APPENDIX I

NOTATION AND CONVENTIONS

Natural units are used throughout so that $c = \hbar = 1$.

The space-time coordinates are denoted by the 4-vector:

$$x^\mu \equiv (x^0, x^1, x^2, x^3) = (t, x, y, z) = (t, \vec{x})$$

Also

$$x_\mu \equiv (x_0, x_1, x_2, x_3) = (t, -x, -y, -z) = \varepsilon_{\mu\nu} x^\nu$$

where

$$\varepsilon_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Summation over repeated indices is implied unless otherwise specified. The inner product is

$$x^2 = x_\mu x^\mu = t^2 - \vec{x}^2$$

Momenta are defined

$$p^\mu = (E, p_x, p_y, p_z)$$

with the inner product

$$p_1 \cdot p_2 = p_1^\mu p_2_\mu = E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2$$

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Also \( x \cdot p = t E - \vec{x} \cdot \vec{p} \)

The momentum operator in the coordinate representation is

\[ p^\mu = i \frac{\partial}{\partial x_\mu} = i \partial_\mu = (i \partial_t, -i \nabla) \]

with \( p^\mu p_\mu = -\partial_\mu p^\mu = -\Box \)

Dirac matrices:

The following representation is chosen for the Dirac \( \gamma \)-matrices.

\[
\gamma^0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \gamma^k = \begin{bmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{bmatrix}, \quad k = 1, 2, 3
\]

where

\[
\begin{align*}
\sigma^1 &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \\
\sigma^2 &= \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \\
\sigma^3 &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}
\end{align*}
\]

are the 2x2 Pauli matrices, and \( 1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \) is the 2x2 unit matrix. One of the useful combinations is

\[
\gamma^5 = -i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = -\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}
\]

Also, we have

\[
(\gamma^0)^+ = \gamma^0, \quad (\gamma^k)^+ = -\gamma^k, \quad \gamma^\mu \gamma^5 = -\gamma^5 \gamma^\mu.
\]
APPENDIX II

WEAK INTERACTION HAMILTONIAN

In the GIM scheme the weak hadron current is given by

\[ j^h_\mu = \bar{q} C_H \gamma_\mu (1 + \gamma_5)q \]

where \( q = \begin{pmatrix} c \\ u \\ d \\ s \end{pmatrix} \)

\[ C_H = \begin{pmatrix} 0 & 0 & U \\ 0 & 0 & 0 \\ -U & -U & -I \end{pmatrix} \]

\[ U = \begin{pmatrix} -\sin \theta_c & \cos \theta_c \\ \cos \theta_c & \sin \theta_c \end{pmatrix} \]

\( \theta_c \) being the Cabibbo angle.

Thus

\[ j^h_\mu = -\sin \theta_c \bar{\delta} \gamma_\mu (1 + \gamma_5)d + \cos \theta_c \bar{\nu} \gamma_\mu (1 + \gamma_5)d \]

\[ + \cos \theta_c \bar{\delta} \gamma_\mu (1 + \gamma_5)s + \sin \theta_c \bar{\nu} \gamma_\mu (1 + \gamma_5)s \]

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\[ (J^h_\mu)^+ = -\sin \theta_c \, d \, \gamma_\mu (1 + \gamma_5)c + \cos \theta_c \, \bar{d} \, \gamma_\mu (1 + \gamma_5)u \\
+ \cos \theta_c \, \bar{s} \, \gamma_\mu (1 + \gamma_5)c + \sin \theta_c \, \bar{s} \, \gamma_\mu (1 + \gamma_5)u \]

In the current x current form, the Hamiltonian is given by

\[ H = \frac{G}{\sqrt{2}} (J^h_\mu)^+ (J^h_\mu) \]

i.e.,

\[ H = \frac{G}{\sqrt{2}} [\sin^2 \theta_c \left\{ \bar{d} \, \gamma_\mu (1 + \gamma_5)c \, \bar{c} \, \gamma_\mu (1 + \gamma_5)d - \bar{d} \gamma_\mu (1 + \gamma_5)c \, \bar{u} \, \gamma_\mu (1 + \gamma_5)s - \bar{s} \, \gamma_\mu (1 + \gamma_5)u \, \bar{c} \, \gamma_\mu (1 + \gamma_5)d + \bar{s} \, \gamma_\mu (1 + \gamma_5)u \, \bar{u} \, \gamma_\mu (1 + \gamma_5)s \right\} \]

\[ + \cos^2 \theta_c \left\{ \bar{d} \, \gamma_\mu (1 + \gamma_5)u \, \bar{u} \, \gamma_\mu (1 + \gamma_5)d + \bar{d} \, \gamma_\mu (1 + \gamma_5)u \, \bar{c} \, \gamma_\mu (1 + \gamma_5)s + \bar{s} \, \gamma_\mu (1 + \gamma_5)c \, \bar{u} \, \gamma_\mu (1 + \gamma_5)d + \bar{s} \, \gamma_\mu (1 + \gamma_5)c \, \bar{c} \, \gamma_\mu (1 + \gamma_5)s \right\} \]

\[ + \sin \theta_c \cos \theta_c \left\{ \bar{d} \, \gamma_\mu (1 + \gamma_5)c \, \bar{u} \, \gamma_\mu (1 + \gamma_5)d - \bar{d} \, \gamma_\mu (1 + \gamma_5)c \, \bar{c} \, \gamma_\mu (1 + \gamma_5)s - \bar{d} \, \gamma_\mu (1 + \gamma_5)u \, \bar{c} \, \gamma_\mu (1 + \gamma_5)d + \bar{d} \, \gamma_\mu (1 + \gamma_5)u \, \bar{u} \, \gamma_\mu (1 + \gamma_5)s \right\} \]
We are interested in the charm-changing and strangeness-changing parts of \( H \) only. Consequently terms involving \( cc \) may be left out. Also terms which do not contain \( c \) or \( \bar{c} \) may be left out for the same reason. We are thus left with 8 terms. They correspond to \( \Delta c = \pm 1 \), and \( \Delta s = \pm 1, 0 \).

From these we pick out terms satisfying particular selection rules:

1. \( \Delta c = -1 \) and \( \Delta s = -1 \),
   \[ \Delta c = +1 \) and \( \Delta s = +1 \),

so that \( \Delta s / \Delta c = +1 \)

\[
H_W = \frac{g}{\sqrt{2}} \cos^2 \Theta \left\{ \bar{d} \gamma_\mu (1 + \gamma_5) u \bar{c} \gamma_\mu (1 + \gamma_5) s + \bar{s} \gamma_\mu (1 + \gamma_5) c \bar{u} \gamma_\mu (1 + \gamma_5) d \right\}
\]
2. \( \Delta c = -1 \), and \( \Delta s = +1 \),

\( \Delta c = +1 \) and \( \Delta s = -1 \),

so that \( \Delta s / \Delta c = -1 \).

\[ \Delta s / \Delta c = -1 \]

\[ H_W = \frac{G}{\sqrt{2}} (-\sin^2 \theta) \left\{ \bar{d} \gamma_\mu (1 + \gamma_5) c \bar{u} \gamma_\mu (1 + \gamma_5) \right\} \]

\[ + \bar{s} \gamma_\mu (1 + \gamma_5) u \bar{c} \gamma_\mu (1 + \gamma_5) d \]