CHAPTER - FIVE

Development of an RMS Detector

5.1) INTRODUCTION

In this chapter the details of the developed rms detector are presented. The detector has been designed for the rms measurement on low and medium frequency, high crest factor signals. The direct rms computation technique is used. A squarer, a squarerooter and an averager are designed, fabricated and tested. The squarer design is carried out specially for high crest-factor signals, however it is useful in other rms measurements also. A systematic analysis of the squarerooter is carried out and three adjustments are introduced to compensate for the squarerooter errors. These adjustments have helped in extending the performance of the squarerooter over a wider dynamic range.

5.2) SQUARER

Among the various squaring techniques, the function generator is chosen because it provides flexibility in shaping the error characteristic.

5.2.1) THE ABSOLUTE - VALUE DETECTOR

The well known absolute-value circuit as shown in (Fig. 5.1) was used along with the function generator to reduce the complexity. The absolute-value circuit needed three adjustments. (1) Zero Adjustment - by $R_x$ (Fig. 5.1) (2) Two adjustments were introduced to make the circuit perform identically on both positive and negative signals; low-level performance was adjusted by $R_1$, and high-level performance was adjusted by $R_3$.

For stable operation of the absolute-value circuit as well as other blocks, using operational amplifier, it is essential that the amplifier should have
sufficiently low noise and drift. One method of compensating for the difference in temperature coefficients of the input bias currents and thereby controlling the resulting drift was worked out by the Author while in Russia (Ref. 1).

The DC tests on the fabricated absolute-value circuit showed that the circuit could be trimmed to have a dynamic range of 3 decades (4mV to 4V) with a percentage error of ± 0.3 %. The frequency tests showed that the circuit had a flat response up to 10 kHz.

5.2.2) THE SQUARE -- FUNCTION GENERATION

The function generator approximates the curve $y = x^2$, (curve 3, Fig. 5.2), by a series of straight lines which must lie in the region bounded by the two curves

$$y_1 = x^2 + 2 \cdot C \cdot x$$
$$y_2 = x^2 - 2 \cdot C \cdot x$$

curves 2 and 4 in Fig. 5.2)

'\(C\)' is the maximum permissible absolute error in rms computation corresponding to the approximation error.

Different constraints, like the maximum permissible error of approximation and least mean square error, appear (Ref. 2) in the function generator designs. In the present case however, the constraint is unique; the minimum number of segments for a given squarer-error characteristic, or the peak segment error proportional to the input level.

It is a common practice (Ref. 2) to use graphical methods for approximating the well-defined functions by a set of straight lines. On the other hand, the analytical methods provide more design flexibilities. A good compromise is made between the two. The straight line giving the maximum segment length is chosen graphically; whereas its constants are determined analytically. The design is first carried out for a convenient input span of 0 to 10 units with a corresponding output span of 0 to 100 units. Suitable scale factors can then be introduced to use the design for the actual input and the corresponding output spans.
5.2.3) THE DESIGN OF THE \( i \)th SEGMENT

It is obvious that the straight line, with its end points on one boundary curve and just touching the other boundary curve (as shown in Fig. 5.2, curve 1) is the one with maximum segment length. Assuming the starting point \( a \) to be known the straight line \( ab \) can be completely determined (Appendix VI). The abscissa \( x_{1i} \) of the tangent point is given by

\[
x_{1i} = x_{i-1} + 2 \sqrt{C} \cdot x_{i-1}
\]

The slope \( a_i = 2x_{1i} + 2C \)

The segment length \( L = 4 \sqrt{(Cx_{1i} + C^2)} \)

The abscissa \( x_i \) of the end point of the \( i \)th segment

\[
x_i = x_{i-1} + L
\]

Thus the \( i \)th segment is completely designed to have (i) maximum segment length and an error characteristic. Such that the resulting peak rms measurement error is exactly \( C \) units (the derivations of the formulae are given in the appendix VI).

This design procedure is followed for all the segments except the first one. The first segment needs a special consideration. Firstly, the squarer error must be zero at zero input, Secondly the boundary curves should be modified as per the expected squarer threshold error \( C_0 \) caused by the noise and drift.

5.2.4) THE DESIGN OF THE FIRST SEGMENT

The boundary curves are (Curves 2 & 4 in Fig. 5.3)

\[
y_1 = x^2 - C_0
\]

and

\[
y_2 = x^2 + C_0
\]
where \( C_\alpha \) = constant threshold error. The approximating straight line, \( ob' \), is fully defined by (Please see the appendix VII)

1. the slope \( a_1 = 2x_{11} \).

2. the abscissa of the tangent point \( x_{11} = C_\alpha \)

and the abscissa of the end point \( x_1 = (1 + \sqrt{2}) C_\alpha \)

5.2.5) THE RELATIONSHIP BETWEEN THE NUMBER OF SEGMENTS (\( n \)) AND THE PERCENTAGE RMS MEASUREMENT ERROR (\( \varepsilon_r \)).

In case of a linearly segmented approximator for a square function, the relationship between (\( n \)) and the percentage squarer error (\( \varepsilon_s \)) is well-known. (Ref. 3). In the case of a nonlinearly segmented design the relationship is not that simple. The Author has worked out the relationship (Ref. 4) in the tabular form (table T. 5.1). A flow-chart used in determination of this table is given in the appendix VIII. A squarer design with 0.3\% rms measurement error was selected for the 0.5\% rms measurement.

5.2.6) THE CIRCUIT IMPLEMENTATION

In the actual circuit the input to the squarer (\( V_x \)) ranged from 0 to 8 volts, limited by the saturation characteristics of the absolute-value circuit used. The output of the squarer (\( V_y \)) was limited to 6 volts by the maximum input characteristics of the averager block used. The following scale factors were introduced:

\[
K_x = \frac{V_x}{x} = 0.8
\]

\[
K_y = \frac{V_y}{y} = 0.06
\]

\[
K_a = \frac{a_{wi}}{a_i} = 0.075
\]
The resulting modified version of the design was then as given below:

1) Break Point of the i'th segment

\[ V_{Bi-1} = 0.8 x_{i-1} \]

2) Slope of the i'th segment

\[ a_{vi} = 0.075 a_i \]

3) The length of the first segment

\[ V_{Bi} = 0.8 x_1 \]

4) The slope of the first segment

\[ a_{v1} = 0.075 a_1 \]

The function generator was implemented by using precision feedback limiters (9 Nos.) and an adder as shown in figures (Fig. 5.4 & Fig. 5.5). In this design, the precision feedback limiters PFL (1) to PFL (i) were in conducting state when \( V_x \) satisfied the condition, \( V_{Bi-1} \leq V_x \leq V_{Bi} \); whereas PFL (i + 1) to PFL (n) were in the cutoff state. Therefore, the slope \( S_i \) of the i'th PFL is given by: \( S_i = (a_{vi} - a_{vi-1}) \). The table (T. 5.2) gives the complete design values of all the components.

5.2.7) THE PERFORMANCE OF THE SQUARER

The fabricated function generator had 18 adjustments. The use of stabilized power supply to control the break points resulted in successful setting up of the squarer.

The function generator was subjected to various tests. The results of d.c. tests showed that the actual nature of the squarer error was very much similar to the designed one (Curves 1 and 2 respectively, in the figure Fig. 5.6.) The square function generator had a dynamic range of 0.1 volt to 8.4 volts (d.c.) i.e about three decades with an rms measurement error of not more than 0.4 %. The frequency tests showed that the frequency error was less than 5 % upto 10 kHz and 3 dB point was at 50 kHz. The pulse - input test, was carried out at repetition period of 20ms. The duty cycle was changed in steps and the filtered output of the squarer was found to be directly proportioned to the duty cycle (D) over the range of D equal to 0.25 % to 50 %.
This test proved that the filtered output of the squarer was directly proportional to the mean square value irrespective of the duty cycle.

It must be noted that 0.25 % duty cycle is equivalent to the crest factor of 20 thus the pulse test results proved that the squarer permitted the RMS measurement of signals with crest factors as high as 20.

5.3) THE AVERAGER

Two identical second order Butter Worth’s gain low pass filters (Fig. 5.7) were cascaded. The cut off frequency was selected as 4 Hz.

The averager was subjected to a frequency test, by applying 6 V (peak) sinusoidal signal. The frequency was varied from 1 Hz to 100 Hz. The ripple was less than a few millivolts for signals of 10 Hz and of higher frequency. The response time of the averager was about 2 seconds on 5 V step signal.

5.4 THE SQUARE-ROOTER

It performs the two functions; square-rooting and AD conversion. The circuit is shown in the figure (Fig. 5.8) and the working of the circuit is presented by its time diagram in the figure (Fig. 5.9).

5.4.1) THE SQUARE-ROOTER OPERATION

The basic equation is

\[ V_{in} = V_2 \text{ at } t = t_p \]

therefore,

\[ t_p = \left( \frac{2 \tau_1 \cdot \tau_2}{V_R} \cdot V_{in} \right)^{\frac{1}{2}} \]

Where (i) \( V_R \) is the known constant voltage; \( \tau_1, \tau_2 \) are the integration time constants of the two integrators.

thus \[ t_p = K \sqrt{V_{in}} \]
where \[ K = \left( \frac{2 \tau_1 \tau_2}{V_R} \right)^{0.5} \]

The circuit was designed for 10 ms output pulse width for \( V_{in} = 3 \) volts.

In practice, the output pulse width will deviate from the ideal or theoretical value because of various imperfections in the actual working of the three blocks; two integrators and a comparator. Certain adjustments are therefore required to get the desired performance.

5.4.2) THE ADJUSTMENTS ON SQUARE - ROOTER

The square-rooter is a nonlinear device consisting of two integrators and a comparator. The main sources of errors in an integrator are (Ref. 5) (1) Finite open-loop gain \( A_0 \), (2) the d.c offset \( V_{os} \) and the bias current, and (3) the limited band width of the operational amplifier.

The expressions given by Tobey & others (Ref. 5) are reproduced below:

1) Error caused by the open-loop gain \( A_0 \) in case of a step input \( E/s \):

\[
\Delta e_o = A_0 E \left( 1 - e^{-t / A_0 RC} \right) - \frac{Et}{RC}
\]

with \[ \frac{t}{A_0 RC} < < 1 \]

\[
\Delta e_o = \frac{Et^2}{2A_0 R^2 C^2}
\]

2) Error caused by d.c. offset \( V_{os} \) and the bias current \( I_B \) :

\[
\Delta e_o = \frac{1}{RC} \int V_{os} \, dt + \frac{1}{C} \int I_B \, dt + V_{os}.
\]
3) Error caused by the finite bandwidth in case of a step input $E/s$:

$$\triangle e_o = E \left( \frac{\tau_o}{RC} + e^{-t/\tau_o} \cdot \frac{\tau_o}{RC} \right)$$

Where the open-loop frequency response is approximated by a single pole at $1/\tau_o$.

Thus, in general one can group the integrator errors in three groups:
(1) Constant errors. (2) errors proportional to the first power of $t$ and (3) errors proportional to the second power of $t$.

In the square-root circuit the net error at the output of the second integrator is due to the second integrators error, and the integrated error of the first integrator. The error in $V_2$ can be considered to have four complements; (1) constant error (2) error component proportional to $t$, (3) proportional to $t^2$ and (4) proportional to $t^3$.

The zero adjustment as shown in the figure (Fig. 5.10) compensates for the constant error component, the fullscale adjustment compensates for the error component of $V_2$ proportional to $t^2$ (Fig. 5.10) and the half scale adjustment (at 50% of rated rms input) as shown in the figure Fig. 5.10 for the error component proportional to the first power of time. It should be noted that the perfect compensation is difficult in practice because these error components are not entirely independent.

5.4.3) THE SQUARE-ROOTER PERFORMANCE

The square-rooter was fabricated and tested. The three adjustments were introduced and carried out. The desired performance was possible only after the proper adjustments. The square-rooter possessed an input dynamic range of 15 mv to 6 v, i.e. 400:1, with an accuracy of 0.3%. The d.c. input-output characteristic is shown in the figure (Fig. 5.11)

It must be noted that the dynamic range of the square-rooter limits basically the range of measurement and it has nothing to do with the crest factor specification of RMS measurements.
5.5) THE RMS VOLTMETER

A complete digital rms voltmeter was developed by the Author while in Russia (Ref. 6) by using the described squarer, the averager and a feedback type square-rooter. The suitable input amplifier and a standard pulse width measuring circuitry were also incorporated. The voltmeter had a d.c. measurement span of 0.3 V to 10 V: (extendable to 15 V i.e. (50:1) using 3½ digit display); and a.c. measurement span of 0.3 to 18 V for a 2 kHz sinusoidal signal. The error was not more than ± 0.5 % (Ref. 6). The D.C. error characteristic is plotted in the figure (Fig. 5.12).

5.6) CONCLUSIONS

The theoretical and experimental work has resulted in the successful development of RMS detector circuitry for RMS measurement on high crest factor signals of low and medium frequency. The squarer was designed specially for high crest factor signals. The filtered squarer output was a measure of the mean square value for signals with crest factor as high as 20. The development of a square-rooting A/D converter resulted in the successful application of the simple direct rms computation technique in the digital rms measurement.
References


4) Deo P. V., Melnikov A. G. 'Function Generator for RMS Voltmeter.' Izv. VUZ. priborostroyeniya (Russian) Vol. No. 5, 1975 p. 72 (Russian)


### TABLE T. 5.1

**Approximation Segment VS RMS Measurement Error**

<table>
<thead>
<tr>
<th>( \varepsilon_r ) %</th>
<th>0.1</th>
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<th>0.3</th>
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<td>( n )</td>
<td>16</td>
<td>11</td>
<td>9</td>
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<td>7</td>
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</table>

### TABLE T. 5.2

**Squarer Design**

<table>
<thead>
<tr>
<th>( i )</th>
<th>( \theta_{i-1} )</th>
<th>( R_{zi} ) kohm</th>
<th>( S_{iz} )</th>
<th>( R_{zi} ) kohm</th>
<th>( s_i )</th>
<th>( R_o / R_{xi} )</th>
<th>( R_{xi} ) kohm</th>
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<td>1</td>
<td>0</td>
<td>-</td>
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<td>0.137</td>
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<td>2</td>
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<td>400</td>
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<td>3</td>
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</tr>
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<td>5</td>
<td>1.8209</td>
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<td>6.7015</td>
<td>13.4</td>
<td>0.2871</td>
<td>51</td>
<td>3.4</td>
<td>0.085</td>
<td>23.7</td>
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FIG. 5.4 [A]  
PRECISION FEEDBACK LIMITER  

FIG. 5.5  SQ. FUNCTION APPROXIMATOR
FIG. 5.7 ACTIVE LOW PASS FILTER

FIG. 5.8 A SQUARE ROOT A/D CONVERTER
Fig. 3.9 Time diagram of the square root circuit
FIG 5.10 SQUARE-ROOTER ADJUSTMENTS
Figure 11: Input/Output Characteristics of the Square Rooter