Chapter 4

Feature Extraction in Spatially Extended Reaction-Diffusion-Convective Systems

A widely studied class of complex systems are the spatially extended systems where the methodologies for analysis of one point time-series has to be built-up to incorporate the spatial extent of the system. These systems are described by partial differential equations involving nonlinearities along with diffusive and/or convective coupling and can show intricate and complex patterns including turbulence [Cro93]. The most prominent examples are autocatalytic reaction-diffusion in chemical systems [Gra90], Navier-Stokes equation of hydrodynamics [Hol96], complex Ginzburg-Landau equation [Kur84], heterogeneous catalytic reaction on Pt catalyst [Bar94], etc. Analysis of the dynamics of these systems is not an easy task because of the large attractor dimensions involved and extension of approaches designed for low dimensional systems to high dimensional cases is not straightforward [Kan97]. For this reason, identification of pattern forming mechanisms and their characterization techniques is a contemporary research topic. In fact, it would be desirable to develop ways of studying spatiotemporal systems using reduced model descriptions in terms of fewer degrees of freedom. The method can be used in conjunction with subsystem dynamics especially
when it is known that extensive scaling relationships in dynamics exist as a function of subsystem size [Car99, Par98]. The Karhunen-Loève decomposition captures the most energetic coherent structures by projecting the spatiotemporal data on an optimum set of empirical basis functions. Thus it is possible to condense complex spatiotemporal data in terms of few significant eigenfunctions leading to appreciable degree of data reduction in many cases. Here, the methodology has been developed for coupled map lattices (CML), a discrete prototype that possesses the basic reaction-diffusion and convection mechanisms that give rise to complex patterns including spatiotemporal chaos and convective turbulence [Kan93], and suitably extended to systems governed by partial differential equations having one or two spatial dimensions. Moreover, it will be illustrated that given such a set of empirical eigenfunctions the method can give an unbiased estimate of marred (i.e. partially masked images as appears in cloud covered satellite images) spatiotemporal data in a reasonable manner.

The organization of the Chapter is as follows: In Section 4.1 we exemplify the Karhunen-Loève decomposition and demonstrate the methodology in Section 4.2 for a discrete system (Section 4.2.1), and continuous systems in one and two spatial dimensions in Section 4.2.2 and Section 4.2.3 respectively. We further extend the KLD procedure for analyzing image processing and show the application in Section 4.3.

4.1 Karhunen-Loève Decomposition

The Karhunen-Loève decomposition (KLD) provides a basis of ensemble of functions for decomposing data collected in course of experiments. The superiority of such basis functions lie in the fact that they are optimal and an infinite-dimensional process can be represented in terms of very few “modes”. KLD, originally introduced by Karhunen, Loève, is popular in various fields and forms the basis for the methods like proper orthogonal decomposition, principal component analysis. KLD can be
used to analyze experimental data in order to extract dominant coherent structures and in case of spatiotemporal fields to study patterns in space and time. Here, the emphasis is on identifying key spatial features of spatiotemporal data forming patterns and obtaining a low-dimensional accurate dynamical model of the spatially extended system. The starting point for KLD is in obtaining the fluctuating components \( v(t, x) \) by subtracting the time-averaged mean from the field variables \( u(t, x) \),

\[
v(t, x) = u(t, x) - \lim_{T \to \infty} \frac{1}{T} \int_0^T u(t, x) dt.
\]

The general idea of KLD is to obtain a separable series expansion scheme for \( v(t, x) \) in the form,

\[
v(t, x) = \sum_{k=1}^{\infty} a_k(t) \phi_k(x),
\]

where by truncating the index \( k \) to a suitable order it is possible to achieve reduction with required accuracy. Here \( \phi(x) \) are the coherent structures present in the system and \( a(t) \) are time dependent coefficients. Keeping in mind the fact that experimental measurements yield only a discrete and finite set of data, from now on \( v(t, x) \) will be represented as an ensemble of snapshots \( \{v_n(x)\} \) where \( n = 1, 2, \cdots, M \), \( M \) is the total number of snapshots. To get efficient reduction, one projects \( v_n(x) \) onto an optimal subspace which captures maximum information of the system. \( \phi \)s are orthonormal empirical basis functions which contain maximum projection of \( v_n(x) \) and maximizes

\[
\lambda \equiv \langle (v_n, \phi)^2 \rangle.
\]

Here, \( \langle \cdot \rangle \) represents averaging while \( (, ) \) is the inner product in infinite dimensional Hilbert space

\[
(f, g) = \int f(x) g^*(x) dx
\]

Then extremising \( \langle (v, \phi)^2 \rangle \), subjected to the constraint \( ||\phi||^2 = 1 \), can be treated as a problem of calculus of variations and let the functional be

\[
J[\phi] = \langle (v, \phi)^2 \rangle - \lambda(||\phi||^2 - 1)
\]
and a necessary condition for extrema is that the functional derivative vanish for all variations \( \phi + \delta \psi \), i.e.,

\[
\frac{d}{d\delta} J[\phi + \delta \psi]|_{\delta = 0} = 0
\]  

(4.6)

From Eq. (4.5) and (4.6) we have,

\[
\frac{d}{d\delta} J[\phi + \delta \psi]|_{\delta = 0} = \frac{d}{d\delta} [[(u, \phi + \delta \psi)(\phi + \delta \psi, u)] - \lambda(\phi + \delta \psi, \phi + \delta \psi)]|_{\delta = 0}
= 2\text{Re}[(u, \psi)(\phi, u)] - \lambda(\phi, \psi) = 0
\]  

(4.7)

where, we use the property \((f, g) = (g, f)^*\) Since, \(\psi(x)\) is an arbitrary variation and noting the commutativity of \(\cdot\) and \(\int \cdot dx\), our condition reduces to

\[
\int_0^1 \langle u(x)u^*(x')\rangle \phi(x')dx' = \lambda \phi(x).
\]  

(4.8)

Thus the optimal basis is given by the eigenfunctions \(\{\phi_j\}\) of the integral equation Eq. (4.8) whose kernel is the averaged autocorrelation function \(\langle u(x)u^*(x')\rangle = R(x, x')\). The above optimization problem is equivalent of solving the eigenvalue problem for the spatial correlation matrix \(R(x, x')\)

\[
\int R(x, x')\phi(x')dx' = \lambda \phi(x).
\]  

(4.9)

Eigenvalue spectrum \(\{\lambda_i\}\) determine the importance of basis functions and eigenfunctions corresponding to the largest eigenvalue is the most important and so on. A factor may be defined as

\[
\eta_N = \frac{\sum_{i=1}^N \lambda_i}{\sum_{k=1}^M \lambda_k}
\]  

(4.10)

to quantify the energy content for increasing mode index \(k\) and decides an index \(N < M\) for which the series in Eq. 4.2 may be truncated. From a practical point of view most often one deals with a very large spatial domain for \(x\), and computation
of $R(x, x')$ becomes quite an involved procedure. The method of snapshots enables to
get over this practical difficulty [Sir87]. Eq.(4.9) gives us

$$\int R(x, x')\phi(x')dx' = \frac{1}{M} \sum_{n=1}^{M} v_n(x) \int v_n(x')\phi(x')dx' = \sum_{n=1}^{M} \gamma_n v_n(x).$$ (4.11)

Thus the span of the eigenspace of $R$ is restricted to $M$ dimensional space. Combining
Eq.(4.9) and (4.11) empirical eigenfunctions can be written as

$$\phi(x) = \sum_{n=1}^{M} \Gamma_n v_n(x),$$ (4.12)

for some constants $\Gamma_1, \Gamma_2, \ldots, \Gamma_M$ and Eq.(4.9) takes the form

$$C\Gamma = \lambda \Gamma$$ (4.13)

where,

$$C = \frac{1}{M} \int v_n(x)v_m(x')dx'.$$ (4.14)

Thus the computation of eigenvalues and eigenfunctions of a spatial correlation matrix
can be replaced by calculating that from the matrix $C$ which has a dimension $(M \times M)$
and is independent of the spatial extent. It is to be noted that $C$ and $R$ are not
identical even though they have same eigenvalues.

4.2 Illustrative Examples

4.2.1 Discrete System: Coupled Map Lattice

Because of their computational simplicity, CMLs are a popular and convenient paradigm
for studying fully developed turbulence [Kan93, Wil95, Hil99], chaos [Kan89], and
pattern formation [Cro93] in spatially extended systems. A CML model is a discrete
space-time system with continuous state space and studies the effects of local nonlin-
ear reaction dynamics, the coupling arising from diffusion due to state space gradients
as well as convective effects by asymmetric coupling [Wil95]. Here, we consider a CML involving a single spatial dimension and incorporating these mechanisms as

\[
    u(n+1,j) = (1 - D_d - D_c)f(u(n,j)) + D_c f(u(n,j - 1)) \\
    + \frac{D_d}{2} [f(u(n,j + 1)) + f(u(n,j - 1))],
\]  

(4.15)

where, \( u(n,j), j = 1, 2, \ldots, L \) is the state of the variable located at site \( j \) at time \( n \) for a lattice of size \( L \), \( D_d \) the nearest neighbor diffusive coupling strength and \( D_c \) denoting the asymmetric coupling constant. For \( D_c = 0 \) the system represents a reaction-diffusion system while for \( D_c \neq 0 \) mimics one with convective effects included. We assume the reaction dynamics on the lattice sites is governed by the nonlinear logistic function \( f(u) = 1 - Fu^2 \), where, \( F \) is the nonlinearity parameter. Thus, depending on the parameter values for \( F, D_d \) and \( D_c \), a variety of dynamical patterns may be observed in Eq. (4.15) and characterized as in [Wil95]. We bring out the methodology for Karhunen-Loève decomposition for selected dynamics covering a broad range of complexity, \( \text{viz.,} \)

- weak chaos
- traveling wave
- fully developed chaos
- convective turbulence

Spatiotemporal data for the different cases are obtained by evolving Eq. (4.15). All the sites are given random initial conditions at \( n = 0 \) and snapshots are stored after eliminating initial transients. Cases \( (a,b,c) \) are evolved with periodic boundary conditions, \( \text{i.e.,} \ u(n,1) = u(n,L) \) while for the convective case \( (d) \) the left boundary is assumed fixed, \( \text{i.e.,} \ u(n,1) = 1, \) with the right boundary open. The gray-scale
Figure 4.1: Evolved spatiotemporal data $u(n,j)$ for the CML ($j$ spatial grid with $L = 60; M = 20$ snapshots. (a) Weak chaos ($F = 1.73, D_d = 0.4, D_c = 0.0$); (b) Traveling wave ($F = 1.5, D_d = 0.5, D_c = 0.0$); (c) Fully developed chaos ($F = 2.0, D_d = 0.4, D_c = 0.0$); (d) Convective turbulence ($F = 2.0, D_d = 0.4, D_c = 0.3$).
Table 4.1: Significance of KL modes in CML.

<table>
<thead>
<tr>
<th>case</th>
<th>Mode no.</th>
<th>$\lambda_k$</th>
<th>$\eta_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Weakly chaotic</td>
<td>1</td>
<td>10.5467</td>
<td>0.9408</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.3639</td>
<td>0.9733</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.2862</td>
<td>0.9988</td>
</tr>
<tr>
<td>(b) Traveling wave</td>
<td>1</td>
<td>13.2207</td>
<td>0.9337</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.3712</td>
<td>0.9600</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.1098</td>
<td>0.9957</td>
</tr>
<tr>
<td>(c) Fully chaotic</td>
<td>1</td>
<td>3.8130</td>
<td>0.2690</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.0661</td>
<td>0.9912</td>
</tr>
<tr>
<td></td>
<td>19</td>
<td>0.0217</td>
<td>0.9999</td>
</tr>
<tr>
<td>(d) Convective turbulence</td>
<td>1</td>
<td>4.0071</td>
<td>0.2950</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.0458</td>
<td>0.9929</td>
</tr>
<tr>
<td></td>
<td>19</td>
<td>0.0106</td>
<td>0.9999</td>
</tr>
</tbody>
</table>

images of the spatiotemporal data along with the parameter values yielding the data for a lattice size of $L = 60$ and for $M = 20$ snapshots, is shown in Figure 4.1.

We obtain a KL decomposition for the spatiotemporal data $v(n, j)$ and Table 4.1 shows the corresponding eigenvalues $\lambda_k$, and the energy content $\eta_k$, for the data shown in Figure 4.1 (a-d). The results show that for the CML exhibiting weak chaos and traveling wave, a smaller number of basis modes $N = 3$ and $N = 5$, respectively, are required to capture and reconstruct 99% of the data. For the more complex patterns, viz., fully developed chaos and convective turbulence the number of basis modes significantly rise to 15 for $\approx 99\%$ and 19 for $\approx 100\%$ accuracy.

4.2.2 One-dimensional Reaction-Diffusion System: Gray-Scott Model

Pattern formation in reaction-diffusion systems has been extensively studied both theoretically and experimentally [Kur84, Fie85]. Now we undertake a prototype reaction-diffusion model where one chemical species grows autocatalytically on another species [Gra90, Maz96]. This model is a simplification of the model of glycolysis proposed
Figure 4.2: Spatiotemporal data for the variable $u^{(1)}(t,x)$ in the autocatalytic reaction-diffusion system with parameter values $f = 0.029$, $k = 0.0535$, $D_u = 0.00002$, $D_v = 0.0001$ with spatial length $L = 1$ spanning 160 spatial sites and $M = 128$ snapshots recorded at a time step $\Delta t = 0.1$ is shown.

by Selkov [Sel68]. It follows the reaction mechanism $U + 2V \rightarrow 3V$; $V \rightarrow P$ with a continuous supply of the reactant $U$ and removal of product $P$.

A two variable PDE model for a spatially linear system (1-D) involving concentrations $u_1(t,x)$, $u_2(t,x)$ of $U$, $V$ respectively, may be written as:

$$\frac{\partial u_1(t,x)}{\partial t} = D_u \nabla^2 u_1(t,x) - u_1(t,x)u_2^2(t,x) + f[1 - u_1(t,x)]$$

$$\frac{\partial u_2(t,x)}{\partial t} = D_v \nabla^2 u_2(t,x) + u_1(t,x)u_2^2(t,x) - [f + k]u_2(t,x)$$

where, $D_u$ and $D_v$ are diffusion coefficients of $U$ and $V$ respectively. Parameters $f$ and $k$ are related to the flow of reactant into the system and the kinetic rate constant. The parameters $f,k$ form a set of bifurcation parameters for studying spatiotemporal dynamics of the system. In a particular bifurcation parameter space and for unequal diffusion coefficients, a host of spatiotemporal patterns have been identified and char-
Figure 4.3: The empirical eigenfunctions for the Gray-Scott model (a) $\phi_1^{(1)}$ and (b) $\phi_1^{(2)}$. (c) The eigenvalue spectra $\lambda_i$ vs. KL-modes $i$.

cacterized, for our study we consider that corresponding to spatiotemporal chaos as shown in Figure 4.2.

For obtaining the spatiotemporal data $u^{(1)}(t, x), u^{(2)}(t, x)$, Eq. (4.16) is solved numerically with Euler discretization in the spatial domain, with spatial length $L = 1$ spanning 160 spatial sites and $M = 40$ snapshots are stored at a time step $\Delta t = 0.1$ and with periodic boundary conditions $u^{(1)}(t, 0) = u^{(1)}(t, L)$ and $u^{(2)}(t, 0) = u^{(2)}(t, L)$ imposed. The initial conditions correspond to the stationary solution $u^{(1)}(0, x) = 1$ and $u^{(2)}(0, x) = 0$ except for a few central sites which are given a random perturbation to break the symmetry. On subjecting the spatiotemporal data to KL-decomposition the most significant empirical basis functions $\phi_1^{(1)}$ and $\phi_1^{(2)}$ corresponding to the variable $u^{(1)}$ and $u^{(2)}$ is shown in Figure 4.3(a) and (b) respectively. The eigenvalues $\lambda_i$ of the correlation matrix indicating the importance of KL-modes is shown in Figure 4.3(c).
4.2.3 Two-dimensional Reaction-Diffusion System: Activator-Inhibitor Model

A most fascinating aspect of biological systems is the generation of complex features in each round of the life cycle. This involves the processes like cell differentiation, cell movement, change of shape of cells and tissues, region specific control of cell division and cell death. A crucial problem is that in spite of genetic information coded in each cell though same how spatial patterns are generated that specifies functionality of cells. Modeling of biological systems exhibiting complex patterns has been a subject of active research. Pattern formation is certainly based on the interaction of many components and interactions are expected to be nonlinear. Two features that play a crucial role in pattern formation are local self-enhancement and long-range inhibition [Koc81].

The patterns that can be generated are graded concentration profiles, local concentration maxima and stripe like distribution of substances. A possible interaction between an activator \( u^{(1)} \) and its rapidly diffusing antagonist \( u^{(2)} \) can be expressed...
Figure 4.5: The first three significant spatial basis functions for the two-dimensional activator-inhibitor model.

![Figure 4.5](image)

Figure 4.6: Eigenvalue spectra vs. the KL-modes for the two-dimensional activator-inhibitor model.

![Figure 4.6](image)

As:

\[
\frac{\partial u^{(1)}}{\partial t} = D_1 \Delta u^{(1)} + \rho_1 \frac{u^{(1)}u^{(1)}}{(1 + \kappa_{1}u^{(1)}u^{(1)})u^{(2)}} - \mu_{1}u^{(1)} + \sigma_{1}
\]

\[
\frac{\partial u^{(2)}}{\partial t} = D_2 \Delta u^{(2)} + \rho_{2}u^{(2)}u^{(2)} - \mu_{2}u^{(2)} + \sigma_{2}.
\]

(4.17)

Here \( \Delta = \partial^2 / \partial x^2 + \partial^2 / \partial y^2 \) is the Laplace operator in a two-dimensional \((x, y)\) orthonormal coordinate system. The parameters of the system are the diffusion constants \(D_1, D_2\), removal rates \(\mu_1, \mu_2\), cross-reaction terms \(\rho_1, \rho_2\), basic production terms \(\sigma_1, \sigma_2\) and the saturation constant \(\kappa_1\).

The patterns on simulation are shown in Figure 4.4 which when subjected to the Karhunen-Loève decomposition yields the two-dimensional basis functions as shown in Figure 4.5.
4.3 Application of Karhunen-Loéve decomposition for Image Analysis

Two-dimensional images obtained from various sources are often found to be marred and noisy. As is the case of satellite images in the presence of cloud cover as a natural obstruction. Thus recovering a full image from a marred image when an ensemble of like images are provided is essential for further image analysis studies. Moreover, an even more demanding exercise is retrieving an image when the ensemble provided is itself marred and noisy. Here we utilize the methodology of Karhunen-Loéve decomposition to identify an image by characterizing the significant coherent structures present in the data.

Let \( \{\psi_n(x)\} \) be an ensemble of empirical eigenfunctions obtained from an ensemble of snapshots (images) given by \( v_n(x) \). Let an image \( \phi(x) \) be represented as

\[
\phi(x) \approx \sum_{n=1}^{N} a_n \psi_n(x) \tag{4.18}
\]

where, the coefficients, \( a_n \) are obtained from the usual inner product,

\[
a_n = (\phi, \psi_n) \tag{4.19}
\]

and \( N \) represents the number of basis functions needed to reconstruct with sufficient accuracy. Let a masked image be described by

\[
\tilde{\phi}(x) = m(x)\phi(x) \tag{4.20}
\]

where, \( m = 0 \) on the mask and \( m = 1 \) elsewhere. Our aim is to write \( \tilde{\phi}(x) \) as

\[
\tilde{\phi}(x) \approx m(x) \sum_{n=1}^{N} \tilde{a}_n \psi_n(x) \tag{4.21}
\]

and determine the optimum set of coefficients \( \tilde{a}_n \) which can no longer be evaluated via Eq. (4.19) since \( \psi_n \) are not necessarily orthogonal over the support of \( \tilde{\phi} \), \( s[\tilde{\phi}] \). We
intend to minimize the least square error defined as

$$E = \int_{s[\phi]} dx \left( \bar{\phi} - \sum_{n=1}^{N} \tilde{a}_n \psi_n \right)^2$$

(4.22)

and the minimization of $E$ requires

$$\left( \bar{\phi} - \sum_{n=1}^{N} \tilde{a}_n \psi_n, \psi_k \right)_{s[\phi]} = 0$$

(4.23)

which requires that the residual be orthogonal to $\psi_k$ for $k = 1, \cdots, N$, and inner product is evaluated over the support of $\bar{\phi}$, $s[\phi]$. The Hermitian matrix $M$ is

$$M_{kn} = \langle \psi_k, \psi_n \rangle_{s[\phi]}$$

(4.24)

is non-negative and we seek the unknown coefficients $\tilde{a}_k$ from

$$Ma = f$$

(4.25)

and we write

$$f_k = \langle \phi, \psi_k \rangle_{s[\phi]}.$$  

(4.26)

In the present case if we denote the eigenvalues by $\mu_n$ and corresponding eigenvectors by $v_n$ and the solution is given by

$$\tilde{a} = \sum_{k=1}^{N} \frac{1}{\mu_n} \langle v_n, f \rangle v_n.$$  

(4.27)

In our study we have considered images obtained from the two-dimensional Gray-Scott model described in Eq. (4.16). To illustrate the nature of this construction we have considered an ensemble of $K = 35$ snapshots. Figure 4.7 shows the snapshot 1, 15 and 30 each having $100 \times 100$ pixels. This data set is subjected to KL decomposition and the corresponding ensemble of 35 eigenfunctions $\{\psi\}$ are obtained. The test/original image $\phi$ is masked by a randomly generated mask $m(x)$ and an
image with 12% mask is shown in Figure 4.7. It is to be noted that masked image is not an element in the set of snapshots considered for generating the eigenfunctions. We determine the coefficients $\hat{a}$ by the procedure described above and the image is reconstructed via Eq. (4.21) for a choice of $N = 10$ and the reconstructed image is shown in Figure 4.7.

The success of the above mentioned procedure owns to the fact that only a limited number of fitting functions are required in order to approximate the full image as the underlying empirical eigenspace is optimum. It is intuitively clear that an image with fairly high extent of masking cannot be reconstructed at finer scales. Since
eigenfunctions corresponding to higher KL-modes (i.e. increasing $N$) resolving at smaller scales using too many eigenfunctions can thus result in deterioration during reconstruction.

Now we explore the situation when the data set provided is obscured and hence the empirical eigenfunctions thus obtained from them are marred. Let the ensemble of marred faces be $\{\bar{\phi}(x)\}$ and each of the marred face is of the form

$$\bar{\phi}(x) = m_n(x)\phi(x)$$

(4.28)

where $\phi(x)$ is chosen from the original set of images. Here, we will numerically demonstrate the procedure of reconstructing images from the ensemble of marred images. Let us start by calculating the average value at pixel location $x$ by

$$\langle \bar{\phi}(x) \rangle = \frac{1}{M(x)} \sum_{n \in S[x]} \bar{\phi}_n(x)$$

(4.29)

where, $S[x]$ is the set of indices at which $m_n(x)$ is unity and $M(x)$ is the number of indices in this set at pixel location $x$. We obtain the repaired ensemble $\{\tilde{\phi}_n^{(1)}(x)\}$ by missing values of $\bar{\phi}_n(x)$ and subsequently apply Karhunen-Loève decomposition on this repaired set to generate $\{\psi_n^{(0)}(x)\}$. Next we obtain $\{\tilde{\phi}_n^{(1)}(x)\}$ by a superposition of $R$ eigenfunctions as

$$\tilde{\phi}^{(1)} = \sum_{k=1}^{R} a_k^{(1)} \psi_k^{(0)}(x)$$

(4.30)

and the set $\{a_k^{(1)}\}$ by minimization of the error function given by

$$\tilde{E}_n = \int (\bar{\phi}_n - \tilde{\phi}_n^{(1)})^2 m_n(x) dx. $$

(4.31)

The repaired snapshot, is obtained by

$$\tilde{\phi}_n^{(1)}(x) = \begin{cases} \bar{\phi}_n x & \text{if } m_n(x) = 1 \\ \hat{\phi}_n x & \text{if } m_n(x) = 0 \end{cases}$$

(4.32)
Figure 4.8: Reconstruction of a masked image using an ensemble of 35 masked images generated from the 2-D Gray-Scott model.
Figure 4.9: RMS error vs. KL-modes for masked image (100 x 100) using masked (A) and unmasked (B) ensemble of 35 snapshots for various percentage of masking.
Figure 4.10: Reconstruction of a noisy image using an ensemble of 35 noisy images generated from the 2-D Gray-Scott model.
Thus the refined values of the eigenfunctions are obtained on an iterative basis and on reconstruction the image is retrieved. The results of this iterative scheme is exemplified for an ensemble of marred images as shown in Figure (4.8). The efficiency in reconstructing the images can be quantified by calculating the RMS error. The variation of RMS error with the number of KL-modes for masked images taken for analysis is shown in Figure (4.9). Another commonly encountered problem in surface characterization of various physical and chemical spatiotemporal systems is noisy images. The ensemble of snapshots considered for our study have been artificially made noisy by addition of randomly generated numbers. Extraction of information in the form of an cleaned image is also possible by the method outlined above. Here we take an ensemble of noisy images and try to extract the original image and the results are illustrated in Figure (4.10).

4.4 Conclusion

Introducing spatial coupling to simple logistic maps, a variety of dynamical features ranging from traveling wave to chaos can be seen. Even reaction diffusion systems modeling autocatalytic reaction or activation-inhibition in species have pattern forming capabilities for certain parameter ranges. By the Karhunen-Loève decomposition, we have demonstrated a methodology to extract the spatial features by identifying the coherent spatial structures in the data by suitable transformations to obtain a space-time separable form both for CMLSs and continuous time domain systems. This feature can be suitably used to obtain reduced description of high-dimensional systems as will be shown in Chapter 5. As a further application of this procedure we have addressed the problem of image analysis where marred or noisy images is encountered. We show that once the coherent structures are identified it is possible to extract the image within reasonable error bounds.