CHAPTER 2

ON THE FURTHER PROBLEM OF NONADDITIVITY
IN TWO WAY-ANOVA

2.1 Introduction

The difficulties in using analysis of variance (ANOVA) F-test for comparing the efficiency of fishing gear have been discussed by Nair (1982) and Nair & Alagaraja (1982). Broadly, these problems arose from the lack of satisfaction of the assumptions underlying analysis of variance. The importance of each assumption has been clearly discussed by Eisenhart (1947). Kempthorne (1967) has indicated that the main requirements on the usefulness of a model are the additivity of treatment effects and homogeneity of errors and that of these two additivity is more important. Treatment of nonadditivity in two-way classification has received much attention (Tukey, 1949; Mandel, 1961; Daniel, 1976; Johnson and Graybill, 1972a, b; Krishnaiah and Yochmowitz, 1980; Marasinghe and Johnson, 1981, 1982; Bradu and Gabriel, 1978 and Snee, 1982). Snedecor and Cochran (1968) describe the usefulness of Tukey's (1949) test of additivity "(i) to help decide if a transformation is necessary (ii) to suggest a suitable transformation and (iii) to learn if a transformation has been successful in producing additivity". Federer (1967) has observed that
Tukey's sum of squares for nonadditivity is increased when one or more observations are usually discrepant and when the row and column effects are not additive and that nonadditivity could arise from more than one source.

Johnson and Graybill's (1972b) and Rao's (1974) methods of derivation and interpretation of Tukey's test show that when the above type of nonadditivity is present, the model is:

\[ X_{ij} = \mu + \alpha_i + \beta_j + \lambda \alpha_i \beta_j + \epsilon_{ij} \]

and that Tukey's test correspond to testing \( \lambda = 0 \).

\( X_{ij} \) stands for catch on the \( i^{th} \) day for the \( j^{th} \) gear,

\( \mu \) is the overall mean catch, \( \alpha_i \) and \( \beta_j \) are the effects due to the \( i^{th} \) day and \( j^{th} \) gear respectively, \( \lambda \) a constant and \( \epsilon_{ij} \) is the error term. Mandel, as quoted by Krishnaiah and Yochmowitz (1980), identified this model as the concurrent model and the concurrent model can be tested effectively by using Tukey's test for nonadditivity.

Johnson and Graybill (1972b) and Hegemann and Johnson (1976b) have discussed that when Tukey's test shows significant nonadditivity, that is when the model given above describes the data, then the best way to analyse the data may be to find a transformation that will restore additivity. Bartlett (1947) gives a number of transformations suitable for various forms of relationship between the variance in terms of the
mean and the distribution for which those are appropriate. He recommended logarithmic transformation for certain type of data with considerable heterogeneity. Nair (1982) has found that for data on fishing experiments with trawl nets logarithmic transformation did not stabilize the variance. Also application of Tukey's test to the data after logarithmic transformation showed highly significant nonadditivity ($p < 0.001$). Cochran (1947) has observed that nonadditivity tends to produce heterogeneity of the error variance. Snee (1982) discusses procedures to examine whether nonadditivity is caused due to nonhomogeneous variance or interaction between row and column factors. These show the relative importance of the assumption of additivity and this chapter presents the results of an investigation on nonadditivity in trawl net—catch data on comparative fishing efficiency studies and procedures to tackle the problem using graphical analysis and transformation.

2.2 Materials and Methods

To decide whether a transformation is necessary and if required what would be the appropriate one, Tukey's (1949) test of additivity was applied to the four sets of data given in Nair (1982). Graphical analysis of nonadditivity (Tukey, 1949) was applied to these data to
check whether the nonadditivity was due to analysis in
the wrong form or due to one or more usually discrepant
values. Tukey's test of additivity leads to transformation
of the form $Y = X^p$ in which $X$ is the original scale. The
procedure followed in Snedecor and Cochran (1968) was
applied to determine $p$ to which $X$, the observation must
be raised to produce additivity. $p$ is estimated by
$(1 - B \bar{x}.)$, where $B$ is the regression coefficient in the linear
regression of the residual $(\hat{X}_{ij} - \bar{x}_{ij})$ on the variate
$(\bar{x}_i - \bar{x}_{..}) (\bar{x}_j - \bar{x}_{..})$. An estimate of $B$ is obtained from
$$B = \frac{N}{D},$$
where $N = \sum w_i d_i$, $w_i = \sum X_{ij} d_j$, $d_i = (\bar{x}_i - \bar{x}_{..})$,
$d_j = (\bar{x}_j - \bar{x}_{..})$ and
$$D = \left( \sum d_i^2 \sum d_j^2 \right) \bar{x}_{..} \bar{x}_{..} \bar{x}_j \bar{x}_j$$
$\bar{x}_{..}$ refer to the row (block) means, column (treatment)
means and grand mean respectively. Tests for nonadditivity
is given by $F$, where $F$ follows Snedecor's $F$ distribution
with 1 and $r-1 (c-1)-1$ degrees of freedom, $r$ and $c$
indicating numbers of rows and columns, respectively.
Tukey (1949) discusses transformations which are additive
for $0 \leq p \leq 1$, $P = 1$ and $1 < p$ and log $(x+a)$ corresponding
to none of these. Snedecor and Cochran (1968) stated
that when $p = -1$, it is a reciprocal transformation
analysing $1/X$, instead of $X$. ($p = 0$ corresponds to logarithmic
transformation because for $p$ very small $X^p$ behaves like log $X$).
2.3 Results and Discussion

Application of Tukey's test of additivity for the four sets of data on trawl catch (Nair, 1982) showed that there was significant nonadditivity in all the sets (Table 1). For sets 1-3 (that is for the actual data), nonadditivity was found to be very highly significant with $p < 0.001$.

Table 1. Test for nonadditivity of the four sets of data

<table>
<thead>
<tr>
<th>Set</th>
<th>F for nonadditivity</th>
<th>Degrees of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 1</td>
<td>38.64***</td>
<td>1,67</td>
</tr>
<tr>
<td>Set 2</td>
<td>63.87***</td>
<td>1,67</td>
</tr>
<tr>
<td>Set 3</td>
<td>87.70***</td>
<td>1,67</td>
</tr>
<tr>
<td>Set 4</td>
<td>4.80*</td>
<td>1,18</td>
</tr>
</tbody>
</table>

Tukey's (1949) procedure was followed to check whether nonadditivity was caused by the presence of one or more discrepant observations or due to the need for a transformation. His method of graphical analysis for detecting the discrepant observations (outliers) was applied to the four sets of data. The method involves in plotting $W_i$ against the block means. According to Tukey, "a usually discrepant observation will tend to be reflected by one point high or low and the others distributed around
a nearly horizontal line. An analysis in the wrong terms will tend to be reflected by a slanting regression line."

To determine the points high or low Tukey provided a 2s limit, namely,

\[(\text{Average cross product}) \pm 2 \left( \frac{\text{sums of squares}}{\text{of deviations}} \right)^{\frac{1}{2}} \left( \frac{\text{Means square}}{\text{for balance}} \right)^{\frac{1}{2}} = \frac{\text{Means column}}{\text{of deviations for balance}} \]

The plots of \( w_i \) against the row means with the 2s limits for sets 1-4 are presented in Figs. 1-4. The figures show the presence of outliers in all the four sets ranging from 1 to 5 in number. It is clear from the figures that the points excluding the outliers are distributed on a nearly horizontal line for set 1 and on a slanting regression line for sets 2 to 4. This shows that no transformation is required for set 1 after removing the outliers while it is required for the other sets. This was confirmed by applying Tukey's test to the outlier-eliminated data (Table 1). Sets 2-4 showed the presence of nonadditivity indicating the need for a transformation for these sets.

The power transformation \( Y = X^p \), suggested by Tukey's test of additivity were worked out for sets 2-4. These have been presented in Table 3 along with the estimated values of B and P. For set 2, the transformation worked out to \( Y = X^{-0.31} \), which is a reciprocal transformation.
Table 2. Test for nonadditivity of the outlier-eliminated data

<table>
<thead>
<tr>
<th>Set</th>
<th>F for nonadditivity</th>
<th>Degrees of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.02</td>
<td>1.59</td>
</tr>
<tr>
<td></td>
<td>(not significant)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>9.90**</td>
<td>1.61</td>
</tr>
<tr>
<td>3</td>
<td>34.37***</td>
<td>1.57</td>
</tr>
<tr>
<td>4</td>
<td>15.23**</td>
<td>1.17</td>
</tr>
</tbody>
</table>

* Significant at 5% level; ** Significant at 1% level; *** Significant at 0.1% level

Table 3. Tukey's transformation after eliminating the outliers

<table>
<thead>
<tr>
<th>B</th>
<th>P</th>
<th>Y = X^P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 1</td>
<td>Data additive after exclusion of outliers</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.1594</td>
<td>-0.31</td>
</tr>
<tr>
<td>3</td>
<td>1.0335</td>
<td>0.0618</td>
</tr>
<tr>
<td>4</td>
<td>0.0166</td>
<td>0.1594</td>
</tr>
</tbody>
</table>
Fig.1. PLOT OF $w_i$ ON ROW MEANS WITH THE 2S LIMITS FOR SET I.
Fig. 2. PLOT OF $w_i$ ON ROW MEANS WITH THE 2S LIMITS FOR SET 2.
Fig. 3. PLOT OF $w_i$ ON ROW MEANS WITH 2 S LIMITS FOR SET 3.
Fig. 4: PLOT OF $w_i$ ON ROW MEANS WITH 2 S LIMITS FOR SET 4.
For set 3, the transformation obtained was $Y = x^{0.0618}$ and for set 4, $Y = x^{0.1594}$.

The data were analysed after carrying out these transformations. Tukey's test of additivity now showed, nonadditivity to be insignificant for all the sets (Table 4). The reduction by 4 in the lower d.f.

Table 4. Test for nonadditivity of the outlier-eliminated and transformed data

<table>
<thead>
<tr>
<th>Set 1</th>
<th>Not applicable as data is additive after exclusion of outliers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 2</td>
<td>2.55 Not significant</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Set 3</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Set 4</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

for set 2 is due to omission of two rows where one observation each was zero. Though $p$ was as small as 0.0618 for set 3, logarithmic transformation did not remove nonadditivity, $F$ for nonadditivity being 12.97**, which is highly significant for 1 and 57 degrees of freedom. Thus application of the power transformation suggested by Tukey's test to the data after eliminating
the outliers has been found to be effective in making the
data additive. In case where nonadditivity is not accounted
for by Tukey's transformation and outlier elimination by
graphical analysis or in other words where the concurrent
model does not describe the data, there are other methods
for testing the structure of interaction and testing the
main effects, for instance, methods mentioned by Marasinghe
and Johnson (1982) (for a multiplicative interaction
structure) and Krishnaiah and Yochmowitz (1980).

Daniel (1976) points out that nonadditivity is often
associated with a few rows or columns of the two-way table.
Snee (1982) states that nonadditivity in a two-way
classification with one observation per cell may be either
due to nonhomogeneous variance or interaction and the data
may not be sufficient to distinguish between these two.
However, ways and means for interpretation of the observed
nonadditivity has been discussed by this author. Federer
(1967) states that the sum of squares associated with
Tukey's one degree of freedom for nonadditivity gives the
linear row by linear column interaction. Nair (1982)
reported the dependence of standard error per unit on the
average catch. A look at the model considered in this
paper will show that when the availability of fish changes
over period of days, the $\alpha_i$'s may change, for different
periods causing this situation. (The dependence of variance on the mean also suggests nonnormality).

Apart from graphical procedure, much work has been done on the rejection of outliers. Rules for rejection has been discussed by Anscombe (1960), Anscombe and Tukey (1963) and Snedecor and Cochran (1968). Lately, Gaplin and Hawkins (1981) have presented bounds for the fractiles of maximum normed residuals (MNR). The present procedure is convenient to apply along with additivity test because the steps involved in testing provide the material for graphical analysis.

The present study shows that elimination of the outliers by graphical analysis and application of Tukey's test of additivity can be adopted to tackle the problem of nonadditivity in the analysis of catch data. Nair and Alagaraja (1982) suggested Wilcoxon matched-pairs signed-rank test as an appropriate procedure for comparing the efficiency of two fishing gear and illustrated with a set of data the superiority of this method over usual ANOVA. (Ordinary ANOVA was less sensitive in this case). The same set of data was analysed using the above procedure (that is outlier-elimination and application of Tukey's test of additivity and the consequent transformation as introduced and discussed in this chapter) and the same result as that given by
Wilcoxon test was obtained. This shows the usefulness of this combination of procedures in statistical comparison of the efficiency of fishing gear.