CHAPTER 3

A BAG MODEL STUDY OF CHARMED MESONS

3.1 Introduction

The discovery of the narrow resonances $J/\psi$ (3.1) in 1974 [6,7] was a remarkable event in high energy physics. This was followed by the observation of the other members of the "\(\psi\) family" in a relatively short period. These heavy hadrons with extremely narrow widths were interpreted as bound $c\bar{c}$ systems, $c$ being the "charmed" quark whose existence was predicted earlier on theoretical grounds [12] and $\bar{c}$ the corresponding antiquark. Their extreme narrowness or long lives implied highly suppressed decays to ordinary light hadrons with no charm content. However, direct experimental evidence for the existence of particles carrying "charm" was lacking until the charmed $D^0$ and $D^+$ mesons were discovered in 1976 by the SLAC-LEL group [16,17]. These are narrow resonances observed near 1.87 GeV in $e^+e^-$ annihilation experiments, coupled
predominantly to weak hadronic decay channels. Further, evidence for the existence of a meson having both charm and strangeness, called $F$, became available [97,98] hardly a year after the discovery of the D's.

With a view to exploring the phenomenological content of the MIT bag model in applications falling outside the low mass hadron world we propose to attempt a bag model study of the charmed mesons. The model in its spherical cavity approximation has been quite successful in predicting several of the static properties of light hadrons in reasonable agreement with experimental observations [63,64,99,100]. The model has also been applied with some success to the study of weak nonleptonic decays of baryons and mesons [101,102] and the radiative decays of some of the vector mesons [103], although the bag amplitudes for electromagnetic and weak leptonic decays [104] have not been in good agreement with experiment. In view of the relative simplicity of the ideas on which the bag model is based, its successes should be regarded as remarkable. Clearly the situation warrants efforts to extend the application of the model to the charm regime which serve two purposes:

(i) To enlarge the scope of the model by testing its validity beyond the three-flavour sector of hadron spectroscopy.

(ii) To understand the new hadrons in terms of this model.
We are thus motivated to carry out the work [105] presented in this chapter and the next. In the present chapter we are concerned with the masses of the charmed pseudoscalar mesons: \( D_0^0, D^+, \) and their vector counterparts: \( L^0, D^{*+}, F^{*+}. \) Our mass predictions are in substantial improvement over earlier estimates [106] and in good agreement with experimental values. The pseudoscalar vector mass splittings are, however, poorly predicted.

The material presented in this chapter is a revised and extended version of Ref.[105]. The present investigation differs from Ref.[105] in two respects:

(i) The study, which was restricted to \( D \) mesons only is now extended to cover the mass spectrum of the entire low lying charmed mesons.

(ii) The nonstrange quark mass which was taken to be zero in the earlier work is now given a non-zero value.

With the availability, after the publication of Ref.[105], of confirmed experimental results on \( F, D^* \) and \( F^* \) mesons, the extension of the study became necessary. Better overall agreement of the mass predictions with the observed masses has been obtained for the new choice of the nonstrange quark mass.
3.2 Charmed Mesons

The charmed D mesons were first observed by the SIA-C-LBL collaboration [16,17] in $e^+e^-$ annihilation at center-of-mass energies 3.9 to 4.6 GeV. Subsequently they were detected in neutrino [106-108], hadron [109] and photon [110-112]–induced reactions. The invariant mass spectra for the sum of all observed $D^0$ and $D^+$ decay modes show peaks at 1865 and at 1876 MeV respectively. The D's were produced primarily in association with the D*'s.

The $F^+$ and $F^{**}$ were discovered by the DASP collaboration [97] at DESY in $e^+e^-$ annihilation at c.m. energy 4.414 GeV. From events containing a charged pion, and $\eta$ and a low energy photon the $F$ and $F^*$ masses were found to be 2.03 ± 0.06 and 2.14 ± 0.06 GeV respectively. All of these observations were made at the peaks in the annihilation cross section: 3.772, 4.028, 4.16 and 4.414 GeV which are charmonium resonances above threshold [113]. Even before the discovery of the charmed hadrons, an elaborate SU(4) classification of charmed mesonic and baryonic states and their possible decay modes were worked out by Gaillard, Lee and Rosner [114] by extending the familiar notions of the colour triplet quark model to the four-quark scheme of Glashow, Iliopoulos and Maiani [12].
A charmed meson is composed of a charmed quark (c) and an ordinary light antiquark (\( \bar{q} \)) forming a spin singlet or a triplet. The \( D^+ \) and \( D^0 \) form an isospin doublet and the \( F \) meson an isospin singlet. They have \( J^P = 0^- \). The \( D^{*+}, D^{*0} \) and \( F^{*-} \) have \( J^P = 1^- \). The presence of a fourth quark \( c \) besides the familiar \( u, d, \) and \( s \) quarks implies that the SU(3) nonet of 8+1 mesons will be replaced by a hexadecimet of 15+1 states. The SU(4) multiplet 15 contains, in addition to the ordinary SU(3) resonances with \( c = 0 \), six states with open charm, \( c = \pm 1 \). Thus in the pseudoscalar case we have

\[
c = +1 : \quad D^+, D^0, F^+
\[
c = -1 : \quad D^-, D^0, F^-
\]

and in the vector case

\[
c = +1 : \quad D^{*+}, D^{*0}, F^{*+}
\]
\[
c = -1 : \quad D^{*-}, D^{*0}, F^{*-}
\]

States with hidden charm (\( c = 0 \)), namely the \( J/\psi \) and the \( \eta_c \) in the vector and pseudoscalar cases respectively are, however, excluded from the present investigation.

The masses and quantum numbers of the charmed mesons together with their quark contents are presented in Table 3.1.
### Table 3.1 The $0^-$ and $1^-$ charmed mesons

<table>
<thead>
<tr>
<th>Particle</th>
<th>Quark content</th>
<th>$J^P$</th>
<th>$I, I_3$</th>
<th>$S$</th>
<th>Mass (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^+$</td>
<td>$\bar{c}d$</td>
<td>$0^-$</td>
<td>$\frac{1}{2}, \frac{1}{2}$</td>
<td>0</td>
<td>1868.3 ± 0.9</td>
</tr>
<tr>
<td>$D^0$</td>
<td>$c\bar{u}$</td>
<td>$0^-$</td>
<td>$\frac{1}{2}, -\frac{1}{2}$</td>
<td>0</td>
<td>1863.1 ± 0.9</td>
</tr>
<tr>
<td>$F^+$</td>
<td>$c\bar{s}$</td>
<td>$0^-$</td>
<td>0, 0</td>
<td>+1</td>
<td>2039.5 ± 60</td>
</tr>
<tr>
<td>$D^{*+}$</td>
<td>$\bar{c}d$</td>
<td>$1^-$</td>
<td>$\frac{1}{2}, \frac{1}{2}$</td>
<td>0</td>
<td>2008.6 ± 1.5</td>
</tr>
<tr>
<td>$D^{*0}$</td>
<td>$c\bar{u}$</td>
<td>$1^-$</td>
<td>$\frac{1}{2}, -\frac{1}{2}$</td>
<td>0</td>
<td>2006.0 ± 1.0</td>
</tr>
<tr>
<td>$F^{*+}$</td>
<td>$\bar{c}s$</td>
<td>$1^-$</td>
<td>0, 0</td>
<td>+1</td>
<td>2140.0 ± 60</td>
</tr>
</tbody>
</table>
3.3 The Charmed Bag

Here we consider the parameters of the hadron bag containing a charmed quark $c$ and a nonstrange antiquark $\bar{u}$ or $\bar{d}$ forming the $D^0$ or $D^+$ meson. The bag is assumed to be a fixed sphere of radius $R$ (cavity approximation). The field equations and the bag boundary conditions determine the quark and antiquark wave functions [63,64]. For the lowest frequency mode we have

$$\Psi (r,t) = \frac{N}{\sqrt{4\pi}} \begin{pmatrix} i\alpha j_0 \left( \frac{2\pi}{R} \right) u \\ -i\beta j_1 \left( \frac{2\pi}{R} \right) \vec{\sigma} \cdot \hat{r} u \end{pmatrix} e^{-i\omega t/R} \quad (3.1)$$

for quarks, and

$$\varphi (r,t) = \frac{N}{\sqrt{4\pi}} \begin{pmatrix} -i\beta j_1 \left( \frac{2\pi}{R} \right) \vec{\sigma} \cdot \hat{r} u \\ \alpha j_0 \left( \frac{2\pi}{R} \right) u \end{pmatrix} e^{i\omega t/R} \quad (3.2)$$

for antiquarks, where

$$\alpha = [(\omega + m)/\omega]^{1/2}$$

$$\beta = [(\omega - m)/\omega]^{1/2}$$

The symbols have the same significance as explained in Ch.2.
The functions \( N \), \( x \) and \( \omega \) are given respectively by the equations (2.30), (2.32) and (2.33).

We choose all the bag parameters except the charmed quark mass \( m_c \) from Ref. 64. In that paper results are presented for two different nonstrange quark masses: \( m_n = 0 \), and \( m_n = 108 \) MeV. However, we set \( m_n = 110 \) MeV. The strange quark is given a different mass with a view to breaking the SU(3) degeneracy. In the literature one comes across a wide spectrum of values for the strange quark mass ranging from 100 MeV to 500 MeV or even more. Our choice of the strange quark mass is \( m_s = 300 \) MeV, which is roughly the same as the MIT value. The bag pressure parameter \( B \) which determines the stability of the bag has been found to give best fit to the hadronic masses [64] for the choice \( 1/4 \) = 145 MeV. Following Gaillard, Lee and Rosner [114], and Donoghue and Golowich [116] we fix the charmed quark mass \( m_c = 1.5 \) GeV. The bag size is determined by minimizing the hadron mass with respect to the bag radius \( R \), a procedure demanded by the quadratic bag boundary condition.

3.4 Charmed Meson Masses

The mass of a hadron of bag radius \( R \) is given by the equation

\[
H(R) = E_v + E_q + E_0 + E_m + E_e
\]  

(3.3)
The various phenomenological contributions to $H(R)$ are the ones discussed in Ch.2 [Eqs. (2.35) - (2.37) and (2.43)]. The major contribution comes from the relativistic motion of the quarks in the cavity. This may be explicitly written as

$$E_q = N_n \omega (m_n R) + N_s \omega (m_s R) + N_c \omega (m_c R)$$  \hspace{1cm} (3.4)

where $N_n$, $N_s$ and $N_c$ are respectively the number of nonstrange, strange and charmed quarks/antiquarks contained in the hadronic bag in question. $\omega$ is the frequency defined by Eq. (2.33).

$E_m$ and $E_e$ in Eq. (3.3), are the 'magnetic' and 'electric' quark-gluon interaction energies. The magnetic spin-spin interaction energy [See Eq. (2.43)] is given by

$$E_m = -\frac{3\alpha_e}{36R} \sum_a \sum_i j (\vec{\sigma}_i \lambda_a^i \cdot (\vec{\sigma}_j \lambda_a^j))$$

$$\times \mu'(m_i R) \mu'(m_j R) \cdot I(m_i R, m_j R)$$  \hspace{1cm} (3.5)

where

$$\mu'(mR) = \frac{4\omega R + 2mR - 3}{2\omega R(\omega R - 1) + mR}$$  \hspace{1cm} (3.6)

and $I(m_i R, m_j R)$ is a slowly varying function of $m_i R$ and $m_j R$ given by Eq. (2.44). The Casimir invariants $\lambda_i^a \lambda_j^a$ of the colour $SU(3)$ for the colour triplet quarks $q_i$ and colour antitriplet
quarks $\bar{q}_1$ forming colour singlet hadrons are given by

$$\sum_\alpha \lambda_i^\alpha \lambda_j^\alpha (i \neq j) = -16/3 \text{ for } q\bar{q} \text{ mesons}$$  \hspace{1cm} (3.7)

$$\sum_\alpha \lambda_i^\alpha \lambda_j^\alpha (i \neq j) = -8/3 \text{ for } qqq \text{ baryons}$$  \hspace{1cm} (3.8)

Thus we have

$$D_m = \sum_{i>j} a_{ij} N_{ij}$$  \hspace{1cm} (3.9)

where $\lambda = 2$ for mesons and 1 for baryons. The coefficients $a_{ij}$ are determined by the spin vectors $\vec{\sigma}_i$ and $\vec{\sigma}_j$ of the quarks $i$ and $j$:

$$a_{ij} = \vec{\sigma}_i \cdot \vec{\sigma}_j$$  \hspace{1cm} (3.10)

and

$$M_{ij} = \frac{8\alpha_c}{36R} \mu'(m_iR)\mu'(m_jR)I(m_iR,m_jR)$$  \hspace{1cm} (3.11)

$\alpha_c$ is the colour coupling constant of quarks. For $\alpha_c$, we choose the phenomenological value obtained by De Grand et al. [64], namely

$$\alpha_c = 0.55$$

The spin factors $a_{ij}$ (i, j-flavour indices) for the cases of interest, namely, for the charmed pseudoscalar states with flavour-spin content: $(c \bar{q}_{\psi})$, and the charmed vector states
with flavour-spin content: \((c \bar{q} \bar{q})\), are evaluated by noting that here we have two spin-\(\frac{1}{2}\) particles with their spins anti-parallel in one case and parallel in the other. Considering, for example, the case of the \(D\) meson we have the quark \(c\) and the anti-quark \(\bar{u}\) or \(\bar{d}\) in a spin-0 state. Now the spin vectors

\[
\vec{S}_1 = \frac{1}{2}\vec{\sigma}_1 \quad \text{and} \quad \vec{S}_2 = \frac{1}{2}\vec{\sigma}_2
\]

(3.12)

add up to give

\[
\vec{S}_1 + \vec{S}_2 = 0,
\]

(3.13)

so that from the identity

\[
(\vec{S}_1 + \vec{S}_2)^2 = (\vec{S}_1)^2 + (\vec{S}_2)^2 + 2\vec{S}_1 \cdot \vec{S}_2
\]

(3.14)

it follows that

\[
\vec{S}_1 \cdot \vec{S}_2 = -3
\]

(3.15)

The values of \(a_{ij}\) for the various states with specific non-strange (n), strange (s) and charm (c) contents are given in Table 3.2.

Knowledge of the bag radius \(R\) is now required for estimating the various contributions to the hadronic mass \(M\). This is accomplished by following the standard procedure [64,117] of minimising \(M\) with respect to \(R\) in the zero quark mass limit. In this limit \(M\) can be expressed as an explicit function
Table 3.2 Values of $a_{ij}$

<table>
<thead>
<tr>
<th></th>
<th>$D^+$</th>
<th>$D^0$</th>
<th>$F^+$</th>
<th>$D^*$</th>
<th>$L^*$</th>
<th>$L^**$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{cn}$</td>
<td>-3</td>
<td>-3</td>
<td>0</td>
<td>+1</td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>$a_{cs}$</td>
<td>0</td>
<td>0</td>
<td>-3</td>
<td>0</td>
<td>0</td>
<td>+1</td>
</tr>
</tbody>
</table>

---
of $R$ and the bag parameters:

$$M_k(R) = \frac{4}{3} \pi R^2 B + (N x(0) - Z + a_k M_{oo})/R \quad (3.16)$$

in which

$N$ = Total number of quarks and anti-quarks in the hadron bag,

$x(0) = x(n)$ in the limit $m \to 0$,

$M_{oo} = M_{ij}$ with $m = 0$

$$= \frac{8e}{36\pi} [\mu'(0)]^2 I(0,0), \quad (3.17)$$

$$a_k = \sum_{ij} \lambda(a_{ij})_k , \quad (3.18)$$

where the index $k$ designates the hadron under consideration.

Evidently $a_k$ has the same value for all the $0^-$ states and a different value for the $1^-$ states. All other factors on the r.h.s. of Eq.(3.16) being identical for the entire class of charmed mesons considered, it follows that we have different bag radii for pseudoscalar and vector multiplets, while the individual members of either multiplet have the same size. Requiring

$$\frac{\partial M}{\partial R} = 0 \quad (3.19)$$

we get

$$R_k = \sqrt[4]{\frac{H x(0) - Z + a_k M_{oo}}{4\pi B}} \quad (3.20)$$
The functions $x(m_R)$ and $I(m_i R, m_j R)$ are readily evaluated in the limit of vanishing quark mass. They have the numerical values

\[ x(0) = 2.04 \]
\[ I(0,0) = 1.44 \]  \hspace{1cm} (3.21)

Using these and the parameter values

\[ Z = 1.84 \]
\[ B^{1/4} = 0.145 \text{ GeV} \]  \hspace{1cm} (3.22)

we get for the bag radii

\[ R_0 = 3.3 \text{ GeV}^{-1} \text{ for the } 0^- \text{ mesons,} \]

and \[ R_1 = 4.72 \text{ GeV}^{-1} \text{ for the } 1^- \text{ mesons}. \]

Knowing $R$, the functions $I(m_i R, m_j R)$ and hence $M_{ij}$ are determined for all relevant sets of $m_i R$ and $m_j R$ values. These are listed in Tables 3.3 and 3.4.

Finally we come to the colour 'electric' interactions between quarks in the bag. With quarks of the same kind as in $J/\psi$ this contribution is zero. It was for this reason that this was not taken into account in computing the bag size which was done in the zero quark mass limit. For quarks of different masses the interaction energy has been determined [64] to be

\[ E_c = \frac{8\alpha_s}{2R} \left( \lambda \sum_{i>j} f(x_i, x_j) - \sum_i f(x_i, x_i) \right) \]  \hspace{1cm} (3.23)
Table 3.3 Values of $I(m_i R, m_j R)$ and $M_{ij}$, for $m_n = 0$, $m_s = 0.3$ GeV, $m_c = 1.5$ GeV.

<table>
<thead>
<tr>
<th>$R$ (GeV$^{-1}$)</th>
<th>$m_i R$</th>
<th>$m_j R$</th>
<th>$I(m_i R, m_j R)$</th>
<th>$M_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.441</td>
<td>0.0788</td>
</tr>
<tr>
<td>0.99</td>
<td>0</td>
<td>0</td>
<td>1.490</td>
<td>0.0679</td>
</tr>
<tr>
<td>3.3</td>
<td>4.95</td>
<td>0</td>
<td>1.593</td>
<td>0.0362</td>
</tr>
<tr>
<td></td>
<td>4.95</td>
<td>0.59</td>
<td>1.669</td>
<td>0.0313</td>
</tr>
</tbody>
</table>

Table 3.4 Values of $I(m_i R, m_j R)$ and $M_{ij}$, for $m_n = 0.11$ GeV, $m_s = 0.3$ GeV, $m_c = 1.5$ GeV.

<table>
<thead>
<tr>
<th>$R$ (GeV$^{-1}$)</th>
<th>$m_i R$</th>
<th>$m_j R$</th>
<th>$I(m_i R, m_j R)$</th>
<th>$M_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3</td>
<td>4.95</td>
<td>0.363</td>
<td>1.6535</td>
<td>0.0353</td>
</tr>
<tr>
<td>3.3</td>
<td>4.95</td>
<td>0.990</td>
<td>1.6690</td>
<td>0.0317</td>
</tr>
<tr>
<td>4.72</td>
<td>7.08</td>
<td>0.519</td>
<td>1.6634</td>
<td>0.0182</td>
</tr>
<tr>
<td>4.72</td>
<td>7.08</td>
<td>1.416</td>
<td>1.7303</td>
<td>0.0160</td>
</tr>
</tbody>
</table>
where \( \lambda = 2 \) for mesons and 1 for baryons, and

\[
\bar{r}(x_i, x_j) = \int_0^{T} \frac{dr}{r^2} P_i(r) P_j(r)
\]

(3.24)

\( P_i(r) \) being the fraction of the quark charge density within a radius \( r \) and is a function of \( m_q, \omega, x_i \) and \( r \).

\[
P_i(r) = \frac{\omega (x_i r - \sin^2 x_i r / x_i r) - m (\sin x_i r \cos x_i r - \sin^2 x_i r / x_i r)}{\omega (x_i - \sin^2 x_i / x_i) - m (\sin x_i \cos x_i - \sin^2 x_i / x_i)}
\]

(3.25)

The functions \( f(x_i, x_j) \) have been evaluated numerically on a computer. These are displayed in Tables 3.5 and 3.6.

Putting together various contributions to the bag mass (Eq. 3.3), the masses of the charmed mesons are obtained. Our results are presented in Table 3.7. Experimental masses are given alongside for comparison. (Predicted masses are in the 8th column).

3.5 Discussion

Ref. [105], using the nonstrange quark mass \( m_n = 0 \), gave a somewhat good prediction for the \( D \) meson mass. \( M_D \) was found to be 1.805 GeV with the inclusion of a colour electric contribution of 165 MeV. This may be compared with an earlier
Table 3.5 Values of $f(x_i, x_j)$ for charmed pseudoscalar mesons. Parameters: $R = 3.3$ GeV$^{-1}$, $m_n = 0.110$ GeV, $m_s = 0.3$ GeV, $m_c = 1.5$ GeV.

<table>
<thead>
<tr>
<th>$m_i$</th>
<th>$\omega_i$</th>
<th>$x_i$</th>
<th>$m_j$</th>
<th>$\omega_j$</th>
<th>$x_j$</th>
<th>$f(x_i, x_j)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.110</td>
<td>0.675</td>
<td>2.198</td>
<td>0.110</td>
<td>0.675</td>
<td>2.198</td>
<td>9.6426</td>
</tr>
<tr>
<td>0.110</td>
<td>0.675</td>
<td>2.198</td>
<td>1.500</td>
<td>1.731</td>
<td>2.850</td>
<td>8.8012</td>
</tr>
<tr>
<td>0.300</td>
<td>0.784</td>
<td>2.390</td>
<td>0.300</td>
<td>0.784</td>
<td>2.390</td>
<td>8.1865</td>
</tr>
<tr>
<td>0.300</td>
<td>0.784</td>
<td>2.390</td>
<td>1.500</td>
<td>1.731</td>
<td>2.850</td>
<td>8.1918</td>
</tr>
<tr>
<td>1.500</td>
<td>1.731</td>
<td>2.850</td>
<td>1.500</td>
<td>1.731</td>
<td>2.850</td>
<td>8.3661</td>
</tr>
</tbody>
</table>
Table 3.6  Values of $f(x_i,x_j)$ for charmed vector mesons.

Parameters: $R = 4.72 \text{ GeV}^{-1}$, $m_n = 0.110 \text{ GeV}$,
$m_s = 0.3 \text{ GeV}$, $m_c = 1.5 \text{ GeV}$.

<table>
<thead>
<tr>
<th>$m_i$</th>
<th>$\omega_i$</th>
<th>$x_i$</th>
<th>$m_j$</th>
<th>$\omega_j$</th>
<th>$x_j$</th>
<th>$f(x_i,x_j)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.110</td>
<td>0.490</td>
<td>2.255</td>
<td>0.110</td>
<td>0.490</td>
<td>2.255</td>
<td>20.2517</td>
</tr>
<tr>
<td>0.110</td>
<td>0.490</td>
<td>2.255</td>
<td>1.500</td>
<td>1.623</td>
<td>2.930</td>
<td>18.9929</td>
</tr>
<tr>
<td>0.300</td>
<td>0.607</td>
<td>2.490</td>
<td>0.300</td>
<td>0.607</td>
<td>2.490</td>
<td>17.1517</td>
</tr>
<tr>
<td>0.300</td>
<td>0.607</td>
<td>2.490</td>
<td>1.500</td>
<td>1.623</td>
<td>2.930</td>
<td>17.6211</td>
</tr>
<tr>
<td>1.500</td>
<td>1.623</td>
<td>2.930</td>
<td>1.500</td>
<td>1.623</td>
<td>2.930</td>
<td>18.3403</td>
</tr>
</tbody>
</table>
Table 3.7  Masses of charmed mesons in the MIT bag model.

Parameters: $m_n = 110 \text{ MeV, } m_s = 300 \text{ MeV, } m_c = 1500 \text{ MeV, } B^{' \dagger} = 145 \text{ MeV, } Z = 1.84, \alpha_c = 0.55$ (All masses are in MeV).

<table>
<thead>
<tr>
<th>Particle</th>
<th>$M_{\text{Expt.}}$</th>
<th>$E_q$</th>
<th>$E_v$</th>
<th>$E_o$</th>
<th>$E_m$</th>
<th>$E_e$</th>
<th>$M_{\text{Theor.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D$^+$</td>
<td>1868</td>
<td>2406</td>
<td>67</td>
<td>-557</td>
<td>-212</td>
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<td>D$^0$</td>
<td>1863</td>
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<tr>
<td>F$^+$</td>
<td>2039±60</td>
<td>2515</td>
<td>67</td>
<td>-557</td>
<td>-190</td>
<td>75</td>
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<td>D$^{*+}$</td>
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<td>-390</td>
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<td>2142</td>
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<td>F$^{*+}$</td>
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<td>2230</td>
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<td>-390</td>
<td>32</td>
<td>78</td>
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bag model estimate of D meson mass by Szwed [118] employing the bag model with surface tension [68] which yielded a value as low as 1.564 GeV. However, with $m_n = 0$, the bag model predictions for the other charmed mesons turned out to be very bad. The new choice of the non-strange quark mass is consistent with the second set of parameters used by De Grand et al. in their pioneering work on the phenomenological bag model [64].

The colour electric interaction is found to yield a significant contribution to the charmed meson mass. It is to be noted that this part of the hadron mass is usually neglected in bag model calculations of masses of ordinary (light) hadrons for which this turns out to be negligibly small ($\lesssim 5$ MeV) as a result of the nearly equal quark masses. For charmed mesons the quark masses are substantially different from each other: $m_n \sim 0$ or 0.1 GeV, while $m_c \sim 1.5$ GeV, and it was conjectured in Ref.[64] that the electric contribution in such cases might be appreciable. Our computation has confirmed this speculation. That this contribution to $M$ is quite sensitive to the difference in the masses of the constituent quarks is evident from the fact that magnitudes of $E_e$ in the case of mesons with nonstrange-charm content are considerably larger than those in the case of mesons with strange-charm quark content (See Table 3.7).

The vector states are split from the pseudo scalar states by the magnetic coupling of gluons to quarks which
contributes a spin-spin interaction to the bag energy operator. The colour magnetic interaction has the effect of reducing the energy content of a pseudoscalar bag while lifting up the energy of a vector bag. Thus the $D - D^*$ and $F - F^*$ mass splittings are in the right direction as observed experimentally. However, the quantitative agreement between the model predictions and the experimental values for the above mass splittings is not quite good.

The agreement between model prediction and experimental result is excellent in the case of $D$ and $F^*$ masses while there is discrepancy of about 6-7% in the case of $D^*$ and $F$ masses. The overall agreement between theory and experiment can be considered as good regarding the meson masses, while it is not so with the mass splittings.

The reason for this failure is not quite obvious. The nonstrange and charmed quarks have a wide mass separation. They may have to be treated differently as their velocities in the spherical cavity too may differ widely. The heavy quark may not be a truly relativistic object.