APPENDIX

REPRINTS OF RESEARCH PAPERS PUBLISHED
EFFECT OF MELTING AND THERMAL DISPERSION ON MIXED CONVECTION FLOW FROM VERTICAL PLATE EMBEDDED IN NON-NEWTONIAN FLUID SATURATED NON-DARCY POROUS MEDIUM

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ABSTRACT: We analyzed in this paper the problem of mixed convection along a vertical plate in a non-Newtonian fluid saturated non-Darcy porous medium in the presence of melting and thermal dispersion effects for aiding and opposing external flows. Similarity solution for the governing equations is obtained for the flow equations in steady state. The equations are numerically solved by using Runge-kutta fourth order method coupled with shooting technique. The effects of melting (M), thermal dispersion (D), inertia (F), mixed convection (Ra/Pe) and Nusselt number on velocity distribution and temperature are examined for aiding and opposing external flows.

Keywords: Modal Expansion Method, L-Shaped Microstrip Antenna, Return Loss.

1. INTRODUCTION
In many processes associated with phase change, heat transfer occurs with melting. Studies of this phenomenon have applications such as in casting, welding and magma solidification. The melting of permafrost, the thawing of frozen grounds, and the preparation of semiconductor materials are some more areas involving heat transfer with melting. In studies of heat transfer associated with melting in a porous medium Kazmierczak et al. [1] examined the velocity, temperature and Nusselt number in the melting region from a flat plate in the presence of steady natural convection. Bakier [2] studied the melting effect on mixed convection from a vertical plate of arbitrary wall temperature both in aiding and opposing flows in a fluid saturated porous medium. It was observed that, the melting phenomena decrease the local Nusselt number at solid-liquid Interface. The problem of mixed convection in melting from a vertical plate of uniform temperature in a saturated porous medium has been extensively studied by Gorla et al. [3]. It was noticed that, the melting process is analogous to mass injection and blowing near the boundary and thus reduces the heat transfer through solid liquid interface. In [4] Tashtoush analyzed the magnetic and buoyancy effects on melting from a vertical plate by considering the Frochhei-mer’s extension. It was noticed that, the velocity and temperature profiles and heat transfer rate of melting phenomena associated with uniform wall temperature using the collocation finite element method. In such an analysis, the effect of inertial forces on flow and heat transfer was noticed prominently. Cheng and Lin [5] studied the melting effect on mixed convective heat transfer from a solid porous vertical plate with uniform wall temperature embedded in the liquid saturated porous medium by using Runge-kutta Gill method and Newton’s iteration for similarity solutions. They had established the criteria for \( \left( \frac{Gr}{Re} \right) \) values for forced mixed and free convection from an isothermal vertical plate in porous media with aiding and opposing external flows in melting process. Recently A.Y.Bakier et al.[6] studied Group method analysis of melting effect on MHD mixed convection flow from a radiative vertical plate embedded in a saturated porous medium. He developed linear transformation group approach to simulate problem of hydro magnetic heat transfer by mixed convection along vertical plate in a liquid saturated porous medium in the presence of melting and thermal radiation effects for opposing external flow. He studied the effects of the pertinent parameters on the rate of the heat transfer in terms of the local Nusselt number at the solid-liquid interface.

Further non-Newtonian power law fluids are so wide spread in industrial process and in the environment. The melting phenomena on free convection from a vertical front in a non-Newtonian fluid saturated porous matrix is analyzed by Poulikakos and Spatz[7].Nakayama and Koyama[8] studied the more general case of...
free convection over a non-isothermal body of arbitrary shape embedded in a porous medium. Rastogi and Poulikakos [9] examined the problem of double diffusive convection from a vertical plate in a porous medium saturated with a non-Newtonian power law fluid. Shenoy [10] presented many interesting applications of non-Newtonian power law fluids with yield stress on convective heat transport in fluid saturated porous media. Considering geothermal and oil reservoir engineering applications, Nakayama and Shenoy [11] studied a unified similarity transformation for Darcy and non-Darcy forced, free and mixed convection heat transfer in non-Newtonian inelastic fluid saturated porous media. Later Shenoy [12] studied non-Darcy natural, forced and mixed convection heat transfer in non-Newtonian power law fluid saturated porous media. Effect of melting and thermo-diffusion on natural convection heat mass transfer in a non-Newtonian fluid saturated non-Darcy porous medium was studied by R.R.Kairi and P.V.S.N.Murthy [13]. It is noted that the velocity, temperature and concentration profiles as well as the heat and mass transfer coefficients are significantly affected by the melting phenomena and thermal-diffusion in the medium. The non-linear behavior of non-Newtonian fluids in a porous matrix is quite different from that of Newtonian fluids in porous media. The prediction of heat or mass transfer characteristics for mixed or natural convection of non-Newtonian fluids in porous media is very important due to its practical engineering applications such as oil recovery and food processing. More recently Melting and radiation effects on mixed convection from a vertical surface embedded in a non-Newtonian fluid saturated non-Darcy porous medium for aiding and opposing external flows is analyzed by Ali, J.Chamka et al. [14]. They obtained representative flow and heat transfer results for various combinations of physical parameters. Study of thermal dispersion effects becomes prevalent in the porous media flow region. Hong and Tien [15] examined analytically the effect of transverse thermal dispersion on natural convection from a vertical, heated plate in a porous medium. Their results show that due to the better mixing of the thermal dispersion effect, the heat transfer rate is increased. Plumb [16] modeled thermal dispersion effects over a vertical plate. Murthy [17] analyzed the non-Darcy mixed convection flow and heat transfer about an isothermal vertical wall The present paper is aimed at analyzing the effect of melting and thermal dispersion on steady mixed convective heat transfer from a vertical plate embedded in a non-Newtonian power law fluid saturated non-Darcy porous medium for aiding and opposing external flows. The mathematical model of thermal dispersion adopted in this paper is that of Plumb [16]. The inclusion of thermal dispersion modifies energy equation and dispersion adopted in this paper is that of Plumb [16]. The inclusion of thermal dispersion modifies energy equation and also the condition at the plate. By denoting the x-component of velocity u at large distance of the plate as \( u_w \), we can recover the melting conditions with out thermal dispersion from the present mathematical formulation of the problem.

2. MATHEMATICAL FORMULATION:

A mixed convective heat transfer in a non-Darcy porous medium saturated with a homogeneous non-Newtonian fluid adjacent to a vertical plate, with a uniform wall temperature is considered. This plate constitutes the interface between the liquid phase and the solid phase during melting inside the porous matrix at steady state. The plate is at a constant temperature \( T_m \) at which the material of the porous matrix melts. Fig.1 shows the coordinate and the flow model. The x-coordinate is measured along the plate and the y-coordinate normal to it. The solid phase is at temperature \( T_0 \), \( T_m \). A thin boundary layer exists close to the right \( \partial \) vertical plate and temperature changes smoothly through this layer from \( T_m \) to \( T_0 \) \( (T_m < T_0) \) which is the temperature of the fluid phase.

The continuity equation is

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

(1)

The momentum equation is

\[
\frac{\partial u}{\partial t} + \frac{\partial (\rho u^2)}{\partial y} = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial y} \left[ \mu \frac{\partial u}{\partial y} + \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right) \right]
\]

\[
\frac{\partial v}{\partial t} + \frac{\partial (\rho u v)}{\partial y} = -\frac{\partial P}{\partial y} + \frac{\partial}{\partial y} \left[ \mu \frac{\partial v}{\partial y} + \frac{\partial}{\partial y} \left( \frac{\partial v}{\partial y} \right) \right]
\]

\[
\frac{\partial T}{\partial t} + \frac{\partial (\rho u T)}{\partial y} = \frac{\partial}{\partial y} \left[ \kappa \frac{\partial T}{\partial y} + \frac{\partial}{\partial y} \left( \frac{\partial T}{\partial y} \right) \right] + \frac{\partial}{\partial y} \left( \frac{\partial \phi}{\partial y} \right)
\]

\[
\frac{\partial \phi}{\partial t} + \frac{\partial (\rho u \phi)}{\partial y} = \frac{\partial}{\partial y} \left[ \beta \frac{\partial \phi}{\partial y} + \frac{\partial}{\partial y} \left( \frac{\partial \phi}{\partial y} \right) \right]
\]

(2)

\[
\frac{\partial \phi}{\partial t} + \frac{\partial (\rho u \phi)}{\partial y} = \frac{\partial}{\partial y} \left[ \beta \frac{\partial \phi}{\partial y} + \frac{\partial}{\partial y} \left( \frac{\partial \phi}{\partial y} \right) \right]
\]

(3)
In the above equations, the term which represents the buoyancy forced effect on the flow field has \( \pm \) signs. The plus sign indicates aiding buoyancy flow whereas the negative sign stands for buoyancy opposed flow. Here \( u \) and \( v \) are the velocities along \( x \) and \( y \) directions respectively, \( \beta \) is the power-law fluid viscosity index, \( T \) is temperature in the thermal boundary layer, \( K \) is permeability, \( C \) is Forchheimer empirical constant, \( \alpha \) is coefficient of thermal expansion, \( \nu \) is Karman constant, \( \rho \) is density, \( \eta \) is specific heat at constant pressure, \( g \) is acceleration due to gravity, and thermal diffusivity \( \alpha = \alpha_m + \alpha_d \), where \( \alpha_m \) is the molecular diffusivity and \( \alpha_d \) is the dispersion thermal diffusivity due to mechanical dispersion. As in the linear model proposed by Plumb [11] the dispersion thermal diffusivity \( \alpha_d \) is proportional to the velocity component i.e \( \alpha_d = \gamma u_d \), where \( \gamma \) is the dispersion coefficient and \( d \) is the mean particle diameter.

The physical boundary conditions for the present problem are

\[
q=0, T=T_m, \quad k \frac{\partial T}{\partial y} = \left[ \hat{h} \alpha_c + c_v (T_m - T_0) \right] v \quad \text{(4)}
\]

and \( y \to \infty, T \to T_m, u=0 \) \text{ (5)}

Where \( \hat{h} \) and \( \alpha_c \) are latent heat of the solid and specific heat of the solid phases respectively and \( u_0 \) is the assisting external flow velocity, \( k=\alpha_c \rho \) is the effective thermal conductivity of the porous medium. The boundary condition (4) means that the temperature on the plate is constant and thermal flux of heat conduction to the melting surface is equal to the sum of the heat of melting and the heat required for raising the temperature of solid to its melting temperature \( T_m \).

Introducing the stream function \( \psi \) with \( u=\frac{\partial \psi}{\partial y} \)

and \( v=-\frac{\partial \psi}{\partial x} \)

The continuity equation (1) will be satisfied and the equations (2) and (3) transform to

\[
\frac{\partial^2 \psi}{\partial y^2} + \frac{C \sqrt{K}}{\nu} \frac{\partial \psi}{\partial y} \left( \frac{\partial \psi}{\partial y} \right)^2 = \frac{\pm \hat{h} \alpha_c + c_v (T_m - T_0)}{\nu} \frac{\partial T}{\partial y} \quad \text{(6)}
\]

\[
\frac{\partial \psi}{\partial y} T_x - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left[ \left( \alpha_m + \gamma \frac{\partial \psi}{\partial y} \right) \frac{\partial T}{\partial y} \right] \quad \text{(7)}
\]

Introducing the similarity variables as

\[
\psi = f(\eta)(\eta_m \eta x)^{1/2}, \quad \eta = \left( \frac{\hat{h} \alpha_c + c_v (T_m - T_0)}{\nu} \right)^{1/2} \frac{\partial T}{\partial y} \quad \text{(8)}
\]

where \( \theta(\eta) = \frac{T - T_m}{T_m - T_0} \), the momentum equation (6) and energy equation (7) are reduced to

\[
f q^{11} f^{11} = 2 f q^{11} \frac{R}{P_e} \quad \text{(9)}
\]

Where the prime symbol denotes the differentiation with respect to the similarity variable \( \eta \) and \( Ra/Pe_\alpha \) is the mixed convection parameter,

\[
Ra = \frac{\alpha c_v (T_m - T_0)}{K \nu} \quad \text{(10)}
\]

\[
where \( \alpha \) is the local Rayleigh number, \( Pe_\alpha = \frac{u_0 x}{\alpha} \) is the local Peclet number, \( F=0 \) is the non-darcian parameter. \( Pe_d \) is the pore diameter dependent Peclet number. \( D= \frac{\hat{h} \alpha_c}{\alpha_m} \) is the dispersion parameter.

Taking into consideration, the thermal dispersion effect together with melting, the boundary conditions (4) and (5) take the form

\[
\eta=0, \theta=0, f(0)=1+D f(0), 2M(0)=0 \quad \text{ (11)}
\]

\[
\text{Where } M= \frac{c_f(T_m - T_0)}{h_f + c_s(T_m - T_0)} \text{ is the melting parameter.}
\]

The local heat transfer rate from the surface of the plane is given by

\[
q_w = -k \frac{\partial T}{\partial y} \text{ at } y=0, \quad \text{ (12)}
\]

The Nusselt number is

\[
Nu = \frac{h x}{k} = \frac{q_w x}{k(T_m - T_0)} \quad \text{(12)}
\]

Where \( h \) is the local heat transfer coefficient and \( k \) is the effective thermal conductivity of the porous medium, which is the sum of the molecular thermal conductivity \( k_m \) and the dispersion thermal conductivity \( k_d \) [15].

The modified Nusselt number is obtained as

\[
\frac{N_u}{(Pe_d)^{1/2}} = \frac{1+D f(0)}{Pe_\alpha} \quad \text{(12)}
\]

3. SOLUTION PROCEDURE

The dimensionless equations eq.(8) and eq.(9) together with the boundary conditions (10) and (11) are solved numerically by means of the fourth order Runge-Kutta method coupled with double shooting technique. The solution thus obtained is matched with the given values of \( f(\infty) \) and \( \theta(0) \). In addition the boundary condition \( \eta \to \infty \) is approximated by \( \eta_{\max} = 8 \) which is found sufficiently large for the velocity and temperature to approach the relevant free stream properties. Numerical computations are carried out for \( F=0, 0.5, 1; \quad D=0, 0.5, 1; \quad Ra/Pe=1; M=0, 0.4, 0.8, 1.2, 2, 6.2; \eta=0.5, 1, 1.5 \).

4. RESULTS AND DISCUSSION

In order to get clear insight on the physics of the problem, a parametric study is performed and the
obtained numerical results are displayed with the help of graphical illustrations. The parameters governing the physics of the present study are the melting (M), the mixed convection (Ra/Pe), the inertia (F), thermal dispersion (D), and Fluid viscosity index n. The numerical computations were carried out for the fixed value of buoyancy parameter Ra = 1 for both the aiding and opposing external flows. The results of the parametric study are shown in figures 2-11.

Different behavior exists between the velocity profiles in the presence of solid phase melting effect in the case of aiding and opposing external flows. This difference is represented in the increase of the velocity profiles as a result of increasing the melting parameter M for the case of aiding flow; whereas the opposite behavior for the velocity fields of field as M increases is found opposing external flow case. As for the temperature of the liquid, increasing the value of the melting parameter M causes decrease in the temperature distributions for both aiding and opposing flow conditions.

![Fig.2](image1)

**Fig.2**

![Fig.3](image2)

**Fig.3 (a)**

![Fig.3(b)](image3)

**Fig.3 (b)**

Figures 2 and 3 shows the effect of the melting parameter M on the velocity profiles and temperature distributions for aiding and opposing external flows respectively. It is observed that
The effects of the thermal dispersion parameter on the velocity profiles and temperature distributions for the case of aiding external flow and the case of opposing external flow are plotted in figures 4 and 5. It is clear from figure 4 that, for aiding external flow conditions the velocity increases with increase in thermal dispersion parameter. But this effect is found opposite in opposing external flow conditions. In addition from figure 5 it is clear that increasing the values of thermal dispersion parameter leads to decrease in the liquid temperature distribution for both cases of aiding and opposing external flow.

Figures 6 and 7 show the effects of the non-Darcy porous medium parameter $F$ on velocity profiles and temperature distributions for both aiding and opposing flow cases respectively. In the aiding flow case both of the velocity profiles and temperature distributions decrease with increasing values of the non-Darcy porous medium parameter but in the opposing flow case, the fluid velocity and temperature increase as the non-Darcy porous medium parameter increases.
Effect of dispersion parameter on Nusselt number for different values of melting parameter.

The effect of melting strength and thermal dispersion on heat transfer rate is shown in fig. 10 for both aiding and opposing external flows in terms of Nusselt number defined in equation (12).

This shows that Nusselt number decreases significantly with the increasing melting strength (M) and increases with increase in thermal dispersion parameter for both aiding and opposing flows.

Fig.9 (b)
Figures 8 and 9 show the effect of power law fluid viscosity index on velocity profiles and temperature distributions for both aiding and opposing flows respectively. In aiding flow case the velocity profiles decreases as power law fluid viscosity index n increases but this effect are found opposite in the opposing flow case and the effect on temperature distributions is negligible for both the cases.

Fig.10 (a)

Fig.11 (b)
Fig. 11 Nusselt number variation with melting parameter for different non-Darcy parameters with and without dispersion effect.

The variation of Nusselt number with melting parameter for different values of non-Darcy parameter $F$ is shown in fig. 11 for both aiding and opposing flows with $D=0$ and $D=0.5$. In aiding flow case the Nusselt number decreases as non-Darcy parameter increases. But in the opposing flow case the effect is found opposite.

5. REFERENCES


Melting and Thermal Dispersion-Radiation Effects on Non-Darcy Mixed Convection

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ABSTRACT

We analyzed in this paper the problem of mixed convection along a vertical plate in a non-Newtonian fluid saturated non-Darcy porous medium in the presence of melting, thermal dispersion-radiation and heat absorption or generation effects for aiding and opposing external flows. Similarity solution for the governing equations is obtained for the flow equations in steady state. The equations are numerically solved by using Runge-kutta fourth order method coupled with shooting technique. The effect of melting under different parametric conditions on velocity and temperature distributions was analyzed for both aiding and opposing flows.

Keywords: Porous medium, Non-Newtonian Fluid, Melting, Thermal Dispersion, Radiation, Heat generation or absorption.

INTRODUCTION

Convection heat transfer in porous media in the presence of melting effect has received some attention in recent years. This stems from the fact that this topic has significant direct application in permafrost melting, frozen ground thawing, casting and bending processes as well as phase change metal. The study of melting effect is considered by many researchers in Newtonian fluids. Non-Newtonian power law fluids are so widespread in industrial process and in the environment. Effect of melting and thermo-diffusion on natural convection heat mass transfer in a non-Newtonian fluid saturated non-Darcy porous medium was studied by R.R.Kairi and P.V.S.N.Murthy¹. It is noted that the velocity, temperature and concentration profiles as well as the heat and mass transfer coefficients are significantly affected by the melting phenomena and thermal-diffusion in the medium. If the
The continuity equation is \( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \) \( (1) \)

The momentum equation is
\[
\frac{\partial u^n}{\partial y} + \frac{c\sqrt{K}}{v} \frac{\partial u^2}{\partial y} = \frac{1}{v} \frac{K\beta}{\partial T} \frac{\partial T}{\partial y}
\]
\( (2) \)

The energy equation is
\[
u \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left( \alpha \frac{\partial T}{\partial y} \right) - \frac{1}{\rho C_p} \frac{\partial q}{\partial y} + \frac{a Q_0}{K} (T - T_m)
\]
\( (3) \)

In the above equations, the term which represents the buoyancy forced effect on the flow field has ± signs. The plus sign indicates aiding buoyancy flow where as the negative sign stands for buoyancy opposed flow. Here \( u \) and \( v \) are the velocities along \( x \) and \( y \) directions respectively, \( n \) is the power-law fluid viscosity index, \( T \) is Temperature in the thermal boundary layer, \( T_m \) is melting temperature, \( K \) is Permeability, \( C \) is Forchheimer empirical constant, \( \beta \) is coefficient of thermal expansion, \( \nu \) is Kinematics viscosity, \( \rho \) is Density, \( C_p \) is Specific heat at constant pressure, \( g \) is acceleration due to gravity, and thermal diffusivity \( \alpha = \alpha_m + \alpha_d \) where \( \alpha_m \) is the molecular diffusivity and \( \alpha_d \) is the dispersion thermal diffusivity due to mechanical dispersion and \( Q_0 \) is the volumetric heat generation or absorption parameter. As in the linear model proposed by Plumb\(^4\), the dispersion thermal diffusivity \( \alpha_d \) is proportional to the velocity component \( i.e \),
\[ \alpha_d = \gamma u d, \]
where \( \gamma \) is the dispersion coefficient and \( d \) is the mean particle diameter. The radiative heat flux term \( q \) is written using the Rosseland approximation (Sparrow and Cess\(^5\), Raptis\(^6\) as
\[
q = -\frac{4\sigma_R}{3a} \frac{\partial T^4}{\partial y}
\]
\( (4) \)

Where \( \sigma_R \) is the Stefan - Boltzmann constant and ‘a’ is the mean absorption coefficient.

The physical boundary conditions for the present problem are
\[
y=0, T=T_m, k \frac{\partial T}{\partial y} = \rho (h_{st} + C_d (T_m - T_0)) \nu \]
\( (5) \)

and \( y \to \infty, T \to T_{m}, u=u_{\infty} \)
\( (6) \)

Where \( h_{st} \) and \( C_d \) are latent heat of the solid and specific heat of the solid phases respectively and \( u_{\infty} \) is the assisting external flow velocity, \( k = \rho C_p \) is the effective thermal conductivity of the porous medium.

The boundary condition (5) means that the temperature on the plate is constant and thermal flux of heat conduction to the melting surface is equal to the sum of the heat of melting and the heat required for raising the temperature of solid to its melting temperature \( T_m \).

Introducing the stream function \( \psi \)
\[
\psi = \frac{\partial v}{\partial y}, \text{and } v = -\frac{\partial \psi}{\partial x}
\]

The continuity equation (1) will be satisfied and the equations (2) and (3) transform to
\[
\frac{\partial}{\partial y} \left( \frac{\partial \psi}{\partial y} \right)^n + \frac{c\sqrt{K}}{v} \frac{\partial}{\partial y} \left( \frac{\partial \psi}{\partial y} \right)^2 = \frac{1}{v} \frac{K\beta}{\partial T} \frac{\partial T}{\partial y}
\]
\( (7) \)

\[
\frac{\partial \psi}{\partial T} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left[ (\alpha_m + \gamma \frac{\partial \psi}{\partial y} (d) \frac{\partial T}{\partial y} \right) + \frac{4\sigma_R}{3\rho C_p a} \frac{\partial}{\partial y} \left( \frac{\partial T^4}{\partial y} \right) + \frac{a Q_0}{K} (T - T_m)
\]
\( (8) \)

Introducing the similarity variables as
\[
\psi = f(\eta) \left( \alpha_m u_{\infty} x \right)^{1/2}, \quad \eta = \left( \frac{\mu_0 x}{\alpha_m} \right)^{1/2} \left( \frac{\gamma}{\chi} \right)
\]
results are displayed with the help of graphical illustrations. The parameters governing the physics of the present study are the melting \((M)\), the mixed convection \((Ra/Pe)\), the inertia \((F)\), thermal dispersion \((D)\), and Fluid viscosity index \((n)\), radiation\((R)\), Heat absorption or generation\((r)\). The numerical computations were carried out for the fixed value of buoyancy parameter \(Ra/Pe = 1\) for both the aiding and opposing external flows. The results of the parametric study are shown in figures 2-19.

The effects of thermal dispersion parameter \(D\) on the velocity profiles for both cases of aiding and opposing external flow conditions are plotted in Fig. 2. It is noted that for the case of aiding flow the velocity profiles increase with increase in the value of \(D\). But this effect is found opposite in the case of opposing flow.

The effects of thermal dispersion parameter \(D\) on temperature distributions for aiding and external flow conditions are plotted in Fig. 3 and Fig. 4 respectively. It is noted that increasing the values of \(D\) leads to decrease in the liquid temperature distributions in both cases.

Effect of heat generation or absorption parameter on velocity profiles without dispersion effect is shown in Fig. 5 and Fig. 6 for aiding opposing flow cases respectively. Fig. 7 and Fig. 8 show the same effect in the presence of dispersion. It is noted that for the case of aiding flow the velocity profile decreases with the increase of absorption parameter with and without dispersion effect. Whereas the opposite result is found in the case of opposing flow.

Effect of heat generation or absorption parameter on temperature profiles without dispersion effect is shown in Fig. 9 and Fig. 10 for aiding, opposing flow cases respectively. Fig. 11 and Fig. 12 show the same effect in the presence of dispersion. It is noted that for both the flows the temperature profile increases with the increase of absorption parameter with and without dispersion effect.

The effect of melting strength and thermal dispersion on heat transfer rate is shown in Fig. 13 for both aiding and opposing flows in terms of Nusselt number defined in eq (13). It is observed that Nusselt number decreases significantly with the increase of melting strength \(M\) and increases with increase in thermal dispersion parameter \(D\) for both aiding and opposing flows.

The variation of Nusselt number with melting parameter for different values of non-Darcy parameter \(F\) is shown in Fig. 14 for both aiding and opposing flows. In aiding flow case the Nusselt number decreases as the non-Darcy parameter \(F\) increases. Whereas in the opposing flow case the effect is found opposite.

The variation of Nusselt number with melting parameter \(M\) for different values of radiation parameter \(R\) is shown in Fig. 15 for both aiding and opposing flows. It is observed that in both cases the Nusselt number increases as the radiation parameter \(R\) increases.

Fig. 16 and Fig. 17 show the effect of heat absorption parameter on the Nusselt number for different values of non-Darcy porous medium parameter for aiding and
Fig. 6 Effect of absorption coefficient on velocity profiles

Fig. 7 Effect of absorption coefficient on velocity profiles in the presence of 'D'

Fig. 8 Effect of absorption coefficient on velocity profiles in the presence of 'D'

Fig. 9 Effect of absorption coefficient on temperature profiles

Fig. 10 Effect of absorption coefficient on temperature profiles

Fig. 11 Effect of absorption coefficient on temperature profiles in the presence of 'D'
We compared our results with those of earlier published works of Chemkha et al.\(^3\) and Cheng and Lin\(^7\) for special cases of the problem under consideration. Tables 1-3 show that our numerical values are in good agreement with the compared results.

Table 1. Comparison of \(f'(0)\) with values obtained by Chemkha et al.\(^3\) and Cheng and Lin\(^7\) for Newtonian fluid \((n=1.0)\) with an aiding external flow.

<table>
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<tr>
<th>M</th>
<th>Ra/Pe</th>
<th>Chemkha et al.(^3)</th>
<th>Cheng and Lin(^7)</th>
<th>Present</th>
</tr>
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<td>2.0</td>
<td>0.0</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
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<td>2.400</td>
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</tr>
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Mixed Convection Flow from Vertical Plate Embedded in Non-Newtonian Fluid Saturated Non-Darcy Porous Medium with Melting Effect

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Abstract

We analyzed in this paper the problem of mixed convection along a vertical plate in a non-Newtonian fluid saturated non-Darcy porous medium in the presence of melting and thermal dispersion-radiation effects for aiding and opposing external flows. Similarity solution for the governing equations is obtained for the flow equations in steady state. The equations are numerically solved by using Runge-kutta fourth order method coupled with shooting technique. The effects of melting (M), thermal dispersion (D), radiation (R), temperature ratio (Cr), inertia (F), mixed convection (Ra/Pe) and Nusselt number on velocity distribution and temperature are examined for aiding and opposing external flows.

Key words: Porous medium, Non-Newtonian Fluid, Melting, Thermal Dispersion, Radiation.

Introduction

Convection heat transfer in porous media in the presence of melting effect has received some attention in recent years. This stems from the fact that this topic has significant direct application in permafrost melting, frozen ground thawing, casting and bending processes as well as phase change metal. The study of melting effect is considered by many researchers in Newtonian fluids. Non-Newtonian power law fluids are so wide spread in industrial process and in the environment. The melting phenomena on free convection from a vertical front in a non-Newtonian fluid saturated porous matrix are analyzed by Poulíkakos and Spatz. Nakayama and Koyama studied the more general case of free convection over a non-isothermal body of arbitrary shape embedded in a porous medium. Rastogi and Poulíkakos examined the problem of double diffusive
ordinate and the flow model. The x-coordinate is measured along the plate and the y-coordinate normal to it. The solid phase is at temperature \( T_0 < T_m \). A thin boundary layer exists close to the right of vertical plate and temperature changes smoothly through this layer from \( T_m \) to \( T_\infty \) \((T_m < T_\infty)\) which is the temperature of the fluid phase.

![Diagram of the problem](image)

In the above equations, the term which represents the buoyancy forced effect on the flow field has \( \pm \) signs. The plus sign indicates aiding buoyancy flow whereas the negative sign stands for buoyancy opposed flow. Here \( u \) and \( v \) are the velocities along \( x \) and \( y \) directions respectively, \( n \) is the power-law fluid viscosity index, \( T \) is Temperature in the thermal boundary layer, \( K \) is Permeability, \( C \) is Forchheimer empirical constant, \( \beta \) is coefficient of thermal expansion, \( v \) is Kinematics viscosity, \( \rho \) is Density, \( C_p \) is Specific heat at constant pressure, \( g \) is acceleration due to gravity, and thermal diffusivity \( \alpha = \alpha_m + \alpha_d \), where \( \alpha_m \) is the molecular diffusivity and \( \alpha_d \) is the dispersion thermal diffusivity due to mechanical dispersion. As in the linear model proposed by Plumb, the dispersion thermal diffusivity \( \alpha_d \) is proportional to the velocity component i.e. \( \alpha_d = \gamma ud \), where \( \gamma \) is the dispersion coefficient and \( d \) is the mean particle diameter.

The radiative heat flux term \( q \) is written using the Rosseland approximation (Sparrow and Cess, Raptis) as

\[
q = -\frac{4\sigma_R}{3a} \frac{\partial T^4}{\partial y}
\]

Where \( \sigma_R \) is the Stefan-Boltzmann constant and 'a' is the mean absorption coefficient.

The physical boundary conditions for the present problem are

\[
y=0, T=T_m, k \frac{\partial T}{\partial y} = \rho [h_{sf} + C_s (T_m - T_0)] v
\]

and

\[
y \to \infty, T \to T_\infty, u = u_\infty
\]

Where \( h_{sf} \) and \( C_s \) are latent heat of the solid and specific heat of the solid phases respectively and \( u_\infty \) is the assisting external flow velocity, \( k=\alpha \rho C_p \) is the effective thermal conductivity.
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conductivity \(k_d\). The modified Nusselt number is obtained as

\[
\frac{N_u}{(P_e \xi)^{1/3}} = \left[1 + \frac{4}{3} R (\theta(0) + C_r)^3 + D f^2(0)\right] \theta^3(0)
\]

(13)

Solution Procedure:

The dimensionless equations eq. (9) and eq. (10) together with the boundary conditions (11) and (12) are solved numerically by means of the fourth order Runge-Kutta method coupled with double shooting technique. The solution thus obtained is matched with the given values of \(f(\infty)\) and \(\theta(0)\). In addition the boundary condition \(\eta \to \infty\) is approximated by \(\eta_{max} = 8\) which is found sufficiently large for the velocity and temperature to approach the relevant free stream properties. Numerical computations are carried out for \(F=0, 0.5, 1; D=0, 0.5, 1; Ra/Pe=1; M=0, 0.4, 0.8, 1.2, 1.6, 2; n=0.5, R=0, 0.5, 1; Cr=0.1, 0.5, 1\).

Results and Discussion

In order to get clear insight on the physics of the problem, a parametric study is performed and the obtained numerical results are displayed with the help of graphical illustrations. The parameters governing the physics of the present study are the melting \(M\), the mixed convection \(Ra/Pe\), the inertia \(F\), thermal dispersion \(D\), and Fluid viscosity index \(n\), radiation\(R\), temperature ratio \(Cr\). The numerical computations were carried out for the fixed value of buoyancy parameter \(Ra/Pe =1\) for both the aiding and opposing external flows. The results of the parametric study are shown in figures 2-25. Fig. 2 shows the effect of melting parameter \(M\) on velocity profiles for both aiding and opposing external flows. It is observed that different behavior exists between the velocity profiles in the presence of solid phase melting effect in the case of aiding and opposing external flows. The increase in the melting parameter \(M\) causes the increase of the velocity profiles for the case of aiding flow. Whereas the opposite behavior for the velocity profiles as \(M\) increases is found in the case of opposing flow case.

Fig. 3 and Fig. 4 show the effect of the melting parameter \(M\) on temperature distributions for aiding and opposing external flows respectively. It is observed that as increasing the value of the melting parameter \(M\), the temperature distributions decrease for both cases of aiding and opposing external flows.
fluid viscosity index \( n \) on velocity profiles for both aiding and opposing flow cases. It is observed that in aiding flow case as \( n \) value increases, velocity profiles decreases. Whereas opposite behavior obtained in velocity profiles in opposing flow case.

Fig. 9 and Fig. 10 show the effects of power law fluid viscosity index \( n \) on temperature distributions for both aiding and opposing flow cases respectively. Significant effect is not found. In aiding flow case as \( n \) increases, the temperature distributions decrease. Whereas opposite result is found in opposing flow case.

Fig. 11 shows the effect of radiation parameter \( R \) on velocity profiles for both aiding and opposing external flows. The increase in the radiation parameter \( R \) causes the increase of the velocity profiles for aiding flow case. Whereas in the case of opposing flow the result is found opposite.

Fig. 12 and Fig. 13 show the effect of radiation parameter \( R \) on temperature distributions for aiding and opposing external
Mixed Convection Flow from Vertical Medium with Melting Effect.

The effects of temperature ratio parameter $Cr$ on temperature distributions are plotted in Fig. 19 and Fig. 20 for aiding and opposing flow cases respectively. Same effect is found in both cases. The increase in temperature ratio parameter $Cr$ results decrease of the temperature distributions in both cases.

The effect of melting strength and thermal dispersion on heat transfer rate is shown in Fig. 21 for both aiding and opposing flows in terms of Nusselt number defined in eq (13). It is observed that Nusselt number decreases significantly with the increase of $Cr$ value.

The effects of temperature ratio parameter $Cr$ on velocity profiles for aiding and opposing external flow cases are plotted in Fig. 18. In aiding flow case, the increase in the temperature ratio parameter $Cr$ results increase of the velocity profiles. Whereas in the opposing flow case opposite effect is found in the velocity profiles.
We compared our results with those of earlier published works of Chemkha et al. and Cheng and Lin for special cases of the problem under consideration. Tables 1-3 show that our numerical values are in good agreement with the compared results.

Table 1. Comparison of $f'(0)$ with values obtained by Chemkha et al. and Cheng and Lin for Newtonian fluid ($n=1.0$) with an aiding external flow.

<table>
<thead>
<tr>
<th>M</th>
<th>Ra/Pe</th>
<th>Chemkha et al.</th>
<th>Cheng and Lin</th>
<th>Present</th>
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<td>4.000</td>
<td></td>
</tr>
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<td>9.008</td>
<td>9.000</td>
<td>9.000</td>
<td></td>
</tr>
<tr>
<td>10.0</td>
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<td>11.00</td>
<td>11.00</td>
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</tbody>
</table>

Table 2. Comparison of $\theta'(0)$ with values obtained by Chemkha et al. and Cheng and Lin for Newtonian fluid ($n=1.0$) with an aiding external flow.

<table>
<thead>
<tr>
<th>M</th>
<th>Ra/Pe</th>
<th>Chemkha et al.</th>
<th>Cheng and Lin</th>
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</table>

Table 3. Comparison of $\theta'(0)$ with values obtained by Chemkha et al. and Cheng and Lin for Newtonian fluid ($n=1.0$) with an opposing external flow.

<table>
<thead>
<tr>
<th>M</th>
<th>Ra/Pe</th>
<th>Chemkha et al.</th>
<th>Cheng and Lin</th>
<th>Present</th>
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References

MHD and Thermal Dispersion-Radiation Effects on Non-Newtonian Fluid Saturated Non-Darcy Mixed Convective Flow with Melting Effect

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Abstract

The problem of mixed convection along a vertical plate in a non-Newtonian fluid saturated non-Darcy porous medium in the presence of melting, thermal dispersion-radiation effects for aiding and opposing external flows is analyzed. Similarity solution for the governing equations is obtained for the flow equations in steady state. The equations are numerically solved by using Runge-kutta fourth order method coupled with shooting technique. The effects of melting (M), thermal dispersion (D)-radiation (R), inertia (F) and mixed convection (Ra/Pe) on velocity and temperature distributions are examined in the presence of magnetic parameter MH.

Keywords: Porous medium, Non-Newtonian Fluid, Melting, Thermal Dispersion, Radiation

Introduction

Convection heat transfer in porous media in the presence of melting effect has received some attention in recent years. This stems from the fact that this topic has significant direct application

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in permafrost melting, frozen ground thawing, casting and bending processes as well as phase change metal. The study of melting effect is considered by many researchers in Newtonian fluids. Non-Newtonian power law fluids are so wide spread in industrial process and in the environment. The melting phenomena on free convection from a vertical front in a non-Newtonian fluid saturated porous matrix are analyzed by Poulikakos and Spatz[1]. Nakayama and Koyama [2] studied the more general case of free convection over a non-isothermal body of arbitrary shape embedded in a porous medium. Rastogi and Poulikakos [3] examined the problem of double diffusive convection from a vertical plate in a porous medium saturated with a non-Newtonian power law fluid. Shenoy[4] presented many interesting applications of non-Newtonian power law fluids with yield stress on convective heat transport in fluid saturated porous media. Considering geothermal and oil reservoir engineering applications, Nakayama and Shenoy[5] studied a unified similarity transformation for Darcy and non-Darcy forced, free and mixed convection heat transfer in non-Newtonian inelastic fluid saturated porous media. Later Shenoy[6] studied non-Darcy natural, forced and mixed convection heat transfer in non-Newtonian power law fluid saturated porous media.

Effect of melting and thermo-diffusion on natural convection heat mass transfer in a non-Newtonian fluid saturated non-Darcy porous medium was studied by R.R.Kairi and P.V.S.N.-Murthy[7]. It is noted that the velocity, temperature and concentration profiles as well as the heat and mass transfer coefficients are significantly affected by the melting phenomena and thermal-diffusion in the medium. The non-linear behavior of non-Newtonian fluids in a porous matrix is quite different from that of Newtonian fluids in porous media. The prediction of heat or mass transfer characteristics for mixed or natural convection of non-Newtonian fluids in porous media is very important due to its practical engineering applications such as oil recovery and food processing. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the effect of thermal radiation and mass diffusion. On the other hand it is worth mentioning that heat transfer simultaneous radiation and
convection is very important in the context of space technology and processes involving high temperatures.

Recently, A.Y. Bakier et al.[8] studied Group method analysis of melting effect on MHD mixed convection flow from a radiative vertical plate embedded in saturated porous medium for Newtonian fluids. He developed linear transformation group approach to simulate problem of hydro magnetic heat transfer by mixed convection along vertical plate in a liquid saturated porous medium in the presence of melting and thermal radiation effects for opposing external flow. He studied the effects of the pertinent parameters on the rate of the heat transfer in terms of the local Nusselt number at the solid-liquid interface. More recently Melting and radiation effects on mixed convection from a vertical surface embedded in a non-Newtonian fluid saturated non-Darcy porous medium for aiding and opposing external flows is analyzed by Ali.J. Chamka et.al.[9]. They obtained representative flow and heat transfer results for various combinations of physical parameters.

The present paper is aimed at analyzing the effect of melting and thermal dispersion-radiation on steady mixed convective heat transfer from a vertical plate embedded in a non-Newtonian power law fluid saturated non-Darcy porous medium for aiding and opposing external flows.

Mathematical formulation

A mixed convective heat transfer in a non-Darcy porous medium saturated with a homogeneous non-Newtonian fluid adjacent to a vertical plate, with a uniform wall temperature is considered. This plate constitutes the interface between the liquid phase and the solid phase during melting inside the porous matrix at steady state. The plate is at a
constant temperature $T_m$ at which the material of the porous matrix melts. Fig.1 shows the co-ordinate and the flow model. The x-coordinate is measured along the plate and the y-coordinate normal to it. The solid phase is at temperature $T_0 < T_m$. A thin boundary layer exists close to the right of vertical plate and temperature changes smoothly through this layer from $T_m$ to $T_\infty$ ($T_m < T_\infty$) which is the temperature of the fluid phase.

Taking into account the effect of thermal dispersion the governing equations for steady non-Darcy flow in a non-Newtonian fluid saturated porous medium can be written as follows.

The continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

The momentum equation is

$$\left( \frac{\sigma B_0^2 K}{\rho \nu} + 1 \right) \frac{\partial u^n}{\partial y} + \frac{C\sqrt{K} \rho u^2}{\nu} \frac{\partial u}{\partial y} = \pm \frac{Kg\beta \partial T}{v \partial y} \quad (2)$$

The energy equation is

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left( \alpha \frac{\partial T}{\partial y} \right) - \frac{1}{\rho C_p} \frac{\partial q}{\partial y} \quad (3)$$

In the above equations, the term which represents the buoyancy forced effect on the flow field has ± signs. The plus sign indicates aiding buoyancy flow where as the negative sign stands for buoyancy opposed flow. Here $u$ and $v$ are the velocities along x and y directions respectively, $n$ is the power-law fluid viscosity index, $T$ is Temperature in the thermal boundary layer, $K$ is Permeability, $C$ is Forchheimer empirical constant, $\beta$ is coefficient of thermal expansion, $\nu$ is Kinematics viscosity, $\rho$ is Density, $C_p$ is Specific heat at constant pressure, $g$ is acceleration due to gravity, $\sigma$ is electrical conductivity, $B_0$ is magnetic field intensity and thermal diffusivity $\alpha = \alpha_m + \alpha_d$, where $\alpha_m$ is the molecular diffusivity and $\alpha_d$ is the dispersion thermal diffusivity due to mechanical dispersion. As in the linear model proposed by Plumb[10], the dispersion thermal diffusivity $\alpha_d$ is proportional to the velocity component i.e $\alpha_d = \gamma ud$, where $\gamma$ is the dispersion coefficient and $d$ is the mean particle diameter. The radiative heat flux term $q$ is written using the Rosseland approximation (Sparrow and Cess[11], Raptis[12]) as

\[ q = -\frac{4\sigma_R}{3a} \frac{dT^4}{dy} \]  \hspace{1cm} (4)

Where \( \sigma_R \) is the Stefan - Boltzmann constant and \('a'\) is the mean absorption coefficient.

The physical boundary conditions for the present problem are

\[ y=0, \ T=T_m, \ k \frac{\partial T}{\partial y} = \rho [h_{sf} + C_s(T_m - T_0)]v \]  \hspace{1cm} (5)

and \( y\rightarrow\infty, \ T\rightarrow T_\infty, \ u = u_\infty \)  \hspace{1cm} (6)

Where \( h_{sf} \) and \( C_s \) are latent heat of the solid and specific heat of the solid phases respectively and \( u_\infty \) is the assisting external flow velocity, \( k=\alpha \rho C_p \) is the effective thermal conductivity of the porous medium. The boundary condition (5) means that the temperature on the plate is constant and thermal flux of heat conduction to the melting surface is equal to the sum of the heat of melting and the heat required for raising the temperature of solid to its melting temperature \( T_m \).

Introducing the stream function \( \psi \) with \( u = \frac{\partial \psi}{\partial y} \), and \( v = -\frac{\partial \psi}{\partial x} \)

The continuity equation (1) will be satisfied and the equations (2) and (3) transform to

\[ \left( \frac{\sigma B_0^2 K}{\rho v} + 1 \right) \frac{\partial (\psi^* y)}{\partial y} + \frac{c\sqrt{K}}{v} \frac{\partial (\psi^* y)}{\partial y} \left( \frac{\partial \psi^* y}{\partial y} \right)^2 = \frac{\rho g}{v} \frac{\partial T}{\partial y} \]  \hspace{1cm} (7)

\[ \frac{\partial \psi^* y}{\partial x} \frac{\partial T}{\partial y} - \frac{\partial \psi^* y}{\partial x} \frac{\partial T}{\partial y} = \frac{\partial \psi^* y}{\partial y} \left[ (\alpha_m + \gamma \frac{\partial \psi}{\partial y}) \frac{\partial T}{\partial y} \right] + \frac{4\sigma_R}{3C_P a} \frac{\partial \partial T}{\partial y} \left[ \frac{\partial T^4}{\partial y} \right] \]  \hspace{1cm} (8)

Introducing the similarity variables as \( \psi = f(\eta)(\alpha_m u_\infty x)^{1/2}, \eta = \left( \frac{u_\infty}{\alpha_m} \right)^{1/2} \frac{y}{x}, \theta(\eta) = \frac{T-T_m}{T_\infty-T_m} \), the momentum equation (7) and energy equation (8) are reduced to

\[ n(1+MH) f^{11} f^{11n-1} + 2F f^{11} = \frac{R_{ax}}{P_{ax}} \left( \frac{R_{ax}}{P_{ax}} \right)^n \theta^1 \]  \hspace{1cm} (9)

and

\[ (1 + Df^{11}) \theta^{11} + \left( \frac{1}{2} f + Df^{11} \right) \theta^1 + \frac{4}{3} R \left[ ((\theta + C_r)^3 \theta^{11} + 3\theta^{12} (\theta + C_r)^2) \right] = 0 \]  \hspace{1cm} (10)

Where the prime symbol denotes the differentiation with respect to the similarity variable \( \eta \) and \( MH = \frac{\sigma B_0^2 K}{\rho v} \) is Magneto Hydro
Dynamic parameter, $\frac{R_{ax}}{P_{ex}}$ is the mixed convection parameter, $R_{ax} = \frac{\chi}{a} \left( \frac{g \beta k (T_\infty - T_m)}{v} \right)^{1/2}$ is the local Rayleigh number, $P_{ex} = \frac{u_{ex}}{a}$ is the local Peclet number. $F = f_0 (P_{ed})^{2-n}$, $f_0 = \left( \frac{\alpha}{d} \right)^{2-n} \left( \frac{C a}{v} \right)$ is the non-darcian parameter. $P_{ed}$ is the pore diameter dependent Peclet number. $D = \frac{\gamma d u_{\infty}}{a_m}$ is the dispersion parameter. $C_r = \frac{T_m}{T_\infty - T_m}$ is the temperature ratio, $R = \frac{4 \sigma R (T_\infty - T_m)^{3}}{k a}$ is the radiation parameter.

Taking into consideration, the thermal dispersion effect together with melting, the boundary conditions (5) and (6) take the form

$$\eta = 0, \theta = 0, f(0) + \left\{ 1 + D f'(0) \right\} 2 M \theta'(0) = 0$$

and

$$\eta \to \infty, \theta = 1, f' = 1$$

where $M = \frac{C_r (T_\infty - T_m)}{n_s f + C_s (T_m - T_0)}$ is the melting parameter.

**Solution Procedure**

The dimensionless equations (9) and (10) together with the boundary conditions (11) and (12) are solved numerically by means of the fourth order Runge-Kutta method coupled with double shooting technique. The solution thus obtained is matched with the given values of $f'(\infty)$ and $\theta(0)$. In addition the boundary condition $\eta \to \infty$ is approximated by $\eta_{max} = 8$ which is found sufficiently large for the velocity and temperature to approach the relevant free stream properties. Numerical computations are carried out for $F = 0, 0.5, 1; D = 0, 0.5, 1; R_{a}/P_{e} = 1, 2.5; M = 0, 0.8, 2; n = 0.5, 2.5, R = 0, 0.5, 1; M_{H} = 0, 1, 5, 10; C_r = 0.1$.

**Results and Discussion**

In order to get clear insight on the physics of the problem, a parametric study is performed and the obtained numerical results are displayed with the help of graphical illustrations. The parameters governing the physics of the present study are the melting ($M$), the mixed convection ($R_{a}/P_{e}$), the inertia ($F$), thermal dispersion ($D$), and Fluid viscosity index ($n$), radiation($R$), temperature ratio($C_r$). The numerical computations were carried out for the fixed value of buoyancy parameter $R_{a}/P_{e} = 1$ for both.
the aiding and opposing external flows. The results of the parametric study are shown in figures 2-22.

Figure 2

Figure 2 shows the effect of melting parameter M on velocity profiles for both aiding and opposing external flows. It is observed that different behavior exists between the velocity profiles in the presence of solid phase melting effect in the case of aiding and opposing external flows. The increase in the melting parameter M causes the increase of the velocity profiles for the case of aiding flow. Whereas the opposite behavior for the velocity profiles as M increases is found in the case of opposing flow case.

Figure 3
Figure 4

Figure 3 and Figure 4 show the effect of the melting parameter $M$ on temperature distributions for aiding and opposing external flows respectively. It is observed that as increasing the value of the melting parameter $M$, the temperature distributions decrease for both cases of aiding and opposing external flow conditions.

Figure 5
The effects of thermal dispersion parameter D on the velocity profiles for both cases of aiding and opposing external flow conditions are plotted in Fig. 5. It is noted that for the case of aiding flow the velocity profiles increase with increase in the value of D. But this effect is found opposite in the case of opposing flow.

The effects of thermal dispersion parameter D on temperature distributions for aiding and external flow conditions are plotted in Fig. 6 and Fig. 7 respectively. It is noted that increasing the values of D leads to decrease in the liquid temperature distributions in both cases.
Fig. 8 shows the effects of power law fluid viscosity index $n$ on velocity profiles for both aiding and opposing flow cases. It is observed that in aiding flow case as $n$ value increases, velocity profiles decreases. Whereas opposite behavior obtained in velocity profiles in opposing flow case.

Fig. 9 and Fig. 10 show the effects of power law fluid viscosity index $n$ on temperature distributions for both aiding and opposing flow cases respectively. Significant effect is not found. In aiding flow case as $n$ increases, the temperature distributions decrease. Whereas opposite result is found in opposing flow case.
Fig. 9 and Fig. 10 show the effects of power law fluid viscosity index $n$ on temperature distributions for both aiding and opposing flow cases respectively. Significant effect is not found. In aiding flow case as $n$ increases, the temperature distributions decrease. Whereas opposite result is found in opposing flow case.

Fig. 11 shows the effect of radiation parameter $R$ on velocity profiles for both aiding and opposing external flows. The increase in the radiation parameter $R$ causes the increase of the velocity.
profiles for aiding flow case. Whereas in the case of opposing flow the result is found opposite.

Fig. 12 and Fig. 13 show the effect of radiation parameter R on temperature distributions for aiding and opposing external flow conditions respectively. It is observed that same phenomena exist in both cases. As R increases the temperature distributions decrease for both cases.
Fig. 14 shows the effect of the MHD parameter on velocity profiles for both opposing flow cases. It is observed that in aiding flow cases, the velocity profile increases, whereas in opposite behavior, the velocity profiles decrease.

Fig. 15

Fig. 16
Fig. 15 and Fig. 16 show the effect of Magneto Hydro Dynamic parameter MH on temperature distributions for aiding and opposing external flow conditions respectively. In aiding flow case as MH increases, the temperature distributions decrease. Whereas opposite result is found in opposing flow case.

Fig. 17 shows the effect of Non-Darcy Porous Medium parameter F on velocity profiles for both aiding and opposing flow cases. It is observed that in aiding flow case as F value increases, velocity profiles decreases. Whereas opposite behavior obtained in velocity profiles in opposing flow case.

Fig. 18 and Fig. 19 show the effect of Non-Darcy Porous Medium parameter F on temperature distributions for aiding and opposing external flow conditions respectively. In aiding flow case as F increases, the temperature distributions decrease. Whereas opposite result is found in opposing flow case.

**Figure 18**

**Figure 19**
Fig. 20

Fig. 20 shows the effect of Mixed Convection parameter Ra/Pe on velocity profiles for both aiding and opposing flow cases. It is observed that in aiding flow case as Ra/Pe value increases, velocity profiles increases. Whereas opposite behavior obtained in velocity profiles in opposing flow case.

Figure 21
Fig. 21 and Fig. 22 show the effect of Mixed Convection parameter Ra/Pe on temperature distributions for aiding and opposing external flow conditions respectively. In aiding flow case as Ra/Pe increases, the temperature distributions increase. Whereas opposite result is found in opposing flow case.

References


