SYNOPSIS

The thesis deals with certain quantum field systems exhibiting spontaneous symmetry breaking (SSB) and their response to temperature. These models are of interest in particle physics, solid state physics and nonlinear optics. The nature of phase transition that these systems may undergo is also investigated. The main theoretical tools employed to understand these properties are the lattice formalism, effective potential method and the renormalisation group.

Lattice gauge theory is a nonperturbative formalism and constitutes an attempt to explain various problems in quantum field theory, which were hitherto unanswered. There exists a close resemblance between this formulation and statistical physics. In fact many of the phenomena discovered independently in particle physics and condensed matter physics bear close resemblance as far as the theoretical questions are concerned. Phase transition is a process associated with the change of symmetry. Sine-Gordon field system is a nonlinear scalar field theory which exhibits SSB. In order to study the nature of phase transition associated with this symmetry breaking, the lattice formulation is used. The present investigation reveals that the sine-Gordon field system in 1+1 dimension undergoes a
second order phase transition from the disordered phase characterised by \( \langle \varphi \rangle = 0 \) to the ordered phase characterised by \( \langle \varphi \rangle \neq 0 \) where \( \langle \varphi \rangle \) denotes the vacuum expectation value. The critical values of the parameters which characterise these two phases are also evaluated. The method is extended to generalised sine-Gordon field systems exhibiting a second order phase transition.

The fact that SSB in relativistic field theories disappears when the temperature of the system is increased above a critical value has significant consequences in particle physics. The most important physical consideration in statistical mechanics is the introduction of temperature. The relationship between SSB and temperature is investigated via the effective potential defined at finite temperature. As an application of this formalism, the effect of temperature on the 1+1 dimensional sine-Gordon field system discussed above is studied. The effective potential in the one-loop order at zero temperature is evaluated with a view to obtaining the renormalised Lagrangian for the system. The advantage of this procedure is that the quantum correction to soliton mass immediately follows. By evaluating the effective potential at finite temperature, the critical temperature above which the system regains the original symmetry is obtained. The soliton energy at finite temperature is found to be less than the classical
soliton energy, and decreases smoothly with temperature and finally vanishes at the critical temperature, signalling a second order phase transition. The calculations are extended to the generalised sine-Gordon models. The study of generalised sine-Gordon models is motivated by the fact that for a particular case, the generalised sine-Gordon becomes the double sine-Gordon model which finds many applications in contemporary physics.

The formalism discussed above is applied to a \( \phi^6 \) field system in 1+1 dimensions, which is a nonlinear field theory exhibiting SSB. This model is of importance in the theory of ferroelectric and structural phase transitions encountered in solid state physics. First, the model under consideration is shown to be renormalisable by evaluating the effective potential at zero temperature. Then the effective potential at finite temperature is evaluated. The calculations are done up to two-loop level. The critical temperature above which the system regains the symmetry property is calculated. Here the nature of phase transition of the model field system is investigated by evaluating the thermal average of the field variable and is shown to be one of first order.

With the advent of the Grand Unified era, studies on finite temperature behaviour of quantum field systems are of prime importance since these studies will help us to
speculate on the evolution of the universe at times as early as $10^{-35}$ sec after the big bang. In these studies the dependence of the coupling constant on temperature is to be taken into account as it can reduce drastically the amount of supercooling associated with the first order phase transition in grand unified theories. Using the vertex renormalisation procedure, the temperature behaviour of the coupling constant in two models is investigated. The massive $\phi^4$ theory is considered first. The present investigation shows that when $m^2 > 0$, the scalar coupling constant decreases with temperature leading to a phase transition to a non-interacting phase. In a model with $m^2 < 0$, the coupling constant increases logarithmically. A renormalisation group study of the problem also supports these results. The temperature behaviour of the gauge coupling constant in scalar electrodynamics is also studied and it is found that the gauge coupling constant uniformly increases with temperature. This study is extended to include the temperature dependence of gauge coupling constants in abelian and non-abelian gauge theories. It is found that the gauge coupling constant in abelian gauge theory increases with temperature while that of non-abelian gauge theory decreases with temperature consistent with the idea of asymptotic freedom. This investigation is of relevance to Grand Unified Models of Cosmological evolution.