CHAPTER 3
Effect of Melting and Thermal Dispersion-Radiation on Mixed Convection Flow from a Vertical Plate Embedded in a Non-Newtonian Fluid Saturated Non-Darcy Porous Medium

3.1 Introduction:

In modern engineering areas, many processes occur at high temperatures and require the knowledge of radiation heat transfer besides the convective heat transfer which becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas.

Recently, Bakier et al.[1] studied Group method analysis of melting effect on MHD mixed convection flow from a radiative vertical plate embedded in saturated porous medium for Newtonian fluids. He developed linear transformation group approach to simulate problem of hydromagnetic heat transfer by mixed convection along with vertical plate in a liquid saturated porous medium in the presence of melting and thermal radiation effects for opposing external flow. He studied the effects of the pertinent parameters on the rate of the heat transfer in terms of the local Nusselt number at the solid-liquid interface. Murthy et al. [2] have studied the combined effect of radiation and mixed convection from a vertical wall with suction/injection in a non-Darcy porous medium. They established that the Nusselt number increases with increase in radiation parameter and also with increase in fluid suction parameter.

More recently, Melting and radiation effects on mixed convection from a vertical surface, embedded in a non-Newtonian fluid saturated non-Darcy porous medium for aiding and opposing external flows is analyzed by Chamkha et al.[3]. They obtained representative flow and heat transfer results for various combinations of physical parameters.

The present chapter aims at analyzing the effect of melting, thermal dispersion and radiation on steady mixed convective heat transfer from a vertical plate embedded in a non-Newtonian power law fluid saturated non-Darcy porous medium for aiding and opposing external flows.
3.2 Mathematical Formulation

A mixed convective heat transfer in a non-Darcy porous medium saturated with a homogeneous non-Newtonian fluid adjacent to a vertical plate, with a uniform wall temperature is considered (as shown in Fig. 2.1).

Taking into account, the effect of thermal dispersion and thermal radiation, the governing equations for steady non-Darcy flow in a non-Newtonian fluid saturated porous medium can be written as follows.

The continuity equation is
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  
(3.1)

The momentum Equation is
\[ \frac{\partial u}{\partial y} - \frac{c_v K \partial u}{u \partial y} = \pm \frac{K \beta}{u \partial y} \frac{\partial T}{\partial y} \]  
(3.2)

The Energy Equation is
\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left( \alpha \frac{\partial T}{\partial y} \right) - \frac{1}{\rho c_p} \frac{\partial q}{\partial y} \]  
(3.3)

In the above equations, the term which represents the buoyancy forced effect on the flow field has \( \mp \) signs. The negative sign indicates aiding buoyancy flow where as the positive sign stands for buoyancy opposed flow. Here \( u \) and \( v \) are the velocities along \( x \) and \( y \) directions respectively, \( n \) is the power-law fluid viscosity index, \( T \) is Temperature in the thermal boundary layer, \( K \) is Permeability, \( C \) is Forchheimer empirical constant, \( \beta \) is coefficient of thermal expansion, \( v \) is Kinematics viscosity, \( \rho \) is Density, \( C_p \) is Specific heat at constant pressure, \( g \) is acceleration due to gravity, and thermal diffusivity \( \alpha = \alpha_m + \alpha_d \), where \( \alpha_m \) is the molecular diffusivity and \( \alpha_d \) is the dispersion thermal diffusivity due to mechanical dispersion. As in the linear model proposed by Plumb [5] the dispersion thermal diffusivity \( \alpha_d \) is proportional to the velocity component i.e \( \alpha_d = \gamma ud \), where \( \gamma \) is the dispersion coefficient and \( d \) is the mean particle diameter. The radiative heat flux term \( q \) is written using the Rosseland approximation (Sparrow and Cess [6], Raptis [7]) as

\[ q = \frac{- \varepsilon \sigma_R}{3a} \frac{\partial T^4}{\partial y} \]  
(3.4)

Where \( \sigma_R \) is the Stefan–Boltzmann constant and \( \varepsilon \) is the mean absorption coefficient

The physical boundary conditions for the present problem are

\[ y=0, T=T_m, k \frac{\partial T}{\partial y} = \rho [h_{st} + C_s (T_m - T_0)] v \]  
(3.5)
and \( y \rightarrow \infty, T \rightarrow T_\infty, u = u_\infty \) \hspace{1cm} (3.6)

Where \( h_s \) and \( C_s \) are latent heat of the solid and specific heat of the solid phases respectively and \( u_\infty \) is the assisting external flow velocity, \( k = \alpha \rho C_p \) is the effective thermal conductivity of the porous medium. The boundary condition (3.5) means that the temperature on the plate is constant and thermal flux of heat conduction to the melting surface is equal to the sum of the heat of melting and the heat required for raising the temperature of solid to its melting temperature \( T_m \).

Introducing the stream function \( \psi \) with \( u = \frac{\partial \psi}{\partial y} \) and \( v = -\frac{\partial \psi}{\partial x} \).

The continuity equation (3.1) will be satisfied and the equations (3.2) and (3.3) transform to

\[
\frac{\partial}{\partial y} \left( \frac{\partial \psi}{\partial y} \right)^n + \frac{C\sqrt{K}}{v} \frac{\partial}{\partial y} \left( \frac{\partial \psi}{\partial y} \right)^2 = \mp \frac{Kg\beta}{v} \frac{\partial T}{\partial y} \hspace{1cm} (3.7)
\]

\[
\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left[ \sigma_m + \gamma \frac{\partial \psi}{\partial y} \right] \frac{\partial T}{\partial y} + \frac{4 \sigma R}{3p C_p a} \frac{\partial \left( \frac{\partial T}{\partial y} \right)}{\partial y} \hspace{1cm} (3.8)
\]

Introducing the similarity variables as

\[
\Psi = f(\eta) (\alpha_m u_\infty x)^{1/2}, \quad \eta = \left( \frac{u_\infty x}{\alpha_m} \right)^{1/2} \left( \frac{y}{x} \right), \quad \theta(\eta) = \frac{T - T_m}{T_\infty - T_m}, \quad \text{the momentum equation (3.7) and energy equation (3.8) are reduced to}
\]

\[
nf^{11} f^{n-1} + 2ff_{11} = \mp \left( \frac{Ra_x}{Pe_x} \right)^n \theta^1 \hspace{1cm} (3.9)
\]

\[
(1+Df^1) \theta_{11} + \left( \frac{1}{2} f + Df_{11} \right) \theta^1 + 4 \frac{R}{3} \theta^1 \left[ (\theta + C_r)^3 - 3 \theta^2 (\theta + C_r)^2 \right] = 0 \hspace{1cm} (3.10)
\]

Where the prime symbol denotes the differentiation with respect to the similarity variable \( \eta \) and \( Ra_x/Pe_x \) is the mixed convection parameter,

\[
Ra_x = \frac{x}{a} \left( \frac{g \beta (T_\infty - T_m)}{\nu} \right)^{1/n} \quad \text{is the local Rayleigh number}, \quad Pe_x = \frac{u_\infty x}{\alpha} \quad \text{is the local Peclet number}.
\]

\[
F = f_0 (Pe_d)^2, \quad f_0 = \left( \frac{\alpha}{d} \right)^{2-n} \left( \frac{C\sqrt{K}}{v} \right) \quad \text{is the non-darcian parameter}, \quad Pe_d \quad \text{is the pore diameter dependent Peclet number}.
\]

\[
D = \frac{\gamma du_\infty}{\alpha_m} \quad \text{is the dispersion parameter}, \quad C_r = \frac{C_m T_m}{T_\infty - T_m} \quad \text{is the temperature ratio parameter}, \quad R = \frac{4 \sigma R (T_\infty - T_m)^3}{ka} \quad \text{is the radiative parameter}.
\]
Taking into consideration, the thermal dispersion effect together with melting, the boundary conditions (3.5) and (3.6) take the form

\[ n=0, \theta=0, f(0)+\{1+Df'(0)\}2M\theta'(0)=0. \] (3.11)

and \( n \to \infty, \theta=1, f'=1. \) (3.12)

where \( M = \frac{c_f(T_m-T_0)}{h_s f + c_s (T_m-T_\infty)} \) is the melting parameter.

The local heat transfer rate from the surface of the plane is given by

\[ q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0}. \]

The Nusselt number is

\[ \text{Nu} = \frac{hx}{k} = \frac{q_w x}{k(T_m-T_\infty)}, \]

Where \( h \) is the local heat transfer coefficient and \( k \) is the effective thermal conductivity of the porous medium, which is the sum of the molecular thermal conductivity \( k_m \) and the dispersion thermal conductivity \( k_d \) [4].

The modified Nusselt number is obtained as

\[ \frac{N_{h_\text{fix}}}{(Pe_\text{c})^{1/2}} = [1+\frac{4}{3} R(\theta(\infty)+C_r)^3 + D f'(0)]\theta'(0) \] (3.13)

### 3.3 Solution Procedure:

The dimensionless equations eq.(3.9) and eq.(3.10) together with the boundary conditions (3.11) and (3.12) are solved numerically by means of the fourth order Runge-Kutta method coupled with double shooting technique. The solution thus obtained is matched with the given values \( f'(\infty) \) and \( \theta(0) \). In addition the boundary condition \( n \to \infty \) is approximated by \( n_{\text{max}} = 3 \) which is found sufficiently large for the velocity and temperature to approach the relevant free stream properties. Numerical computations are carried out for \( F = 0, 0.5, 1; D = 0, 0.5, 1; \text{Ra/Pe}=1, 2, 3, 5; M = 0, 0.8, 2; n = 0.5, 1, 1.5; R = 0, 0.5, 1; \text{C}_r = 0.1, 0.5, 1. \)

### 3.4 Results and Discussion:

In order to get clear insight on the physics of the problem, a parametric study is performed and the obtained numerical results are displayed with the help of graphical illustrations. The parameters governing the physics of the present study are the melting (M), the mixed convection (Ra/Pe), the inertia (F), thermal dispersion (D),
fluid viscosity index $n$, radiation ($R$) and temperature ratio parameter ($C_r$). The numerical computations are carried out for the fixed value of buoyancy parameter $\frac{Ra}{Pe} = 1$ for both the aiding and opposing external flows. The results of the parametric study are shown in figures 3.1.1-3.20.

Fig.3.1.1 The effect of melting parameter on velocity distribution for $n=0.5$.

Fig.3.1.2 The effect of melting parameter on velocity distribution for $n=1$. 

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The effect of melting with dispersion and radiation over the velocity field across the boundary layer in both aiding and apposing flows is illustrated in figures 3.1.1, 3.1.2 and 3.1.3 for different values of \( n = 0.5, 1, 1.5 \) respectively for a given non-Darcy medium fixing \( F = 1 \), for fixed \( \frac{Ra}{Pe} = 1 \). From the figures, it is clear that in aiding flow the velocity is increasing due to increase in melting parameter for different values of \( n = 0.5, 1, 1.5 \). In the opposing flow, the velocity is seen to decrease with the increase of the melting parameter for different values of \( n = 0.5, 1, 1.5 \).
Fig.3.2.1 The effect of thermal dispersion parameter on velocity distribution for $n=0.5$.

Fig.3.2.2 The effect of thermal dispersion parameter on velocity distribution for $n=1$. 

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Figures 3.2.1, 3.2.2 and 3.2.3 show the dependence of dimensionless flow velocity distribution on thermal dispersion with melting and radiation for a fixed inertia (F=1) for different values of \( n = 0.5, 1, 1.5 \) respectively in both aiding and opposing flows. The figures show that the velocity increases with the increase in the dispersion in aiding flow for different values of \( n = 0.5, 1, 1.5 \). In the opposing flow the velocity decreases with the increase in the thermal dispersion for different values of \( n = 0.5, 1, 1.5 \).
Fig.3.3.1 The effect of radiation parameter on velocity distribution for $n=0.5$.

Fig.3.3.2 The effect of radiation parameter on velocity distribution for $n=1$. 
In Figures 3.3.1, 3.3.2 and 3.3.3 the effect of radiation in the presence of thermal dispersion on the velocity profiles for $n = 0.5, 1$ and $1.5$ is presented respectively in both aiding and opposing flows. In aiding flow, increase in radiation increases the velocity of the fluid for different values of $n=0.5, 1$ and $1.5$. From the figures, it is also noticed that the velocity of the fluid is not influenced by the radiation near the wall. The effect is found opposite in opposing flow.
Fig. 3.4.1. The variation in $f'(\eta)$ with $\eta$ for different values of inertia parameter for $n=0.5$.

Fig. 3.4.2. The variation in $f'(\eta)$ with $\eta$ for different values of inertia parameter for $n=1$. 
In Figures 3.4.1, 3.4.2 and 3.4.3 the effect of inertia with melting, thermal dispersion and radiation on velocity distribution in aiding flow is presented for \( n = 0.5, 1 \) and 1.5 respectively. It is observed that the increase in inertia – from Darcian to non-Darcian – decreases the velocity of the fluid. It is also observed from the figures that as the fluid viscosity index increases, the velocity decreases at a fixed inertia value.

Fig.3.4.3. The variation in \( f'(\eta) \) with \( \eta \) for different values of inertia parameter for \( n = 1.5 \).
Fig. 3.4.4. The variation in $f'(\eta)$ with $\eta$ for different values of inertia parameter for $n=0.5$.

Fig. 3.4.5. The variation in $f'(\eta)$ with $\eta$ for different values of inertia parameter for $n=1$. 
In Figures 3.4.4, 3.4.5 and 3.4.6 the effect of inertia with melting, thermal dispersion and radiation on velocity distribution in opposing flow is presented for \( n = 0.5, 1 \) and 1.5 respectively. It is observed that the increase in inertia – from Darcian to non-Darcian – increases the velocity of the fluid. It is also observed from the figures that as the fluid viscosity index increases, the velocity increases at a fixed inertia value.
Fig. 3.5.1 The effect of temperature ratio parameter on velocity distribution for n=0.5.

Fig. 3.5.2 The effect of temperature ratio parameter on velocity distribution for n=1.
In figures 3.5.1, 3.5.2 and 3.5.3 the effect of temperature ratio parameter Cr in the presence of melting, thermal dispersion and radiation on velocity distribution for both aiding and opposing flows is presented for $n=0.5$, 1 and 1.5 respectively. It is noted that increase in temperature ratio parameter increases the velocity of the fluid in aiding flow, and the velocity decreases with increase in temperature ratio parameter in opposing flow for different values of $n$. It is also observed from the figures that the temperature ratio parameter influences the velocity profiles significantly at far away from the wall.
Fig. 3.6.1 Variation of $f^i(\eta)$ with $\eta$ for different values of mixed convection parameter $\frac{Ra}{Pe}$ for $n=0.5$.

Fig. 3.6.2 Variation of $f^i(\eta)$ with $\eta$ for different values of mixed convection parameter $\frac{Ra}{Pe}$ for $n=1$. 

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Figures 3.6.1, 3.6.2 and 3.6.3 show the variation of dimensionless flow velocity for fixed melting strength (M), inertia strength (F), radiation (R) and thermal dispersion (D) for various values of \( \frac{Ra}{Pe} \) in its range for both aiding and opposing flows for different values of \( n = 0.5, 1 \) and 1.5 respectively. The figures show that velocity increases with the increase in mixed convection parameter in aiding flow. In opposing flow, the velocity decreases with the increase in mixed convection parameter for all values of \( n \).
Figure 3.7 shows the variation of dimensionless flow velocity for the non-Darcian medium characterized by the value of the inertial coefficient $F=0.5$ in the presence of radiation and thermal dispersion with varying values of power law fluid viscosity index $n$ in its range both in aiding and opposing flows fixing $\frac{Ra}{Pe}=1$. The figure shows that velocity decreases with increase in fluid viscosity index in aiding flow. In opposing flow, the velocity increases with the increase in fluid viscosity index.

Fig. 3.7 The effect of viscosity index on velocity distribution in aiding and opposing flows.
Fig. 3.8.1 The effect of melting on temperature distribution for $n=0.5$.

Fig. 3.8.2 The effect of melting on temperature distribution for $n=1$. 
Figures 3.8.1, 3.8.2 and 3.8.3 show the melting effect on the temperature distribution in the presence of radiation and thermal dispersion for aiding flow for different values of \( n = 0.5, 1 \) and 1.5 respectively. It is observed that the temperature decreases with the increase in melting parameter and the variation in the temperature is similar to all values of \( n \) in aiding flow.
Fig. 3.8.4 The effect of melting on temperature distribution for \( n=0.5 \).

Fig. 3.8.5 The effect of melting on temperature distribution for \( n=1 \).
Fig. 3.8.6 The effect of melting on temperature distribution for $n=1.5$.

Figures 3.8.4, 3.8.5 and 3.8.6 show the melting effect on the temperature distribution in the presence of radiation and thermal dispersion for opposing flow for different values of $n=0.5$, 1 and 1.5 respectively. It is observed that the temperature decreases with increase in melting parameter and the variations in the temperature are similar to different values of $n$ in opposing flow.
Fig. 3.9.1 The effect of thermal dispersion on temperature distribution for \( n = 0.5 \).

Fig. 3.9.2 The effect of thermal dispersion on temperature distribution for \( n = 1 \).
Figures 3.9.1, 3.9.2 and 3.9.3 show the thermal dispersion effect in the presence of radiation on the temperature distribution for aiding flow for different power law fluid viscosity index values $n=0.5$, 1 and 1.5 respectively. It is observed that the temperature decreases with the increase in thermal dispersion parameter for all power law fluid viscosity index values in aiding flow and the variations in the temperature are similar.
Opposing external flow

\[ F=1, n=0.5, M=2, R=0.5, \text{Ra}/\text{Pe}=1 \]

\[ \text{Temperature, } \theta \]

Fig. 3.9.4 The effect of thermal dispersion on temperature distribution for \( n=0.5 \).

\[ F=1, n=1, M=2, R=0.5, \text{Cr}=0.5, \text{Ra}/\text{Pe}=1 \]

\[ \text{Temperature, } \theta \]

Fig. 3.9.5 The effect of thermal dispersion on temperature distribution for \( n=1 \).
Figures 3.9.4, 3.9.5 and 3.9.6 show the thermal dispersion effect in the presence of radiation on the temperature distribution for opposing flow for different power law fluid viscosity index values $n=0.5$, 1 and 1.5 respectively. It is observed that the temperature decreases with the increase in thermal dispersion parameter for all power law fluid viscosity index values in aiding flow. Comparing with aiding flow, the decrease in the opposing flow is less.
Fig. 3.10.1 The effect of thermal radiation on temperature distribution for $n=0.5$.

Fig. 3.10.2 The effect of thermal radiation on temperature distribution for $n=1$. 
Figures 3.10.1, 3.10.2 and 3.10.3 show the thermal radiation effect in the presence of dispersion on the temperature distribution for aiding flow for different power law fluid viscosity index values \( n = 0.5, 1 \) and 1.5 respectively. It is observed that the temperature decreases with the increase in the thermal radiation parameter for all power law fluid viscosity index values in aiding flow. From the figures, it is noted that as \( n \) value increases the decrease in the temperature is less. It is also observed that the effect of radiation on the temperature near the wall is negligible.

Fig.3.10.3 The effect of thermal radiation on temperature distribution for \( n = 1.5 \).
Fig. 3.10.4 The effect of thermal radiation on temperature distribution for $n=0.5$.

Fig. 3.10.5 The effect of thermal radiation on temperature distribution for $n=1$. 
Figures 3.10.4, 3.10.5 and 3.10.6 show the thermal radiation effect in the presence of dispersion on the temperature distribution for opposing flow for different power law fluid viscosity index values \( n = 0.5, 1 \) and 1.5 respectively. It is observed that the temperature decreases with the increase in thermal radiation parameter for all power law fluid viscosity index values in opposing flow. From the figures, it is also noted that as \( n \) value increases, the decrease in the temperature is less. It is also observed that near the wall the temperature distribution is not influenced by the radiation.
Fig. 3.11.1 The effect of inertia (F) on temperature distribution for \( n=0.5 \).

Fig. 3.11.2 The effect of inertia (F) on temperature distribution for \( n=1 \).
In figures 3.11.1, 3.11.2 and 3.11.3 the effect of inertia in the presence of thermal dispersion-radiation on temperature distribution is presented in aiding flow for different values of power law fluid viscosity index $n=0.5$, 1 and 1.5 respectively. In aiding flow, the increase in inertia from Darcian to non-Darcian decreases the temperature of the fluid for all fluid viscosity index values, but the decrease is less in non-Darcy conditions. It is also observed from the figures that as the thickness of fluid increases, the inertia parameter do not contribute much on the temperature.
Fig. 3.11.4 The effect of inertia ($F$) on temperature distribution for $n=0.5$.

Fig. 3.11.5 The effect of inertia ($F$) on temperature distribution for $n=1$. 
In figures 3.11.4, 3.11.5 and 3.11.6 the effect of inertia in the presence of thermal dispersion-radiation on temperature distribution is presented in opposing flow for different values of power law fluid viscosity index $n=0.5$, 1 and 1.5 respectively. In opposing flow, the increase in inertia from Darcian to non-Darcian contributes to the raise in the temperature of the fluid for all fluid viscosity index values, but the increase is less in non-Darcy conditions. Moreover, it is observed that the effect is found to be negligible as we move from thin fluid to thick fluid.
Fig. 3.12.1 The effect of temperature ratio parameter on temperature distribution for $n=0.5$.

Fig. 3.12.2 The effect of temperature ratio parameter on temperature distribution for $n=1$. 

Aiding external flow

$$F=1, n=0.5, M=2, D=1, R=1, Ra/Pe=1$$

$$F=1, n=1, M=2, D=1, R=1, Ra/Pe=1$$
Figures 3.12.1, 3.12.2 and 3.12.3 illustrate the effect of temperature ratio parameter in the presence of thermal dispersion-radiation on the temperature distribution for aiding flow for different power law fluid viscosity index values $n=0.5$, $1$ and $1.5$ respectively. It is observed that the temperature decreases with the increase in temperature ratio parameter for all power law fluid viscosity index values in aiding flow. However, the effect is found to be zero as we move towards the wall of the plate.
Fig. 3.12.4 The effect of temperature ratio parameter on temperature distribution flow for n=0.5.

Fig. 3.12.5 The effect of temperature ratio parameter on temperature distribution flow for n=1.
Fig. 3.12.6 The effect of temperature ratio parameter on temperature distribution flow for $n=1.5$.

Figures 3.12.4, 3.12.5 and 3.12.6 show the effect of temperature ratio parameter in the presence of thermal dispersion-radiation on the temperature distribution for opposing flow for different power law fluid viscosity index values $n=0.5, 1$ and $1.5$ respectively. It is observed that as $C_r$ value increases, the temperature drops down for all power law fluid viscosity index values in opposing flow. Though not much of significant variation is seen near the wall, the variation is found to be sufficiently large as we move away from the plate.
Fig. 3.13.1 The effect of mixed convection parameter (Ra/Pe) on temperature distribution for \( n=0.5 \).

Fig. 3.13.2 The effect of mixed convection parameter (Ra/Pe) on temperature distribution for \( n=1 \).
Fig. 3.13.3 The effect of mixed convection parameter (Ra/Pe) on temperature distribution for n=1.5.

In figures 3.13.1, 3.13.2 and 3.13.3, the effect of mixed convection parameter in the presence of thermal dispersion-radiation on temperature distribution is presented in aiding flow for different values of power law fluid viscosity index n=0.5, 1 and 1.5 respectively. In aiding flow, the increase in the value of \( \frac{Ra}{Pe} \) increases the temperature of the fluid. Such an increment is found to be more significant in thick fluids than in thin fluids.
Fig. 3.13.4 The effect of mixed convection parameter ($Ra/Pe$) on temperature distribution for $n=0.5$.

Fig. 3.13.5 The effect of mixed convection parameter ($Ra/Pe$) on temperature distribution for $n=1$. 
In figures 3.13.4, 3.13.5 and 3.13.6 the effect of mixed convection in the presence of thermal dispersion-radiation on temperature distribution is presented in opposing flow for different values of power law fluid viscosity index $n=0.5$, 1 and 1.5 respectively. In opposing flow, the increase in the value of $\frac{Ra}{Pe}$ decreases the temperature of the fluid for all values of fluid viscosity index. Such a drop is found to be more significant as the thickness of the fluid increasing.
In figures 3.14.1 and 3.14.2 the effect of viscosity index on temperature distribution in the presence of thermal dispersion-radiation is presented in aiding and opposing flow cases respectively. In both flow cases, the variation of the temperature is negligible. However in aiding flow the temperature of the fluid slightly decreases with the increase of viscosity index and in opposing flow it slightly increases.
The effect of melting strength and thermal dispersion on heat transfer rate in the presence of radiation is illustrated in Figure 3.15 for both aiding and opposing flows, in terms of the Nusselt number defined in Eq.(3.13). This shows that Nusselt number decreases significantly with the increase in melting strength (M) for both flow cases. Further, it is seen that the Nusselt number increases with the increase in thermal dispersion parameter for both flow cases. Also, it is seen that the heat transfer rate is more significant in aiding flow comparing with opposing flow as the value of dispersion parameter increases.
The effect of melting strength and thermal radiation on heat transfer rate in the presence of dispersion is shown in Figure 3.16 for both aiding and opposing flows, in terms of the Nusselt number defined in Eq.(3.13). This shows that Nusselt number decreases significantly with the increase in melting strength (M) for both flow cases. Further, it is seen that the Nusselt number increases with increase in thermal radiation for both flow cases.
Effect of melting strength and inertia on heat transfer rate has been examined in figure 3.17 with thermal dispersion-radiation for both aiding and opposing flow cases in terms of the Nusselt number defined in Eq. (3.13). It is clear that the Nusselt number is decreasing with the increase in inertia effect in aiding flow. Whereas in the opposing flow the Nusselt number leads to increase with the increase of inertia. Further, it is observed that as the melting parameter increases, the Nusselt number decreases significantly in both flow cases.
The effect of melting strength and temperature ratio parameter on heat transfer rate is shown in Figures 3.18.1 and 3.18.2 for aiding and opposing flows respectively in terms of the Nusselt number defined in Eq.(3.13). This shows that Nusselt number decreases significantly with the increase in melting strength (M) for both flow cases. Further, it is noted that the increase in temperature ratio parameter leads to increase in Nusselt number in both flow cases.
The effect of melting strength and fluid viscosity index in the presence of thermal dispersion-radiation on heat transfer rate for aiding and opposing flows in terms of the Nusselt number defined in Eq.(3.13) is noticed in Fig. 3.19. It is observed that the Nusselt number is decreasing with the increase in the value of viscosity index in aiding flow. This indicates that pseudo plastic fluids ($n < 1$) are associated with higher heat transfer rates when compared to dilatants fluids ($n > 1$) in aiding flow. But, in opposing flow the Nusselt number increases with the increase of fluid viscosity index. Further the Nusselt number decreases significantly with increasing melting strength ($M$) for both flow cases.
The effect of melting strength and mixed convection in the presence of thermal dispersion-radiation on heat transfer rate is shown in figure 3.20 in terms of Nusselt number defined in equation (3.13) for both aiding and opposing flows. It is noticed that Nusselt number decreases significantly with the increasing melting strength (M). Further it is seen that in aiding flow as $\frac{Ra}{Pe}$ increases, the Nusselt number increases. But, in opposing flow the Nusselt number is found to decrease as $\frac{Ra}{Pe}$ increases.
3.5 Conclusions:

In the present chapter, the effect of melting on mixed convection flow from a vertical plate embedded in a Non-Newtonian fluid saturated non-Darcy porous medium in the presence of thermal dispersion - radiation is analyzed for different values of fluid viscosity index in aiding and opposing flow cases. The obtained non linear differential equations are solved by 4th order Runge-Kutta method coupled with shooting technique. The results are presented graphically. It is observed that the velocity of the fluid increases / decreases with the increase in the parameter values of melting, thermal dispersion, thermal radiation, temperature ratio parameter and mixed convection in aiding / opposing flow for all fluid viscosity index values. It is also noted that the velocity decreases / increases with the increase in the parameter values of inertia and fluid viscosity index in aiding / opposing flow.

However, the temperature of the fluid decreases with the increase in the parameter values of melting, thermal dispersion, radiation and temperature ratio parameter in both aiding and opposing flow cases. In aiding flow, from Darcian to non-Darcian the temperature decreases but the variations are less as fluid viscosity index increases. In opposing flow, the temperature increases from Darcian to non-Darcian and the variations are less as the thickness of the fluid increases. Also the temperature increases / decreases with the increase in the value of mixed convection parameter in aiding / opposing flow.

Further, it is observed that the heat transfer rate increases with the increase of dispersion, radiation and temperature ratio parameter in both the flow cases. Furthermore, it is noticed that the Nusselt number decreases / increases with the increase in the parameter values of inertia and fluid viscosity index in aiding / opposing flow. The Nusselt number decreases significantly with the increase of melting parameter in both the flow cases. The heat transfer rate increases / decreases with the increase of mixed convection parameter value in aiding / opposing flow.
3.6. References:


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