Preface

The split feasibility problem is one of the most important and applicable problems from nonlinear analysis which is introduced and studied during the last decade. Such problem arises in the intensity-modulated radiation therapy when one attempts to describe physical dose constraints and equivalent uniform dose constraints within a single model. It is a model of several real-world problems, namely, sensor network problem, resolution enhancement problem, antenna design problem, computerized tomography problem, image recovery problem, etc.

The objective of this thesis is to develop and study some iterative methods for finding the approximate solutions of different kinds of split feasibility problems, namely, split common fixed point problems, split hierarchical variational inequality problems and split hierarchical monotone variational inclusion problems in the setting of Hilbert spaces; and to develop and study some iterative methods for computing the approximate solutions of generalized split feasibility problems and split equality fixed point problems in the setting of Banach spaces.

The present thesis comprises eight chapters.

Chapter 1 provides a brief introduction of split feasibility problems and split equality problems.

In Chapter 2, we give some known notations, definitions and results from convex analysis and nonlinear analysis which will be used in the sequel. Some elementary notions, definitions and results from geometry of Banach spaces are presented. We also give a brief introduction of variational inequality problems, variational inclusion problems and equilibrium problems.

Chapter 3 deals with the weak convergence of the relaxed extragradient method with regularization for computing a common element of the solution set of a split feasibility problem and the fixed points set of an asymptotically k-strict psuedo-contractive mapping in intermediate sense.

In Chapter 4, we introduce an implicit algorithm and an explicit algorithm for

solving the split common fixed point problems. Under some mild conditions, we establish strong convergence of the presented algorithms to a solution of the split common fixed point problem which also solves a variational inequality. Furthermore, we apply our iterative algorithms to solve some convex and nonlinear problems, namely, variational problems and equilibrium problems.

In Chapter 5, we study generalized split feasibility problems in the setting of Banach spaces. We propose iterative algorithms to compute the approximate solutions of such problems. The weak convergence of the sequence generated by the proposed algorithms is studied. As applications, we derive some algorithms and convergence results for some problems from nonlinear analysis, namely, split feasibility problems, equilibrium problems, etc.

In Chapter 6, we consider the split equality fixed point problem in the setting of Banach spaces. This problem includes split feasibility problems, split equality problems, split common fixed point problems, split common null point problems, split variational inequality problems and split equilibrium problems as special cases. We propose simultaneous iterative algorithms to compute the approximate solutions of such problems. The weak convergence of the sequence generated by the proposed algorithms is studied either for firmly nonexpansive type mappings or for relative nonexpansive mappings. Several special cases of our algorithms and results are derived.

Chapter 7 deals with the common solution method for finding a fixed point of a nonexpansive mapping and a solution of split hierarchical Minty variational inequality problem. We discuss the weak convergence of the sequences generated by the proposed method to a common solution of a fixed point problem and a split hierarchical Minty variational inequality problem. An example is presented to illustrate the proposed algorithm and result.

In the last chapter, we introduce the split hierarchical monotone variational inclusion problem which includes split variational inequality problems, split monotone variational inclusion problems, etc., as special cases. An iterative algorithm is proposed to compute the approximate solutions of split hierarchical monotone variational inclusion problem. The weak convergence of the sequence generated by the proposed algorithm is studied. We present an example to illustrate our algorithm and convergence result.

At the end of this thesis, we give a fairly large bibliography which covers almost all the papers on split feasibility problems.