Appendix A

Nonlinear Strain Terms \( \{ \tilde{\varepsilon}_{NL} \} \) and Thickness Coordinate Matrix as Appeared in General Mathematical Formulation

Nonlinear strain terms as shown in the Eq. (3.10)

\[
(e^4_1) = \left( (u_x)^2 + (v_x)^2 + (w_x)^2 \right), \quad (e^4_2) = \left( (u_y)^2 + (v_y)^2 + (w_y)^2 \right),
\]

\[
(e^6_1) = 2\left[ u_w u_x + v_w v_x + w_w w_x \right], \quad (e^6_2) = 2\left[ \phi u_x + \phi_2 v_x \right],
\]

\[
(e^8_1) = 2\left[ \phi u_y + \phi_2 v_y \right], \quad (k_2^5) = 2\left[ \phi_1 u_x + \phi_2 v_x - \frac{\phi_3}{R_1} w_x \right],
\]

\[
(k_2^5) = 2\left[ \phi_1 u_y + \phi_2 v_y - \frac{\phi_3}{R_2} w_y \right], \quad (k_2^5) = 2\left[ \phi_1 u_{y,y} + 2\phi_1 u_{x,y} + \phi_2 v_{y,y} - \frac{\phi_3}{R_1} w_{x,y} - \frac{\phi_3}{R_2} w_{y,y} \right],
\]

\[
(k_2^5) = 2\left[ \phi_1 u_{y,y} + 2\phi_1 u_{x,y} + \phi_2 v_{y,y} - \frac{\phi_3}{R_1} w_{x,y} - \frac{\phi_3}{R_2} w_{y,y} \right], \quad (k_2^5) = 2\left[ \phi_1 u_{x,x} + \phi_2 v_{x,x} - \frac{\phi_3}{R_1} w_{x,x} - \frac{\phi_3}{R_2} w_{y,x} \right],
\]

\[
(k_2^5) = 2\left[ \phi_1 u_{x,x} + \phi_2 v_{x,x} - \frac{\phi_3}{R_1} w_{x,x} - \frac{\phi_3}{R_2} w_{y,x} \right], \quad (k_2^5) = 2\left[ \phi_1 u_{x,y} + \phi_2 v_{x,y} - \frac{\phi_3}{R_1} w_{x,y} - \frac{\phi_3}{R_2} w_{y,y} \right],
\]

\[
(k_2^5) = 2\left[ \psi_{x,x}\phi_1 + 2\phi_1 \psi_{x,y} + 2\phi_2 \psi_{y,y} + 2\phi_{x,y} \psi_{y,y} + 2\phi_{x,y} \psi_{y,y} - \frac{\psi_1}{R_1} w_{x,x} - \frac{\psi_2}{R_2} w_{x,y} + \frac{\phi_3}{R_1} \psi_{x,y} \right],
\]

\[
(k_2^5) = 2\left[ \psi_{x,x}\phi_1 + 2\phi_1 \psi_{x,y} + 2\phi_2 \psi_{y,y} + 2\phi_{x,y} \psi_{y,y} + 2\phi_{x,y} \psi_{y,y} - \frac{\psi_1}{R_1} w_{x,x} - \frac{\psi_2}{R_2} w_{x,y} + \frac{\phi_3}{R_1} \psi_{x,y} \right],
\]

\[
(k_2^5) = 2\left[ \psi_{x,x}\phi_1 + 2\phi_1 \psi_{x,y} + 2\phi_2 \psi_{y,y} + 2\phi_{x,y} \psi_{y,y} + 2\phi_{x,y} \psi_{y,y} - \frac{\psi_1}{R_1} w_{x,x} - \frac{\psi_2}{R_2} w_{x,y} + \frac{\phi_3}{R_1} \psi_{x,y} \right],
\]

\[
(k_1^7) = 2\left[ u_{x}\theta_{x,x} + v_{x}\theta_{x,x} + \phi_{x,xx} \psi_{x,x} + \phi_{x,xy} \psi_{y,x} - \theta_{x}\frac{\omega_{x}}{R_1} w_{x,x} + \frac{\phi_3}{R_1} \psi_{x,x} \right],
\]
\[ (k_2^2) = 2 \left[ u_{1,y} \frac{\partial}{\partial x} + v_{1,y} \frac{\partial}{\partial y} + \phi_{1,y} \psi_{1,y} + \phi_{2,y} \psi_{2,y} - \frac{\theta_{2}}{R_2} \frac{\partial}{\partial x} w_{1,y} + \frac{\phi_{1} \psi_{2}}{R_2} \right], \]

\[ (k_6^1) = 2 \left[ u_{1,y} \frac{\partial}{\partial x} + v_{1,y} \frac{\partial}{\partial y} + \phi_{1,y} \psi_{1,y} + \phi_{2,y} \psi_{2,y} - \frac{\theta_{2}}{R_2} \frac{\partial}{\partial x} w_{1,y} + \frac{\phi_{1} \psi_{2}}{R_2} \right], \]

\[ (k_6^3) = 2 \left[ \theta_{1,x} \phi_{1} + \theta_{2,x} \phi_{2} + 2 \psi_{1,x} \theta_{1,y} + 2 \psi_{2,x} \theta_{2,y} + 3 \phi_{1,x} \theta_{1,y} + 3 \phi_{2,x} \theta_{2,y} \right], \]

\[ (k_1^4) = 2 \left[ \phi \theta_{1,y} + \phi \theta_{2,y} + 2 \psi \psi_{1,y} + 2 \psi \psi_{2,y} + 3 \psi \theta_{1,y} + 3 \psi \theta_{2,y} \right], \]

\[ (k_2^1) = \left[ \psi_{1,y}^2 + \psi_{2,y}^2 + 2 \phi_{1,x} \theta_{1,y} + 2 \phi_{2,y} \theta_{2,y} + \frac{\psi_{1}^2}{R_1} + \frac{2 \phi_{1} \theta_{1}}{R_1} \right], \]

\[ (k_2^8) = \left[ \psi_{1,y}^2 + \psi_{2,y}^2 + 2 \phi_{1,x} \theta_{1,y} + 2 \phi_{2,y} \theta_{2,y} + \frac{\psi_{2}^2}{R_2} + \frac{2 \phi_{2} \theta_{2}}{R_2} \right], \]

\[ (k_6^1) = \left[ \psi_{1,y} \psi_{1,y} + \psi_{2,y} \psi_{2,y} + 2 \theta_{1,x} \phi_{1} + 2 \theta_{2,x} \phi_{2} + 2 \theta_{1,x} \psi_{1,y} + 2 \theta_{2,x} \psi_{2,y} + \frac{2 \psi_{1} \psi_{2}}{R_1} + \frac{2 \phi_{1} \theta_{1}}{R_1} + \frac{2 \phi_{2} \theta_{2}}{R_2} \right], \]

\[ (k_8^3) = 2 \left[ 2 \psi_{1,x} \theta_{1,y} + 2 \psi_{2,x} \theta_{2,y} + 2 \theta_{1,x} \psi_{1,y} + 2 \theta_{2,x} \psi_{2,y} + 2 \theta_{1,x} \theta_{1,y} + 2 \theta_{2,x} \theta_{2,y} + \frac{2 \psi_{1} \psi_{2}}{R_1} + \frac{2 \phi_{1} \theta_{1}}{R_1} + \frac{2 \phi_{2} \theta_{2}}{R_2} \right], \]

\[ (k_8^1) = 2 \left[ \psi_{1,x} \frac{\partial}{\partial x} + \psi_{2,x} \frac{\partial}{\partial y} + \psi_{1,y} \frac{\partial}{\partial x} + \psi_{2,y} \frac{\partial}{\partial y} + \frac{\psi_{1} \theta_{1}}{R_1} + \frac{\psi_{2} \theta_{2}}{R_2} \right], \]

\[ (k_8^6) = 2 \left[ \psi_{1,x} \frac{\partial}{\partial x} + \psi_{2,x} \frac{\partial}{\partial y} + \psi_{1,y} \frac{\partial}{\partial x} + \psi_{2,y} \frac{\partial}{\partial y} + \frac{\psi_{2} \psi_{1}}{R_2} + \frac{\psi_{2} \theta_{1}}{R_1} + \frac{\psi_{2} \theta_{2}}{R_2} \right], \]

\[ (k_3^5) = 2 \left[ 3 \phi_{1,x} \psi_{1,y} + 3 \phi_{2,x} \psi_{2,y} \right], \]

\[ (k_4^4) = 6 \left( \theta_{1,x} \psi_{1,y} + \theta_{2,x} \psi_{2,y} \right), \]

\[ (k_1^{10}) = \left[ \theta_{1,x}^2 + \theta_{2,x}^2 + \frac{\theta_{1}^2}{R_1} \right], \]

\[ (k_2^{10}) = \left[ \theta_{1,y}^2 + \theta_{2,y}^2 + \frac{\theta_{1}^2}{R_2} \right], \]

\[ (k_6^{10}) = 2 \left[ \theta_{1,x} \theta_{1,y} + \theta_{2,x} \theta_{2,y} + \frac{\theta_{1} \theta_{2}}{R_1 R_2} \right]. \]

\dots (A.1)
Other coupled terms appeared in the expressions of (A.1):

\[
\begin{align*}
\overline{u}_x &= \frac{\partial \overline{u}}{\partial x} + \frac{\overline{w}}{R_1} \overline{u}_y, \\
\overline{u}_y &= \frac{\partial \overline{u}}{\partial y} + \frac{\overline{w}}{R_{12}} \overline{v}_x, \\
\overline{v}_x &= \frac{\partial \overline{v}}{\partial x} + \frac{\overline{w}}{R_2} \overline{v}_y, \\
\overline{v}_y &= \frac{\partial \overline{v}}{\partial y} + \frac{\overline{w}}{R_{12}} \overline{w}_x, \\
\overline{w}_x &= \frac{\partial \overline{w}}{\partial x} - \frac{\overline{u}}{R_1}, \\
\overline{w}_y &= \frac{\partial \overline{w}}{\partial y} - \frac{\overline{v}}{R_2}.
\end{align*}
\]

\[
\phi_{1,x} = \frac{\partial \phi}{\partial x}, \quad \phi_{1,y} = \frac{\partial \phi}{\partial y}, \quad \phi_{2,x} = \frac{\partial \phi}{\partial x}, \quad \phi_{2,y} = \frac{\partial \phi}{\partial y}, \quad \psi_{1,x} = \frac{\partial \psi}{\partial x}, \quad \psi_{1,y} = \frac{\partial \psi}{\partial y},
\]

\[
\psi_{2,x} = \frac{\partial \psi_2}{\partial x}, \quad \psi_{2,y} = \frac{\partial \psi_2}{\partial y}, \quad \theta_{1,x} = \frac{\partial \theta}{\partial x}, \quad \theta_{1,y} = \frac{\partial \theta}{\partial y}, \quad \theta_{2,x} = \frac{\partial \theta_2}{\partial x}, \quad \theta_{2,y} = \frac{\partial \theta_2}{\partial y}.
\]

Linear and Nonlinear Thickness Coordinate Matrices as Appeared in Eq. (3.10)

\[
\mathbf{H}_L = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0 & z^3 & 0 & 0 & 0 & 0

0 & 1 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0 & 0 & z^3 & 0 & 0

0 & 0 & 1 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0 & 0 & z^3 & 0 & 0

0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0 & 0 & z^3 & 0 & 0

0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0 & 0 & z^3 & 0 & 0
\end{bmatrix}
\]

\[
\mathbf{H}_{NL} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0 & 0

0 & 1 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0

0 & 0 & 1 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0

0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0

0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0

z^3 & 0 & 0 & 0 & 0 & z^4 & 0 & 0 & 0 & 0 & z^5 & 0 & 0 & 0 & 0 & z^6 & 0 & 0

0 & z^3 & 0 & 0 & 0 & 0 & z^4 & 0 & 0 & 0 & 0 & z^5 & 0 & 0 & 0 & 0 & z^6 & 0 & 0

0 & 0 & z^3 & 0 & 0 & 0 & 0 & z^4 & 0 & 0 & 0 & 0 & z^5 & 0 & 0 & 0 & 0 & z^6 & 0 & 0

0 & 0 & 0 & z^3 & 0 & 0 & 0 & 0 & z^4 & 0 & 0 & 0 & 0 & z^5 & 0 & 0 & 0 & 0 & z^6 & 0 & 0

0 & 0 & 0 & 0 & z^3 & 0 & 0 & 0 & 0 & z^4 & 0 & 0 & 0 & 0 & z^5 & 0 & 0 & 0 & 0 & z^6 & 0 & 0
\end{bmatrix}
\]

\ldots \quad (A.2)
Appendix B

Geometric Strain Vector and the Material Property Matrix Derivation

The evaluation steps of the geometric strain vector \( \{ \varepsilon_G \} \) and the material property matrix \( [D_G] \) as appeared in Eq. 3.26 in Chapter 3, is linearized as follows using the steps in Cook et al. (2009):

\[
\{ \varepsilon_G \} = \frac{1}{2} \begin{bmatrix}
\left( \overline{u}_x \right)^2 + \left( \overline{v}_y \right)^2 + \left( \overline{w}_z \right)^2 \\
\left( \overline{u}_y \right)^2 + \left( \overline{v}_x \right)^2 + \left( \overline{w}_z \right)^2 \\
2 \left( \overline{u}_x \right) \left( \overline{u}_y \right) + \left( \overline{v}_x \right) \left( \overline{v}_y \right) + \left( \overline{w}_z \right) \left( \overline{w}_z \right)
\end{bmatrix}
\]

or, \( \{ \varepsilon_G \} = [H_G] \{ \varepsilon_G \} = [H_G] [A_G] \{ \beta_G \} \)

The values of \( [A_G] \) and \( \{ \beta_G \} \) are

\[
[A_G] = \begin{bmatrix}
\left( u_x \right) + \left( v_y \right) + \left( w_z \right) \\
\left( u_y \right) + \left( v_x \right) + \left( w_z \right) \\
\left( u_x \right) \left( u_y \right) + \left( v_x \right) \left( v_y \right) + \left( w_z \right) \left( w_z \right)
\end{bmatrix}
\]

and \( \{ \beta_G \} = \begin{bmatrix}
u_x \\
v_y \\
v_x \\
w_x \\
w_y
\end{bmatrix}\)

The expressions of \( u_x, v_x, w_x, u_y, v_y, w_y \) are:

\[
\overline{u}_x = \frac{\partial \overline{u}}{\partial x} + \frac{\overline{w}}{R_1}, \quad \overline{u}_y = \frac{\partial \overline{u}}{\partial y}, \quad \overline{v}_x = \frac{\partial \overline{v}}{\partial x} + \frac{\overline{w}}{R_2}, \quad \overline{v}_y = \frac{\partial \overline{v}}{\partial y}, \quad \overline{w}_x = \frac{\partial \overline{w}}{\partial x} - \frac{\overline{u}}{R_1}, \quad \text{and} \quad \overline{w}_y = \frac{\partial \overline{w}}{\partial y} - \frac{\overline{v}}{R_2},
\]

respectively.
Appendix

The values of material property matrix are obtained by the following procedure

\[
[D_G] = \sum_{k=1}^{\infty} \left[ H_G \right]^T \{ \delta \}^k \left[ H_G \right]
\]

where,

\[
\{ \delta \}^k = \begin{bmatrix}
N_{AF1} + N_{MC1} & N_{AF12} + N_{MC12} & 0 & 0 & 0 & 0 \\
N_{AF12} + N_{MC12} & N_{AF2} + N_{MC2} & 0 & 0 & 0 & 0 \\
0 & 0 & N_{AF1} + N_{MC1} & N_{AF12} + N_{MC12} & 0 & 0 \\
0 & 0 & N_{AF12} + N_{MC12} & N_{AF2} + N_{MC2} & 0 & 0 \\
0 & 0 & 0 & 0 & N_{AF1} + N_{MC1} & N_{AF12} + N_{MC12} \\
0 & 0 & 0 & 0 & N_{AF12} + N_{MC12} & N_{AF2} + N_{MC2}
\end{bmatrix}
\]
Appendix C

Linear Form of the Nonlinear Strain as Appeared in the General Mathematical Formulation

Individual terms of the matrix $[A]$ as appeared in the Eq. (3.29)

$[A]_{1,1} = u_x$, $[A]_{1,3} = v_x$, $[A]_{1,5} = w_x$, $[A]_{2,2} = u_y$, $[A]_{2,4} = v_y$, $[A]_{2,6} = w_y$

$[A]_{3,1} = u_y$, $[A]_{3,2} = u_x$, $[A]_{3,3} = v_y$, $[A]_{3,4} = v_x$, $[A]_{3,5} = w_y$, $[A]_{3,6} = w_x$

$[A]_{4,1} = \phi_1$, $[A]_{4,3} = \phi_2$, $[A]_{4,22} = u_x$, $[A]_{4,23} = v_x$, $[A]_{5,2} = \phi_1$, $[A]_{5,4} = \phi_2$

$[A]_{5,22} = u_x$, $[A]_{5,23} = v_x$, $[A]_{6,1} = \phi_{1,x}$, $[A]_{6,3} = \phi_{2,x}$, $[A]_{6,5} = -\frac{\phi_1}{R_1}$, $[A]_{6,7} = u_x$

$[A]_{6,9} = v_x$, $[A]_{6,22} = -\frac{w_x}{R_1}$, $[A]_{7,2} = \phi_{1,y}$, $[A]_{7,4} = \phi_{2,y}$, $[A]_{7,6} = -\frac{\phi_2}{R_2}$

$[A]_{7,8} = u_y$, $[A]_{7,10} = v_y$, $[A]_{7,23} = -\frac{w_y}{R_2}$, $[A]_{8,1} = \phi_{1,y}$, $[A]_{8,2} = \phi_{2,y}$, $[A]_{8,3} = \phi_{1,x}$

$[A]_{8,4} = \phi_{2,x}$, $[A]_{8,5} = -\frac{\phi_1}{R_1}$, $[A]_{8,6} = -\frac{\phi_2}{R_2}$, $[A]_{8,7} = u_y$, $[A]_{8,8} = u_x$, $[A]_{8,9} = v_y$

$[A]_{8,10} = v_x$, $[A]_{8,22} = -\frac{w_x}{R_1}$, $[A]_{8,23} = -\frac{w_y}{R_2}$, $[A]_{9,1} = 2\psi_1$, $[A]_{9,3} = 2\psi_2$, $[A]_{9,7} = \phi_1$

$[A]_{9,9} = \phi_2$, $[A]_{9,22} = \phi_{1,x}$, $[A]_{9,23} = \phi_{2,x}$, $[A]_{9,24} = 2u_x$, $[A]_{9,25} = 2v_x$

$[A]_{10,2} = 2\psi_1$, $[A]_{10,4} = 2\psi_2$, $[A]_{10,8} = \phi_1$, $[A]_{10,10} = \phi_2$, $[A]_{10,22} = \phi_{1,y}$

$[A]_{10,23} = \phi_{2,y}$, $[A]_{10,24} = 2u_y$, $[A]_{10,25} = 2v_y$, $[A]_{11,1} = \psi_{1,x}$, $[A]_{11,3} = \psi_{2,x}$
\[
[A]_{1,5} = -\frac{\psi_1}{R_1}, \quad [A]_{1,7} = \phi_{1,x}, \quad [A]_{1,9} = \phi_{2,x}, \quad [A]_{1,11} = u_x, \quad [A]_{1,13} = v_x, \quad [A]_{1,22} = \frac{\phi_y}{R_1}.
\]

\[
[A]_{1,24} = -\frac{w_x}{R_1}, \quad [A]_{1,2} = \psi_{1,y}, \quad [A]_{1,4} = \psi_{2,y}, \quad [A]_{1,6} = -\frac{\psi_2}{R_2}, \quad [A]_{1,8} = \phi_{1,y},
\]

\[
[A]_{12,10} = \phi_{2,y}, \quad [A]_{12,12} = u_y, \quad [A]_{12,14} = v_y, \quad [A]_{12,23} = \frac{\phi_y}{R_2}, \quad [A]_{12,25} = -\frac{w_y}{R_2},
\]

\[
[A]_{13,1} = \psi_{1,y}, \quad [A]_{13,2} = \psi_{1,x}, \quad [A]_{13,3} = \psi_{2,y}, \quad [A]_{13,4} = \psi_{2,x}, \quad [A]_{13,5} = -\frac{\psi_2}{R_2},
\]

\[
[A]_{13,6} = -\frac{\psi_1}{R_1}, \quad [A]_{13,7} = \phi_{1,y}, \quad [A]_{13,8} = \phi_{1,x}, \quad [A]_{13,9} = \phi_{2,y}, \quad [A]_{13,10} = \phi_{2,x}, \quad [A]_{13,11} = u_y,
\]

\[
[A]_{13,12} = u_x, \quad [A]_{13,13} = v_y, \quad [A]_{13,14} = v_x, \quad [A]_{13,22} = \frac{1}{R_1}, \quad [A]_{13,23} = \frac{1}{R_2},
\]

\[
[A]_{13,24} = -\frac{w_y}{R_1}, \quad [A]_{13,25} = -\frac{w_x}{R_2}, \quad [A]_{14,3} = 3\theta_1, \quad [A]_{14,3} = 3\theta_2, \quad [A]_{14,7} = 2\psi_1,
\]

\[
[A]_{14,9} = 2\psi_2, \quad [A]_{14,11} = \phi_1, \quad [A]_{14,13} = \phi_2, \quad [A]_{14,22} = \psi_{1,x}, \quad [A]_{14,23} = \psi_{2,x}, \quad [A]_{14,24} = 2\phi_1,
\]

\[
[A]_{14,25} = 2\phi_2, \quad [A]_{14,26} = 3u_x, \quad [A]_{14,27} = 3v_x, \quad [A]_{15,2} = 3\theta_1, \quad [A]_{15,4} = 3\theta_2,
\]

\[
[A]_{15,8} = 2\psi_1, \quad [A]_{15,10} = 2\psi_2, \quad [A]_{15,12} = \phi_1, \quad [A]_{15,14} = \phi_2, \quad [A]_{15,22} = \psi_{1,y}, \quad [A]_{15,23} = \psi_{2,y},
\]

\[
[A]_{15,24} = 2\phi_1, \quad [A]_{15,25} = 2\phi_2, \quad [A]_{15,26} = 3u_y, \quad [A]_{15,27} = 3v_y, \quad [A]_{16,1} = \theta_1,
\]

\[
[A]_{16,3} = \theta_2, \quad [A]_{16,5} = -\frac{\theta_1}{R_1}, \quad [A]_{16,7} = \psi_{1,x}, \quad [A]_{16,9} = \psi_{2,x}, \quad [A]_{16,11} = \phi_1, \quad [A]_{16,13} = \phi_2,
\]

\[
[A]_{16,15} = u_x, \quad [A]_{16,17} = v_x, \quad [A]_{16,22} = \frac{\psi_y}{R_1}, \quad [A]_{16,24} = \frac{\phi_y}{R_2}, \quad [A]_{16,26} = -\frac{w_x}{R_1},
\]

\[
[A]_{17,2} = \theta_1, \quad [A]_{17,4} = \theta_2, \quad [A]_{17,6} = -\frac{\theta_2}{R_2}, \quad [A]_{17,8} = \psi_{1,y}, \quad [A]_{17,10} = \psi_{2,y},
\]

\[
[A]_{17,12} = \phi_{1,y}, \quad [A]_{17,14} = \phi_{2,y}, \quad [A]_{17,16} = u_y, \quad [A]_{17,18} = v_y, \quad [A]_{17,23} = \frac{\psi_2}{R_2}, \quad [A]_{17,25} = \frac{\phi_y}{R_2},
\]

\[
[A]_{17,27} = -\frac{w_y}{R_2}, \quad [A]_{18,1} = \theta_1, \quad [A]_{18,2} = \theta_2, \quad [A]_{18,3} = \theta_2, \quad [A]_{18,4} = \theta_2,
\]

\[
[A]_{18,5} = -\frac{\theta_1}{R_1}, \quad [A]_{18,6} = -\frac{\theta_2}{R_2}, \quad [A]_{18,7} = \psi_{1,y}, \quad [A]_{18,8} = \psi_{1,x}, \quad [A]_{18,9} = \psi_{2,y},
\]
Appendix

\[
\begin{align*}
[A]_{18,10} &= \psi_{2,x}, \\
[A]_{18,11} &= \phi_{1,y}, \\
[A]_{18,12} &= \phi_{1,x}, \\
[A]_{18,13} &= \phi_{2,y}, \\
[A]_{18,14} &= \phi_{2,x}, \\
[A]_{18,15} &= u_{y}, \\
[A]_{18,16} &= u_{x}, \\
[A]_{18,17} &= v_{y}, \\
[A]_{18,18} &= v_{x}, \\
[A]_{18,22} &= \frac{1}{R_{1}} \frac{\psi_{2}}{R_{2}}, \\
[A]_{18,23} &= \frac{1}{R_{1}} \frac{\psi_{1}}{R_{2}}, \\
[A]_{18,24} &= \frac{1}{R_{1}} \frac{\phi_{2}}{R_{2}}, \\
[A]_{18,25} &= \frac{1}{R_{1}} \frac{\phi_{1}}{R_{2}}, \\
[A]_{18,26} &= -\frac{w_{y}}{R_{1}}, \\
[A]_{18,27} &= -\frac{w_{x}}{R_{2}}, \\
[A]_{19,7} &= 3\theta_{1}, \\
[A]_{19,9} &= 3\theta_{2}, \\
[A]_{19,11} &= 2\psi_{1}, \\
[A]_{19,13} &= 2\psi_{2}, \\
[A]_{19,15} &= \phi_{1}, \\
[A]_{19,17} &= \phi_{2}, \\
[A]_{19,22} &= \theta_{1,x}, \\
[A]_{19,23} &= \theta_{2,x}, \\
[A]_{19,24} &= 2\psi_{1,x}, \\
[A]_{19,25} &= 2\psi_{2,x}, \\
[A]_{19,26} &= 3\phi_{1,x}, \\
[A]_{19,27} &= 3\phi_{2,x}, \\
[A]_{20,8} &= 3\theta_{1}, \\
[A]_{20,10} &= 3\theta_{2}, \\
[A]_{20,12} &= 2\psi_{1}, \\
[A]_{20,14} &= 2\psi_{2}, \\
[A]_{20,16} &= \phi_{1}, \\
[A]_{20,18} &= \phi_{2}, \\
[A]_{20,22} &= \theta_{1,y}, \\
[A]_{20,23} &= \theta_{2,y}, \\
[A]_{20,24} &= 2\psi_{1,y}, \\
[A]_{20,25} &= 2\psi_{2,y}, \\
[A]_{21,11} &= \psi_{1,x}, \\
[A]_{21,13} &= \psi_{2,x}, \\
[A]_{21,15} &= \phi_{1,x}, \\
[A]_{21,17} &= \phi_{2,x}, \\
[A]_{21,22} &= \frac{\theta_{1}}{R_{1}^{2}}, \\
[A]_{21,24} &= \frac{\psi_{1}}{R_{1}^{2}}, \\
[A]_{21,26} &= \frac{\phi_{1}}{R_{1}^{2}}, \\
[A]_{22,8} &= \theta_{1,y}, \\
[A]_{22,10} &= \theta_{2,y}, \\
[A]_{22,12} &= \psi_{1,y}, \\
[A]_{22,14} &= \psi_{2,y}, \\
[A]_{22,16} &= \phi_{1,y}, \\
[A]_{22,18} &= \phi_{2,y}, \\
[A]_{22,23} &= \frac{\theta_{2}}{R_{2}^{2}}, \\
[A]_{22,25} &= \frac{\psi_{2}}{R_{2}^{2}}, \\
[A]_{22,27} &= \frac{\phi_{2}}{R_{2}^{2}}, \\
[A]_{23,7} &= \theta_{1,y}, \\
[A]_{23,8} &= \theta_{1,x}, \\
[A]_{23,9} &= \theta_{2,y}, \\
[A]_{23,10} &= \theta_{2,x}, \\
[A]_{23,11} &= \psi_{1,y}, \\
[A]_{23,12} &= \psi_{1,x}, \\
[A]_{23,13} &= \psi_{2,y}, \\
[A]_{23,14} &= \psi_{2,x}, \\
[A]_{23,15} &= \phi_{1,y}, \\
[A]_{23,16} &= \phi_{1,x}, \\
[A]_{23,17} &= \phi_{2,y}, \\
[A]_{23,18} &= \phi_{2,x}, \\
[A]_{23,22} &= \frac{1}{R_{1}} \frac{\theta_{2}}{R_{2}}, \\
[A]_{23,23} &= \frac{1}{R_{2}} \frac{\theta_{1}}{R_{1}}, \\
[A]_{23,24} &= \frac{1}{R_{1}} \frac{\psi_{2}}{R_{2}}, \\
[A]_{23,25} &= \frac{1}{R_{2}} \frac{\phi_{1}}{R_{2}}, \\
[A]_{23,26} &= \frac{1}{R_{2}} \frac{\phi_{2}}{R_{2}}, \\
[A]_{23,27} &= \frac{1}{R_{1}} \frac{\phi_{1}}{R_{1}}, \\
[A]_{24,11} &= 3\theta_{1}, \\
[A]_{24,13} &= 3\theta_{2}, \\
[A]_{24,15} &= 2\psi_{1}, \\
[A]_{24,17} &= 2\psi_{2}, \\
[A]_{24,24} &= 2\theta_{1,x}, \\
[A]_{24,25} &= 2\theta_{2,x}, \\
[A]_{24,26} &= 3\psi_{1,x}, \\
[A]_{24,27} &= 3\psi_{2,x}, \\
[A]_{25,12} &= 3\theta_{1}, \\
[A]_{25,14} &= 3\theta_{2}, \\
[A]_{25,16} &= 2\psi_{1}, \\
[A]_{25,18} &= 2\psi_{2}, \\
[A]_{25,24} &= 2\theta_{1,x}, \\
[A]_{25,25} &= 2\theta_{2,y}, \\
[A]_{25,26} &= 3\psi_{1,y}, \\
[A]_{25,27} &= 3\psi_{2,y}, \\
[A]_{26,11} &= \theta_{1,x}, \\
[A]_{26,13} &= \theta_{2,x}, \\
[A]_{26,15} &= \psi_{1,x}, \\
[A]_{26,17} &= \psi_{2,x}, \\
[A]_{26,24} &= \frac{\theta_{1}}{R_{1}^{2}}, \\
[A]_{26,26} &= \frac{\psi_{1}}{R_{1}^{2}}, \\
\end{align*}
\]
Appendix

\[ [A]_{27,12} = \theta_{1,y}, \quad [A]_{27,14} = \theta_{2,y}, \quad [A]_{27,16} = \psi_{1,y}, \quad [A]_{27,18} = \psi_{2,y}, \quad [A]_{27,25} = \frac{\theta_2}{R_2}, \]

\[ [A]_{27,27} = \frac{\psi_2}{R_2}, \quad [A]_{28,11} = \theta_{1,y}, \quad [A]_{28,12} = \theta_{1,x}, \quad [A]_{28,13} = \theta_{2,y}, \quad [A]_{28,14} = \theta_{2,x}, \]

\[ [A]_{28,15} = \psi_{1,y}, \quad [A]_{28,16} = \psi_{1,x}, \quad [A]_{28,17} = \psi_{2,y}, \quad [A]_{28,18} = \psi_{2,x}, \quad [A]_{28,24} = \frac{1}{R_1 R_2}, \]

\[ [A]_{28,25} = \frac{\theta_1}{R_1 R_1}, \quad [A]_{28,26} = \frac{\psi_1}{R_1 R_2}, \quad [A]_{28,27} = \frac{\psi_1}{R_2 R_1}, \quad [A]_{29,15} = 3\theta_1, \quad [A]_{29,17} = 3\theta_2, \]

\[ [A]_{29,26} = 3\theta_{1,x}, \quad [A]_{29,27} = 3\theta_{2,x}, \quad [A]_{30,16} = 3\theta_1, \quad [A]_{30,18} = 3\theta_2, \quad [A]_{30,26} = 3\theta_{1,y}, \]

\[ [A]_{30,27} = 3\theta_{2,y}, \quad [A]_{31,15} = \theta_{1,y}, \quad [A]_{31,17} = \theta_{1,x}, \quad [A]_{31,26} = \frac{\theta_1}{R_1}, \quad [A]_{32,16} = \theta_{1,x}, \quad [A]_{32,18} = \theta_{2,y}, \]

\[ [A]_{33,15} = \theta_{1,x}, \quad [A]_{33,16} = \theta_{1,y}, \quad [A]_{33,18} = \theta_{2,x}, \quad [A]_{33,26} = \frac{1}{R_1 R_2}, \quad [A]_{33,27} = \frac{1}{R_2 R_1}. \]

(C1)

Individual terms of the \([G]\) matrix

\[ [G]_{1,1} = \frac{\partial}{\partial x}, \quad [G]_{1,3} = \frac{1}{R_1}, \quad [G]_{2,1} = \frac{\partial}{\partial y}, \quad [G]_{3,2} = \frac{\partial}{\partial x}, \quad [G]_{4,2} = \frac{\partial}{\partial y}, \quad [G]_{4,3} = \frac{1}{R_2}, \]

\[ [G]_{5,1} = -\frac{1}{R_1}, \quad [G]_{5,3} = \frac{\partial}{\partial x}, \quad [G]_{6,2} = -\frac{1}{R_2}, \quad [G]_{6,3} = \frac{\partial}{\partial y}, \quad [G]_{7,4} = \frac{\partial}{\partial x}, \quad [G]_{8,4} = \frac{\partial}{\partial y}, \]

\[ [G]_{9,5} = \frac{\partial}{\partial x}, \quad [G]_{10,5} = \frac{\partial}{\partial y}, \quad [G]_{11,6} = \frac{\partial}{\partial x}, \quad [G]_{12,6} = \frac{\partial}{\partial y}, \quad [G]_{13,7} = \frac{\partial}{\partial x}, \quad [G]_{14,7} = \frac{\partial}{\partial y}, \]

\[ [G]_{15,8} = \frac{\partial}{\partial x}, \quad [G]_{16,8} = \frac{\partial}{\partial y}, \quad [G]_{17,9} = \frac{\partial}{\partial y}, \quad [G]_{18,9} = \frac{\partial}{\partial x}, \quad [G]_{19,1} = 1, \quad [G]_{20,2} = 1, \quad [G]_{21,3} = 1, \]

\[ [G]_{22,4} = 1, \quad [G]_{23,5} = 1, \quad [G]_{24,6} = 1, \quad [G]_{25,7} = 1, \quad [G]_{26,8} = 1, \quad [G]_{27,9} = 1. \]

(C2)
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