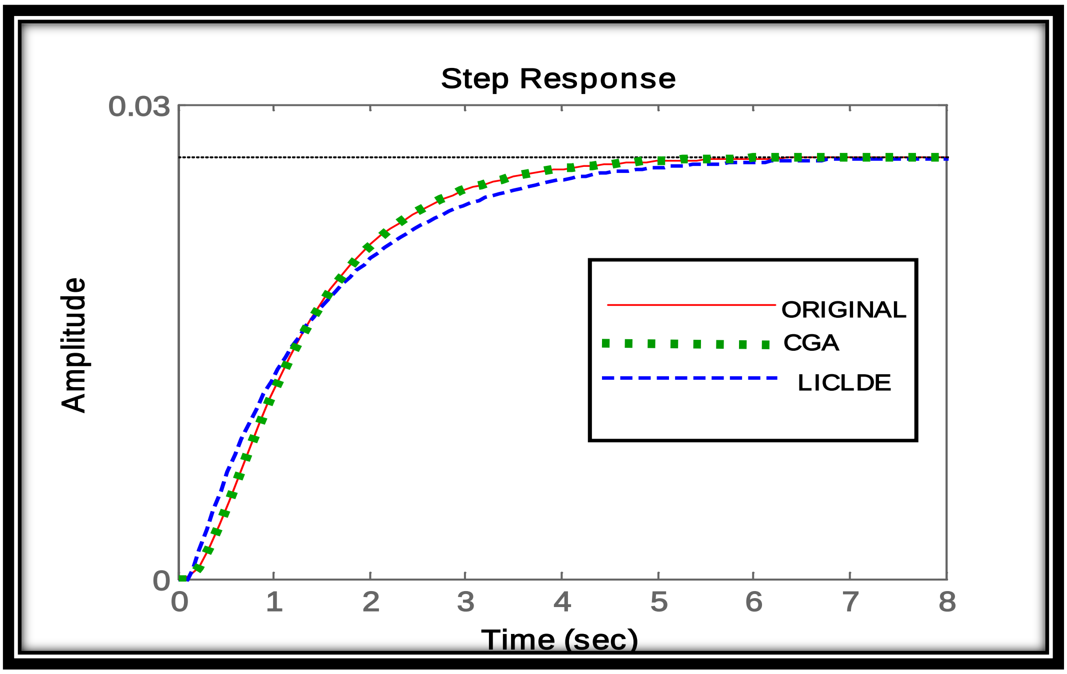

Fig. 4.5: Comparison of step responses for example 3

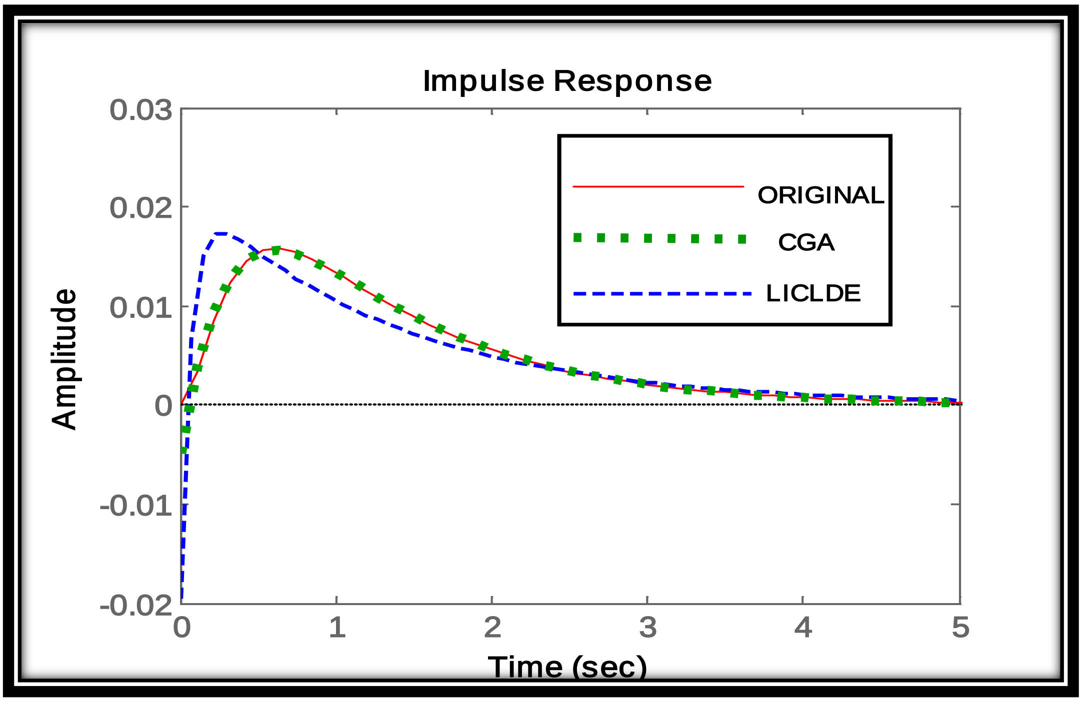

Fig. 4.6: Comparison of impulse responses for example 3

4.4 RESULT ANALYSIS

The different reduced ordered models by CGA and other reported algorithms for the corresponding original model are represented in Table 4.1, 4.2 and 4.3 for examples 1, 2 and 3 respectively. In the corresponding tables, the errors (ISE and IRE) obtained by CGA are also compared with that of all earlier reported results in (Bansal et al., 2012; Bansal and Sharma, 2012) in last two columns. The unit step responses of CGA have been compared only with original system and the best reported result so far which is given in (Bansal et al., 2012) and are shown in the Fig. 4.1, 4.3 and 4.5 respectively. The corresponding impulse responses are plotted in Fig. 4.2, 4.4 and 4.6 for reduction examples 1, 2 and 3 respectively.

It is realized from Fig. 4.1 and 4.2 that there is no significant difference found for ISE between CGA vs. FBDE and step responses vs. impulse responses. They are almost close to the original curve in both the cases.

However, for example 2 (Table 4.2), ISEs obtained by CGA are significantly less in comparison to FBDE, LICLDE and other reported results. It can also be visualized for example 2 (Fig. 4.3), the step response is approximating the original polynomial more closely in comparison to FBDE (Bansal et al., 2012). Also, IRE of the reduced model obtained by CGA is most close to the original model (Fig. 4.4).

Similarly for example 3 (Table 4.3), the ISE and IRE are found to be the minimum as compared to LICLDE (best reported) in (Bansal and Sharma, 2012). The step response as well as impulse response curve (Fig. 4.5 and 4.6) seem to be exactly lying on the original curve.

It may also be noted that the steady state responses of the original and the reduced ordered model by CGA are exactly matching while the transient response matching are also very close as compared to other methods. Thus these examples establish the superiority of CGA over the FBDE, LICLDE as well as all other reported results for those problems. The best reported ISE and IRE are boldfaced in the tables.

4.5 CONCLUSION

In this Chapter, to validate the superiority in the versatile application of CGA, three examples of MOR problem have been picked up and solved by this algorithm and compared with LICLDE, FBDE and other stochastic algorithm. It is shown that CGA outperforms to other results reported so far, in terms of *Integral square error (ISE)* and *Impulse response energy (IRE)*. Based on this study, it is concluded that CGA is a better candidate in order to enhance the performance of GA not only in the field of benchmark function optimization but also in solving real life unconstrained optimization problems.