Abstract—In this paper, we present a time-frequency S-method-based approach for real-time motion compensation, image formation and image enhancement of moving targets in ISAR and SAR. This approach performs better than the Fourier transform by drastically improving images of fast, manoeuvring targets by increasing the SNR in both low and high noise environments. The method also computationally simple, requiring only slight modifications to the existing Fourier transform based algorithm. The effectiveness of this method is demonstrated through the application to simulated and experimental data sets.

Index Terms—Micro-Doppler, S-method, Time-Frequency

I. INTRODUCTION

One proven approach to achieving inverse synthetic aperture radar (ISAR) motion compensation and focused distorted ISAR images is the adaptive joint time-frequency (AJTF) algorithm [1]. However, this algorithm is not without significant weaknesses. One of the problems is the computational burden of the exhaustive search used to extract the motion compensation parameters, which limits its usefulness in an operational situation. Another approach is based on the use of quadratic time-frequency representations [2]. Time-frequency techniques are known to be successful in refocusing blurred ISAR images. This is because the images are obtained at a particular instant in time when the target’s motion can be considered uniform. However, the data are not collected instantaneously. Thus, a large number of refocused ISAR images will be generated that span the entire coherent integration time (CIT). For accurate target recognition, it is imperative to make use of only the best refocused image. It would be very impractical and inefficient to examine all of the images produced in order to identify which is best. Such manual inspection, even with the aid of an automated image searching algorithm, only adds extra complexity to the target recognition process. In this paper, we present a time-frequency s-method-based approach for real-time motion compensation, image formation and image enhancement of moving targets in ISAR and SAR.

II. S-METHOD

The S-method (SM) is derived from the relationship between the short-time Fourier transform (STFT) and the Wigner distribution (WD), which reads [3]:

$$WD(t,\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} STFT(t,\omega + \theta)STFT^*(t,\omega - \theta)d\theta.$$  \hspace{1cm} (1)

A discrete version of the previous relation reads:

$$WD(n,k) = \sum_{i=-N/2}^{N/2} STFT(n,k + i)STFT^*(n,k - i) \hspace{1cm} (2)$$

$$= |STFT(n,k)|^2 + 2\text{Re}\left\{\sum_{i=1}^{N/2} STFT(n,k + i)STFT^*(n,k - i)\right\}.$$  

The mathematical formulation of the SM in the discrete form is:

$$SM(n,k) = \sum_{i=-N/2}^{N/2} P(i)STFT(n,k + i)STFT^*(n,k - i)$$

where $P(i) = 1$ for $|i| \leq L$ and $P(i) = 0$ for other values of $i$. The SM with $L$ terms can be written in the form:

$$SM_L(n,k) = \sum_{i=-L}^{L} STFT(n,k + i)STFT^*(n,k - i). \hspace{1cm} (3)$$

In particular, for $L = 0$, the SM is identical to the spectrogram

$$SM_0(n,k) = |STFT(n,k)|^2 = STFT(n,k)STFT^*(n,k),$$

while for $L = N$, the SM is identical to the WD. It should be noted that SM with $L$ terms is obtained by adding one more term to the SM with $L - 1$ terms:

$$SM_L(n,k) = SM_{L-1}(n,k) + 2\text{Re}\{STFT(n,k + L)STFT^*(n,k - L)\}.$$  

The SM will produce the same auto terms as the WD if we take $L$ such that $(2L+1)$ is equal to the auto terms width in the discrete domain (i.e., to the number of samples within the auto term). In practice it means fewer terms are needed, for example $L \in [3,10]$, since most of the auto-term energy is located around its maximal value. The precise mathematical proof
that the SM produces the WD of each component separately, in those regions of the time-frequency (TF) plane where the components do not overlap, is given in reference [3].

III. RESULTS AND DISCUSSION

In this section we demonstrate the application and effectiveness of the S-method as an ISAR image refocusing technique with simulated and experimental ISAR data.

A. Simulated Data

1) Boeing-727 aircraft: The first model is a Boeing 727 aircraft [1]. The simulation uses a stepped frequency X-band radar operating at a center frequency of 9 GHz. With a total of 64 stepped frequencies, the waveform has a bandwidth of 150 MHz and a range resolution of 1 m. The pulse repetition frequency (PRF) is 20 kHz and the coherent integration time (CIT) is 0.82 seconds. A total of 64 range cells and 256 cross-range cells are used in the imaging.

This simulation makes use of a simple virtual instrument realization of the S-method, done according to (15). For \( L = 0 \), the standard Fourier transform based representation is obtained, as shown in Figure 1a. The general shape of the aircraft can be made out, but it is not possible to locate the range/cross-range cells of individual point-scatterers as they are too smeared in the cross-range dimension. By changing the number of terms, \( L \), we get the S-method based representations with quadratic and higher-order phase errors eliminated. Figure 1b shows the refocused image with \( L = 3 \) and Figure 1c with \( L = 6 \). Compared to Figure 1a, both images show a substantial decrease in the amount of smearing in the cross-range. However, the higher \( L \) value of 6 seems to make further improvements in refocusing the nose and tail point-scatterers of the aircraft. Figure 1c is thus chosen as the best image for this data set.

2) MiG-25 aircraft: The second model is a MiG-25 aircraft with 120 point-scatterers distributed along the edge of the 2D shape of the aircraft [1]. The simulation uses a stepped frequency X-band radar operating at a center frequency of 9 GHz. With a total of 64 stepped frequencies, it has a bandwidth of 512 MHz and a range resolution of 0.293 m. The PRF is 20 kHz and the CIT is 1.64 seconds. A total of 64 range cells and 512 cross-range cells are used for the imaging. The aircraft is at a range of 3,500 m and is rotating at 10 degrees/second, thus giving a cross-range resolution of about 0.058 m. The rotation rate is much higher than the normal rotation rate needed to produce a clear image of the target. We assume that target’s translational motion can be perfectly compensated. However, due to the fast rotation and relatively longer image observation time, even after standard motion compensation, the uncompensated phase error is still large.

The same virtual instrument is used for MiG-25 simulation. The standard Fourier transform based representation is shown in Figure 2a. As with the Boeing-727, the general shape of the aircraft can be made out, but it is not possible to locate the range/cross-range cell of individual point-scatterers as they are, once again, smeared in the cross-range dimension. The S-method representations with quadratic and higher-order phase terms eliminated are shown in Figure 2b with \( L = 3 \) and Figure 2c with \( L = 6 \). As expected, both images show a substantial improvement in the degree of smearing in the cross-range when compared to Figure 2a. The greatest improvement lies in the nose of the aircraft as it now converges to a point. However, the higher \( L \) value of 6 refocuses the nose substantially more and so Figure 2c is chosen as the best image for this data set.

B. Experimental Data

1) Delta-Wing: An ISAR experiment is set up to examine the distortion of ISAR images due to a time-varying rotational motion. A 2-dimensional delta-wing shaped target, the target motion simulator (TMS), is built for the ISAR distortion experiments [1]. The target has a length of 5 m on each of its three sides. Six trihedral reflectors are mounted on the TMS as scattering centres of the target; all the scatterers are located on the \( x - y \) plane. They are designed to always face towards the radar as the TMS rotates. The TMS target is set up so that it rotates perpendicular to the radar line of sight.

The delta-wing data was collected using an X-band radar operating at a center frequency of 10.1 GHz with 300 MHz bandwidth and a range resolution of 0.5 m. The PRF is 2 kHz. Each HRR profile is generated in 0.5 ms and each profile has 41 range bins. The total data set contains 60,000 HRR profiles. The delta-wing is at a range of 2 km and is rotating at 2 degrees/second. Figure 3 shows the ISAR image from 2048 pulses, which corresponds to a CIT of 1.024 seconds. The Figures’ leftmost columns show the Fourier transform-based representations and their rightmost columns show the S-method-based representations. Since significant phase errors due to nonuniform motion exist in the data, the Fourier transform-based images are blurred in the cross-range dimension. In each of the S-method-based images, the cross-range smearing is significantly reduced resulting in a dramatic improvement in image quality. The images are now focused and the six reflectors are visible.

2) Canadian Coast Guard Vessel: Inverse synthetic aperture radar imaging of sea vessels is a challenging task because their 3-D rotational (roll, pitch and yaw motions) motion over the CIT often leads to blurred images. The selection of the duration of the CIT, also known as the coherent processing time window length, is critical because it should be short enough to limit the blurring caused by the 3-D rotational motion and long enough to ensure that the desired cross-range resolution is obtained. The Canadian Coast Guard Vessel (CCGV) data used in this paper were acquired by the EC CV-580 C-band polarimetric system [4].

Figures 4a and 4c show the Fourier transform based representation of the CCGV. The figure clearly shows that the image is severely blurred. This means that the target’s motion contains a substantial amount of rotational error. The S-method based representation, with \( L = 3 \), was selected as a suitable refocused image and is shown in Figures 4b and 4d. The S-method was able to remove most of the blurring caused by the quadratic and higher-order phase effects.

The amount of time it took the Fourier transform and S-method program codes to be executed was measured for each
of the above data sets. The program codes were executed in Matlab using a Pentium-IV 2.66 GHz with 2.1 Gbyte of RAM. In relation to the Fourier transform, the S-method ran 1.5 to 5.5 times more slowly for L values of 1 and 7, respectively. Each L value in the range 1 to 7 was used in producing refocused images of the above data sets. When compared to other transforms, the S-method performs faster. The WD was applied to the above data sets. This gave an average processing time that is 592 times slower than the Fourier transform, and significantly slower than the S-method. The results indicate that the S-method is able to produce well focused images in real-time. For this reason, it is a useful technique for detecting targets.

IV. CONCLUSION

The S-method-based approach can be used to real-time motion compensation, image formation and image enhancement of moving targets in ISAR and SAR. This approach performs better than the Fourier transform by drastically improving images of fast, maneuvering targets. These advantages are a result of the S-method’s ability to automatically compensate for quadratic and all even higher-order terms in phase. Thus, targets with constant acceleration will undergo full motion compensation and their point-scatterers will each be localized.
Fig. 4. ISAR images from Canadian Coast Guard Vessel data.

The method is also computationally simple, requiring only slight modifications to the existing Fourier transform based algorithm. The S-method can be applied to real-time target identification in ISAR systems. This work is especially pertinent to the ISAR imaging capability in military intelligence, surveillance and reconnaissance operations.

REFERENCES

ISAR Imaging of Moving Targets Based on Reassigned Smoothed Pseudo Wigner-Ville Distribution

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Abstract:
High-range resolution inverse synthetic aperture radar (ISAR) imaging is a promising tool for non-cooperative target identification and has attracted wide interest within the scientific and military communities. Most of the existing ISAR algorithms are based on the Fourier transform. However, for long coherent integration time or for fast maneuvering targets, simple Fourier processing without compensation will lead to severe blurred and distorted images. In this paper, we propose a reassignment pseudo Wigner-Ville distribution for focusing distorted ISAR images of moving/rotating targets. The effectiveness of this method has been successfully applied to simulated and experimental data sets.

Key words- Time Frequency analysis, ISAR imaging, reassignment pseudo wigner-ville distribution.

I. Introduction
Ideally, if the target has only uniform rotational motion and data is collected over a small angular aperture, then a simple Fourier transform process would bring a set of range profiles collected over a given dwell time (i.e., the coherent processing interval) into a focused two-dimensional image. However, actual targets observed by operational radar rarely have such an ideal motion. Targets are often engaged in complicated maneuvers that combine the translational and rotational motions. For long imaging times (i.e., long coherent integration time or for fast maneuvering targets), simple Fourier processing without compensation will lead to severe image blurring in cross-range. Unless a good motion compensation algorithm is implemented, serious blurring can result in the ISAR image formed by the Fourier transform [1-3]. Joint Time-Frequency representations can be used for focusing distorted images. The oldest and the most widely used time-frequency representation is the short-time Fourier transform (STFT). In order to improve its concentration, various quadratic representations have been introduced. The most prominent member of this class of representations is the Wigner-Ville distribution (WV), which suffers from cross-term interferences. Considerable efforts have been deployed to design time-frequency distributions (TFDs) which reduce the cross-terms while preserving many desirable properties of WV [4]. One example of such TFD is the smoothed pseudo Wigner-Ville distribution (SPWV). The SPWV has a separable kernel, with a time smoothing window and frequency smoothing window. These two windows are chosen to suppress spurious peaks in both frequency and time domains. The suppression of cross terms is improved with shorter windows. This, however, results in an undesirable loss of resolution and smearing of the image. The reassigned time-frequency distribution allows us to solve the problem of the spectral broadening and cross terms to have readable and localized distribution in the time-frequency plane. Time-frequency reassignment methods are attractive for focusing distorted images of targets exhibiting rotational motion along with the translation motion [5-7].

II. Reassigned Smoothed Pseudo Wigner-Ville Distribution
The WV for a signal \( x(t) \) at time \( \tau \) is defined as [4]

\[
WV(t,\nu) = \frac{1}{2\pi} \int x^*(t - \frac{1}{2} \tau)x(t + \frac{1}{2} \tau)e^{-j2\pi\nu\tau} d\tau
\]

where \( \tau \) is the time lag variable, \( \nu \) is the frequency instant. Although WV has many advantages, it suffers from cross-term interference in the case of multi-component signals. In order to reduce the interference of cross-terms, smoothing kernels are used in the time and frequency domains. One of the consequences of this is that the smoothed pseudo Wigner-Ville distribution (SPWV) suppresses, to some extent, the cross-terms for multi-component signals. The SPWV is defined as [6]
SPWV \( (t, \nu; g, h) = \int_{-\infty}^{\infty} h(\tau) \int_{-\infty}^{\infty} g(t - \tau) \cdot x(t + \frac{\tau}{2})^* (t - \frac{\tau}{2}) \cdot e^{-j2\pi\nu d} d\tau \)  

where \( g \) is the time smoothing window and \( h \) is the frequency smoothing window. Compared with WV, the SPWV greatly suppress the influence of the cross-terms of multi-component signal but at the expense of time-frequency concentration. A method for improved distribution concentration, based on the reassignment of distribution values in the time-frequency plane has been proposed [5]. It has been reintroduced for the readability improvement of time-frequency distribution by Auger and Flandrin [6]. This method is known as the reassignment method. The reassignment method provides both good concentrations of the signal component and minimized misleading interference terms. The key point of the assignment principle is that it moves each value of the distribution computed at any point \((t, \nu)\) to another point \((t', \nu')\), which is the center of gravity of the signal energy distribution around \((t, \nu)\). It is expressed as follows:

\[ \text{RSPWV} \left( t', \nu'; g, h \right) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \text{SPWV} \left( t, \nu; g, h \right) \cdot \delta(t-t') \delta(\nu-\nu') dt d\nu \]\n
where \( \text{SPWV}(t,\nu;g,h) \) smoothed pseudo Wigner-Ville distribution, \( t \) and \( t' \) are time instants, \( \nu \) and \( \nu' \) are frequency instants, \( g \) is the time smoothing window, \( h \) is the frequency smoothing window, \( t' \) is the assigned time instant at which the center of gravity of energy contributions is located, and \( \nu' \) is the assigned frequency instant at which the center of gravity of energy contributions is located. \( t' \) is defined as

\[ t' = t - \frac{\text{SPWV}(t,\nu;\Gamma_g,h)}{2\pi[\text{SPWV}(t,\nu;g,h)]} \]

where \( \Gamma_g \) is the operator of multiplication and \( \Gamma_g(t) = t^*g(t) \)

\[ \nu' = \nu + \frac{\text{SPWV}(t,\nu;g,D_h)}{2\pi[\text{SPWV}(t,\nu;g,h)]} \]

Where \( D_h \) is the operator of multiplication defined as

\[ D_h = \frac{d}{dt} [h(t)] \]

III. Results and Discussion

In this section we demonstrate the application and effectiveness of the reassigned SPWV method as an ISAR image refocusing technique with simulated and experimental ISAR data.

A. Simulated Data

The model considered is a MiG-25 aircraft with 120 point-scatterers distributed along the edge of the 2-dimensional shape of the aircraft [1,3]. The simulation uses stepped frequency X-band radar operating at a center frequency of 9 GHz. With a total of 64 stepped frequencies, it has a bandwidth of 512 MHz and a range resolution of 0.293 m. The pulse repetition frequency (PRF) is 20 kHz and the coherent integration time (CIT) is 1.64 seconds. A total of 64 range cells and 512 cross-range cells are used for the imaging. The aircraft is at a range of 3,500 m and is rotating at 10 degrees/second, thus giving a cross-range resolution of about 0.058 m. The rotation rate is much higher than the normal rotation rate needed to produce a clear image of the target. We assume that target’s translational motion can be perfectly compensated. However, due to the fast rotation and relatively longer image observation time, even after standard motion compensation, the uncompensated phase error is still large.

The standard Fourier transform based representation is shown in Figure 1a. As we can see, the general shape of the aircraft can be made out, but it is not possible to locate the range/cross-range cell of individual point-scatterers as they are, once again, smeared in the cross-range dimension. Figure 1b, 1c and 1d show spectrogram, SPWV and reassigned SPWV distribution respectively. It is observed that the image based on reassigned SPWV shows substantial improvements in the degree of smearing in the cross-range when compared to Fourier transform based method, spectrogram and SPWV.

B. Experimental Data

Delta-Wing: An ISAR experiment is set up to examine the distortion of ISAR images due to a time-varying rotational motion. A 2-dimensional delta-wing shaped target, the target motion simulator (TMS), is built for the ISAR distortion experiments [12]. The target has a length of 5 m on each of its three sides. Six trihedral reflectors are mounted on the TMS as scattering centres of the target; all the scatterers are located on the horizontal x-y plane. They are designed to always face towards the radar as the TMS rotates. The TMS target is set up so that it rotates perpendicular to the radar line-of-sight. The delta-wing data was collected using an X-band radar operating at a center frequency of 10.1 GHz with 300 MHz bandwidth and a range resolution of 0.5 m. The PRF is 2 kHz. Each HRR profile is generated in 0.5 ms and each profile has 41
range bins. The delta-wing is at a range of 2 km and is rotating at 2 degrees/second. Figure 2 shows ISAR images for CIT 0.256s. The Figure 2a shows the Fourier transform-based approach and Figures 2b, 2c and 2d shows spectrogram, SPWV and reassigned SPWV respectively. Figure 3 shows ISAR images for CIT 0.512s. Since significant phase errors due to non-uniform motion exist in the data, the Fourier transform-based images are blurred in the cross-range dimension. In the images formed by Reassigned SPWV cross-range smearing is significantly reduced resulting in a improvement in image quality. By comparing figures 2d and 3d, we can make out that as CIT increased, cross-range resolution is increased. The images are now well focused and the six reflectors are visible. But it is found that for larger values of CIT values, cross-terms come in to picture. Hence there is a tradeoff between resolution and cross terms. It is advisable to have moderate CIT values for cross-terms free and focused image.

Figure 1: ISAR images from simulated Mig-25 data. a) 2-D Fourier transform, b) Spectrogram, c) Smoothed Pseudo Wigner-Ville distribution, and d) Reassigned Smoothed Pseudo Wigner-Ville distribution.
Figure 2: ISAR images of experimental Delta-Wing data for CIT 0.256 sec. a) 2-D Fourier transform, b) Spectrogram, c) Smoothed Pseudo Wigner-Ville distribution, and d) Reassigned Smoothed Pseudo Wigner-Ville distribution.

Figure 3: ISAR images of experimental Delta-Wing data for CIT 0.512 sec. a) 2-D Fourier transform, b) Spectrogram, c) Smoothed Pseudo Wigner-Ville distribution, and d) Reassigned Smoothed Pseudo Wigner-Ville distribution.

IV. Conclusion

The reassignment smoothed Wigner-Ville distribution can be used to refocus distorted ISAR images. The advantage of this method is that it drastically reduces the blurredness in cross-range direction. This method gives better results than the standard Fourier transform, spectrogram and smoothed pseudo Wigner-Ville distribution.

References

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Micro-Doppler analysis of a rotating target in synthetic aperture radar

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Abstract: Rotating targets cause phase modulation of the azimuthal phase history of a synthetic aperture radar (SAR) system. The phase modulation may be seen as a time-dependent micro-Doppler (m-D) frequency. This study presents two approaches for extracting m-D features from SAR images. In order to extract m-D features from SAR images, the time domain radar return is decomposed in two separate ways. One is based on wavelet decomposition in which the returned signal is decomposed into a set of components that are represented at different wavelet scales. The components are then reconstructed by applying the inverse wavelet transform. This wavelet approach has been used in previous m-D analysis work for an inverse SAR (ISAR) system, and it is presented here in the extraction of m-D features for a SAR system. The second approach is based on adaptive chirplet decomposition combined with time–frequency analysis. This new approach is introduced as an alternative to the wavelet approach of decomposing the SAR radar return. The results from the wavelet and adaptive chirplet decomposition procedures are compared, and the chirplet-based approach establishes itself as a viable alternative. The chirplet-based method of m-D extraction has been successfully applied to SAR data scene collected by the US Navy APY-6 radar.

1 Introduction

Synthetic aperture radar (SAR) systems usually produce two-dimensional images. One dimension is called the range and is obtained by precisely measuring the time from transmission of a signal to its return. The second dimension is known as the azimuth and is perpendicular to the range dimension. The azimuthal resolution is obtained by processing the Doppler phase of the radar return. What differentiates SAR from other radars is its ability to produce fine azimuth resolution [1–7].

SAR is a well established and useful technique of acquiring high-resolution images of an area of interest from airborne or space sensors [1, 2]. A particular class of targets that pose a difficult challenge for target recognition is moving targets. If there are moving targets in the scene, SAR cannot simultaneously produce clear images of both of these stationary targets and moving targets [3–7]. Usually, moving targets appear as defocused and spatially displaced objects superimposed on the SAR map. Moving targets cause phase modulation of the azimuth phase history of an SAR collection. The phase modulation can be seen as a time-varying Doppler frequency. It is obvious that in some scenarios it is the moving objects that are of interest. Some targets contain parts that move relative to the target itself. Examples are rotating/vibrating parts such as wheels or engines. A special case is rotating point targets such as a rotating antenna. The resulting class of motions may aid target characterisation and recognition. Rotations and vibrations can be observed by radar when the conditions are right. The phenomenon, as observed by radar, is termed micro-Doppler (m-D). In addition to the m-D, there may be a Doppler shift corresponding to the target body motion.

In this paper, we extract the m-D features relating to a rotating antenna in a SAR target scenario. The m-D for such rotating target may be seen as a sinusoidal phase
modulation of the SAR azimuth phase history. The phase modulation may equivalently be seen as a time-varying Doppler frequency. Time–frequency methods are useful tools in the analysis of such signals. This paper presents two approaches for extracting m-D features from SAR images. In order to extract m-D features from SAR images, the time domain radar return is decomposed in two separate ways. One is based on wavelet decomposition in which the returned signal is decomposed into a set of components that are represented at different wavelet scales. The components are then reconstructed by applying the inverse wavelet transform. This wavelet approach has been used in our previous m-D analysis work for an inverse SAR (ISAR) system [8], and it is presented here in the extraction of m-D features for an SAR system. The second approach is based on the adaptive chirplet decomposition. This approach is introduced as an alternative to the wavelet approach of decomposing the SAR radar return. Furthermore, instead of using the adaptive chirplet decomposition or the high-resolution time–frequency transform alone, as was done in the past [9], the adaptive chirplet decomposition method combined with time–frequency analysis is used in this paper to analyse the time-varying m-D features. The results from the wavelet and adaptive chirplet decomposition procedures are then compared, and the chirplet-based approach establishes itself to be a viable alternative. The chirplet-based method of m-D extraction has been successfully applied to SAR data scene collected by the US Navy APY-6 radar. The rotating frequency of the antenna was calculated using this method and the results agree well with available ground truth. Such parameters may be useful for describing the target, or serve as inputs to automatic target recognition algorithms.

2 Method of m-D feature extraction

Joint time–frequency analysis is the basis of most of the existing methods used to extract m-D features [7–13]. The procedure of wavelet analysis incorporated with time–frequency analysis has been applied in our previous works for an ISAR system [8, 12, 13]. Here, the approach is used in SAR target scenarios. Also, we introduce a procedure for an m-D analysis, namely, the adaptive chirplet decomposition incorporated with the time–frequency distribution series.

2.1 Wavelet analysis

The wavelet transform is a very useful tool in many recent signal processing applications such as detecting the discontinuity of a signal de-noising a signal and image compression. The wavelet transform results in an efficient representation of highly non-stationary signals [14–18]. The main motivation for applying wavelet analysis in the extraction of m-D features is that the micro-motion dynamics of a target that induce m-D features change at a rate much faster than the target body itself. Wavelet analysis has the capability of detecting rapid changes of a signal [14–18]. Therefore wavelet analysis seems ideal for the extraction of m-D features and the wavelet transform can be considered a powerful tool for the task.

The wavelet analysis is actually quite complex and contains a large number of steps. The wavelet transform is applied by using a tree of digital filter banks based upon the multi-resolution analysis theory [14–18]. In order to effectively extract m-D features from the returned signal, a four-level decomposition tree is utilised to represent the returned radar signal. Using the wavelet transform, the signal is first broken down into its constituent parts consisting of the four detail levels containing the m-D features and the left over stationary approximation representing the body vibration. The inverse wavelet transform is then used to reconstruct the decomposed wavelet components. During the reconstruction process of the detail levels, only the wavelet coefficients that are related to the m-D features of the signal are used while the other coefficients are set to zero. The same process is also used in the reconstruction of the stationary part of the target’s body. The result of the wavelet analysis is the separation of the target’s micro-motion dynamics and the extraction of the m-D features. This method has been tested previously and shown to be successful with other data sets [8, 12, 13].

The tree of digital filter banks for computing the discrete wavelet transform is given in Fig. 1 (this is a four-level decomposition tree). \( L \) and \( H \) represent pairs of discrete low-pass and high-pass filters. As demonstrated in this figure, the original signal \( S \) is decomposed into its constituent parts consisting of \( D_1, D_2, D_3, D_4 \) and \( A_4 \). In other words, after decomposition \( S = A_4 + D_1 + D_2 + D_3 + D_4 \). Now, the wavelet transform has decomposed the signal into five parts; the four detail levels and the stationary approximation. Generally, these decomposed parts, by themselves, do not represent motion dynamics of the target in question. Depending on the physical quantity that one wishes to estimate, the decomposed parts of the signal may need to be combined in various ways in order to represent the m-D features for which motion parameters are required.

It should be noted that we are not using the wavelet analysis alone or the time–frequency analysis alone. Rather, we are using the wavelet analysis to separate the rotating part of the signal from the main body signal. The wavelet analysis considerably ‘cleans up’ the signal. Thereafter, the time–frequency analysis is employed to analyse the oscillation and to estimate the motion parameters. In this paper, we use time–frequency distribution series to obtain the time-frequency representation of the analysed signal [14].

2.2 Adaptive chirplet analysis

Here, an alternative to wavelet analysis, the so-called adaptive chirplet analysis, is proposed in the extraction of m-D features from a radar return. A chirplet is a windowed
A Gaussian chirplet is defined as \cite{14, 19} and decomposition is employed to extract m-D features. The excision \cite{9, 14, 19}.

In this work, the adaptive Gaussian chirplet decomposition is employed to extract m-D features. The Gaussian chirplet is defined as \cite{14, 19}

\[
b_k(t) = \frac{a_k}{\pi} \exp \left\{ -\frac{a_k}{2} (t - t_k)^2 ight\} + j \left( \omega_k (t - t_k) + \beta_k^2 (t - t_k)^2 \right) \]

where \((t_k, \omega_k)\) indicates the time and frequency centre of the linear chirp function the variance \(a_k\) controls the width of the chirp function and the parameter \(\beta_k\) determines the rate of change of frequency. When the chirp rate \(\beta_k\) is equal to zero and the parameter \(a_k\) approaches zero, the linear chirplet function \(b_k(t)\) reduces to a sinusoidal signal.

The adaptive spectrogram for the Gaussian chirplet is given as \cite{14}

\[
\text{AS}(t, \omega) = 2 \sum_k |B_k|^2 \times \exp \left\{ -\frac{1}{\alpha_k} (\omega - \omega_k - \beta_k) \right\} \]  

(2)

The algorithm used here estimates the optimal elementary functions in the adaptive signal decomposition. Converting the optimisation process to a traditional curve-fitting problem is the basic idea behind this algorithm. The relationship between the estimated parameters and testing variables is given by the function

\[
P(\alpha_x, t_x, \omega_x, \beta_x; \alpha_k, t_k, \omega_k, \beta_k) \]

as follows

\[
P(\alpha_x, t_x, \omega_x, \beta_x; \alpha_k, t_k, \omega_k, \beta_k) = \frac{|A_j|}{\sqrt{(1/4\alpha_x, \alpha_x)} (\alpha_x + \alpha_k)^2 + (\beta_x - \beta_k)^2} \times \exp \left\{ \frac{\alpha^2_k + \alpha_k \alpha_x}{2((\alpha_x + \alpha_k)^2 + (\beta_x - \beta_k)^2)} (t_k - t_x)^2 \right\} \times \exp \left\{ \frac{\alpha_k \beta_k}{(\alpha_x + \alpha_k)^2 + (\beta_x - \beta_k)^2} (\omega_k - \omega_x)^2 \right\} \]

(4)
This can be written as
\[ P(\alpha_k, t_k, \omega_k, \beta_k; \alpha_{k,0}, t_{k,n}, \omega_{k,m}, \beta_{k,0}) = \langle \langle t_k \beta_{k,0}, \omega_{k,0}, \alpha_{k,0}, \beta_{k,0} \rangle \rangle \]
\[ = a(t_{k,m}) e^{-\sigma(t_{k,m} - \omega_{k,m})^2} \]

where
\[ a(t_{k,m}) = \sqrt{(1/4\alpha_{k,0}\alpha_k)((\alpha_k + \alpha_{k,0})^2 + (\beta_k - \beta_{k,0})^2)} \]
\[ \times \exp \left\{ \frac{(t_k - t_{k,n})^2}{2(\alpha_k^2 + \alpha_{k,0}^2)} \right\} \]
\[ \sigma(t_{k,m}) = \omega_k + \frac{\alpha_k \beta_k + \alpha_{k,0} \beta_{k,0}}{\alpha_k + \alpha_{k,0}} (t_k - t_{k,n}) \]

and
\[ b = \frac{\alpha_k + \alpha_{k,0}}{2[(\alpha_k + \alpha_{k,0})^2 + (\beta_k - \beta_{k,0})^2]} \]

Since both sides of the previous equation are greater than zero, we have
\[ \ln \left| \frac{\langle \langle t_k \beta_{k,0}, \omega_{k,0}, \alpha_{k,0}, \beta_{k,0} \rangle \rangle}{\langle \langle t_k \beta_{k,0}, \omega_{k,0}, \alpha_{k,0}, \beta_{k,0} \rangle \rangle} \right| = b[(\sigma(t_{k,n}) - \omega_k)^2 - (\sigma(t_{k,n}) - \omega_{k,0})^2] \]
\[ = 2b \sigma(t_{k,n})(\omega_k - \omega_{k,0}) - b(\alpha_k^2 - \alpha_{k,0}^2) \]  
(6)

The following three test points are inserted into equation (5)
\[ (\alpha_{k,0}, t_{k,1}, \omega_{k,-1}, \beta_{k,0}) \]
\[ (\alpha_{k,0}, t_{k,1}, \omega_{k,0}, \beta_{k,0}) \]
\[ (\alpha_{k,0}, t_{k,1}, \omega_{k,1}, \beta_{k,0}) \]

This results in the linear system are given below
\[ \begin{bmatrix} (\omega_{k,0} - \omega_{k,-1})(\omega_{k,-1} - \omega_{k,0})^2 \\ (\omega_{k,0} - \omega_{k,1})(\omega_{k,1} - \omega_{k,0})^2 \end{bmatrix} \begin{bmatrix} x_w \end{bmatrix} = \begin{bmatrix} \ln \left| \frac{\langle \langle t_k \beta_{k,0}, \omega_{k,0}, \alpha_{k,0}, \beta_{k,0} \rangle \rangle}{\langle \langle t_k \beta_{k,0}, \omega_{k,0}, \alpha_{k,0}, \beta_{k,0} \rangle \rangle} \right| \\ \ln \left| \frac{\langle \langle t_k \beta_{k,0}, \omega_{k,1}, \alpha_{k,0}, \beta_{k,0} \rangle \rangle}{\langle \langle t_k \beta_{k,0}, \omega_{k,0}, \alpha_{k,0}, \beta_{k,0} \rangle \rangle} \right| \\ \ln \left| \frac{\langle \langle t_k \beta_{k,0}, \omega_{k,-1}, \alpha_{k,0}, \beta_{k,0} \rangle \rangle}{\langle \langle t_k \beta_{k,0}, \omega_{k,0}, \alpha_{k,0}, \beta_{k,0} \rangle \rangle} \right| \end{bmatrix} \]
\[ (8) \]

where \( x_w = 2b \sigma(t_{k,0}), b > 0, \sigma(t_{k,0}) \in R \)

After \( b \) and \( \sigma(t_{k,0}) \) are found from the above system, \( a(t_{k,0}) \) is computed using equation (4)
\[ \ln a(t_{k,0}) = \ln \left| \frac{\langle \langle t_k \beta_{k,0}, \omega_{k,0}, \alpha_{k,0}, \beta_{k,0} \rangle \rangle}{\langle \langle t_k \beta_{k,0}, \omega_{k,0}, \alpha_{k,0}, \beta_{k,0} \rangle \rangle} \right| + b(\sigma(t_{k,0}) - \omega_{k,0})^2 \]  
(9)

Similarly, by applying the test points
\[ (\alpha_{k,0}, t_{k,1}, \omega_{k,-1}, \beta_{k,0}) \]
\[ (\alpha_{k,0}, t_{k,1}, \omega_{k,0}, \beta_{k,0}) \]
\[ (\alpha_{k,0}, t_{k,1}, \omega_{k,1}, \beta_{k,0}) \]

another set of intermediate variables \( \sigma(t_{k,1}) \) and \( a(t_{k,1}) \) are obtained. Letting
\[ r = \frac{\alpha_{k,0}\beta_k + \alpha_{k,0}\beta_{k,0}}{\alpha_k + \alpha_{k,0}} \]

and substituting into equation (4) gives
\[ \sigma(t_{k,1}) = \omega_k - r(t_k - t_{k,n}) \]

Using \( \sigma(t_{k,0}) \) and \( \sigma(t_{k,1}) \), the variable \( r \) can be computed as
\[ r = \frac{\sigma(t_{k,0}) - \sigma(t_{k,1})}{t_{k,0} - t_{k,1}} \]

Now, the parameters \( \alpha_k \) and \( \beta_k \) can be determined from the following
\[ \begin{cases} b = \frac{\alpha_k + \alpha_{k,0}}{2[(\alpha_k + \alpha_{k,0})^2 + (\beta_k - \beta_{k,0})^2]} \\ r = \frac{\alpha_k\beta_k + \alpha_{k,0}\beta_{k,0}}{\alpha_k + \alpha_{k,0}} \end{cases} \]

Using mathematical manipulations, the following is obtained
\[ 2b \left[ \left( \frac{\alpha_k}{r - \beta_{k,0}} \right)^2 + 1 \right] (\beta_k - \beta_{k,0}) - \frac{\alpha_{k,0}}{r - \beta_{k,0}} (\beta_k - \beta_{k,0}) = 0 \]

The solution to the above equation is given by
\[ \beta_k = \beta_{k,0} + \frac{\alpha_{k,0}}{2b(r - \beta_{k,0})[(\alpha_{k,0}/r - \beta_{k,0})]^2 + 1} \]

If \( \beta_k = \beta_{k,0} \), then from (13) one obtains
\[ r = \beta_{k,0} \]

and
\[ \alpha_k = \frac{1}{2b} - \alpha_{k,0}, \quad \alpha_{k,0} > 0 \]
Otherwise
\[ \beta_{k,0} + \frac{\alpha_{k,0}}{2b(r - \beta_{k,0})[(\alpha_{k,0}/r - \beta_{k,0})^2 + 1]} \]  
(19)

\[ \alpha_k = \frac{\beta_k - r}{r - \beta_{k,0}}, \quad \alpha_k > 0 \]  
(20)

Now that \( \alpha_k \) and \( \beta_k \) are known, the time parameter \( t_k \) can be computed from equation (5) as follows

\[ t_k = \frac{\alpha_k^{-1} + \alpha_{k,0}^{-1}}{t_{k,0} - t_{k,1}} \left( \frac{t_{k,0} + t_{k,1}}{2} \right) \]  
(21)

Finally, the frequency parameter \( \omega_k \) can be solved from equation (12)

\[ \omega_k = \omega(t_{k,0}) + r(t_k - t_{k,0}) \]  
(22)

Recent studies show that a radar signal can be properly modelled using the chirplet basis function. Many natural phenomena, for instance, signals encountered in radar systems [20–22], the impulsive signal that is dispersed by the ionosphere [23] and seismic signals [24] are modelled as chirplet-type functions. The chirplet function has many striking characteristics: (i) the chirplet basis is a well understood four-parameter function, \( \alpha_k, \beta_k, t_k \) and \( \omega_k \), localised in the joint time–frequency plane. Only a moderate computation time is needed to search for the basis parameters; (ii) since both the amplitude modulation and frequency modulation are part of the basis function, the chirplet can more efficiently represent the reflected signal from a target with a rotating part. Only a few sets of these bases are needed to approximate the time–frequency structure of the radar signal; and (iii) more importantly, the radar returns from the target body and the rotating part can be more easily separated based on the parameters of the chirplet bases, particularly \( \alpha_k \) and \( \beta_k \). This is because signals from the main body and the rotating part are captured by chirplet bases with different parameters. We separate the body signal from the rotating part using threshold values of \( \alpha_k \) and \( \beta_k \). After the separation, we process the main body signal and the rotating part signal individually for better information extraction. This includes both the extraction of the geometrical features from the main body and the m-D features from the moving parts.

Following the extraction of m-D features from the radar returned signal using either wavelet or chirplet analysis, the time–frequency signatures of the m-D features can be utilised to visualise the oscillation and to extract motion parameters related to the target of interest. The reason for this is that the joint time-frequency analysis can provide time-varying Doppler frequency information. From the joint time–frequency representation of the m-D features, the motion parameters of a target such as the period and amplitude of the oscillation can be estimated [8, 12, 13]. In this paper, we use time–frequency distribution series to obtain the time–frequency representation of the analysed signal [14].

To further the analysis of the extracted m-D features, the autocorrelation of the m-D time sequence signals can be computed and investigated. The autocorrelation of time sequence signals has already been used by Bell and Grubbs to estimate the vibration/rotation rate of micro-motions from a signal containing both vibrational/rotational and stationary parts [25]. It is probable that the estimated rate will be more accurate if this approach is used on data including only vibrational/rotational

<table>
<thead>
<tr>
<th>Table 1 Characteristics of SAR</th>
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<tbody>
<tr>
<td>Centre frequency</td>
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<tr>
<td>Pulse repetition interval (PRI)</td>
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<tr>
<td>Bandwidth</td>
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\[ \text{Figure 3 Top – the original SAR image; bottom – the zoomed in SAR image illustrate the similar plots but at range cell 124} \]

The wavelet transform method consists of decomposing the original signal into components and recombining these components in various groupings in order to obtain the reconstructed signals that represent the desired features. In this case, the features extracted are the body signal and the oscillating signal.
components obtained from the wavelet or chirplet decomposition. This is because the return from the stationary parts strongly overlaps with the return from the vibrational/rotational parts in the Doppler frequency spectrum. Namely, signals from stationary parts and rotating parts superimpose on each other and thus are separable. The extracted m-D features are employed to estimate the target’s motion parameters.

3 m-D analysis

The sample data sets are collected by the APY-6 radar for a rotating antenna. The parameters of the SAR system used to collect this set of data are shown in Table 1. The data have been pulse compressed and motion compensated to the first level. This is an initial data distribution and there may be undiscovered flaws in the processing. A Hamming window is applied to suppress range side lobes. One thousand twenty-four samples were processed. However, the data cuts may not contain edge samples that are reduced by the IF filter. Full data cuts are 72 s long and there are 2048 pulses, 1024 range bins and 8 bytes per complex sample. Using this data set and employing the proposed method, the rotation rate of the antenna is estimated. The rotation rate of the rotating antenna is known to be 4.7 s.

The original SAR image is shown in Fig. 3 (top). The Doppler smearing because of the rotating parts is often well localised in a finite number of range cells. It is reasonable to process the Doppler signal for each range cell independently. Since the ground truth of the target is already known, the data between the range cells 121 and 125 were analysed using the wavelet decomposition method and the adaptive chirplet decomposition method. Therefore the discussion of this paper will deal exclusively with these range cells. Fig. 3 (bottom) shows the zoomed in SAR image between the range cells 115 and 130. We now perform the signal analysis and processing using both the wavelet decomposition method and the adaptive chirplet decomposition method for range cells 123 and 124.

The analysis of the Doppler signals using the wavelet decomposition method is given in Figs. 4–7 for range cells.
123 and 124. Fig. 4 (top) illustrates the time-frequency signature of the original signal at range cell 123, Fig. 4 (middle) illustrates the time-frequency signature of the extracted body signal and Fig. 4 (bottom) illustrates the time-frequency signature of the extracted oscillating signal. The autocorrelation function against time for the extracted body signal is shown in Fig. 5 (top), and the autocorrelation function against time for the extracted oscillating signal is shown in Fig. 5 (bottom). Figs. 6 and 7 demonstrate the overall improvement.

The upper and middle plots in Figs. 4 and 6 show that the target body signature is not extracted well. However, it should be noted that the amplitude at the zero Doppler is much greater than the amplitude at other Doppler frequencies. When the largest peak is removed, other secondary peaks are visible in the middle plot. The relative maximum amplitude of the body signal (middle plot) is smaller than the original signal (upper plot). These plots illustrate that the wavelet decomposition method completely removes the stationary part of the signal corresponding to the zero Doppler but it introduces smaller peaks in other Doppler frequencies. However, the overall procedure improves the extraction of m-D motion parameters. The bottom plots of Figs. 4 and 5 demonstrate the overall improvement.

The analysis of the Doppler signals using the adaptive chirplet decomposition is given in Figs. 8–11 for range cells 123 and 124. We separate contributions from static and dynamic parts of the target based on $\alpha_k$ and $\beta_k$. A threshold of 2200 Hz/s on the Doppler rate $\beta_k$ and $\alpha_k$ value of 100 Hz$^2$ were used to discriminate the static and dynamic parts of the target. Note that $\alpha_k$ is 1/variance. Similar to wavelet analysis, the analysis of each range cell includes time-frequency analysis and autocorrelation function. Fig. 8 (top) illustrates the time-frequency signature of the original signal at range cell 123, Fig. 8 (middle) illustrates the time-frequency signature of the extracted body signal and Fig. 8 (bottom) illustrates the time-frequency signature of the extracted oscillating signal. The autocorrelation function against time for the original signal is shown in Fig. 9 (top). The autocorrelation function against time for the extracted body signal is shown in Fig. 9 (middle), and the
The autocorrelation function against time for the extracted oscillating signal is shown in Fig. 9 (bottom). Figs. 10 and 11 illustrate the similar plots but at range cell 124. As with the wavelet transform method, the features extracted here are the body signal and the oscillating signal.

The results obtained from both the analysis using the wavelet decomposition method and the adaptive chirplet decomposition method allow the rotation rate of the antenna to be estimated. This is done by examining the time–frequency signature of the extracted oscillating signal, and the autocorrelation of the extracted oscillating signal for each of the range cells (using both methods).

Using the time–frequency plot, the rotation rate of the antenna is estimated by measuring the time interval between peaks. The period is the time interval between peaks. As an example, in Fig. 4 (bottom), there are three peaks. The time interval between peak 1 and peak 2, between peak 2 and peak 3, and between peak 1 and peak 3 are measured. The average value is then used to estimate the rotation rate of the antenna. The period of the rotating antenna can be estimated by taking the autocorrelation of the time sequence. The peaks in correlation coefficient correspond to the dominant period of the signal.

The results of range cells 121 and 125 are not clear enough to provide any significant indications towards the rotational rate for either method. However, for the data analysed...
herein, the rotational rate can be estimated for range cells 122, 123 and 124. These values are given in Table 2.

The actual antenna rotational rate is known to be 4.7 s. Clearly, the estimated values for both the wavelet decomposition method and the adaptive chirplet decomposition method for range cells 122, 123 and 124 correspond directly with the known value.

Overall, range cell 123 gives the best results for both methods. This is evident by looking at the time–frequency signatures of the extracted signals. Specifically, the oscillations of the rotating antenna can be clearly seen by looking at the time–frequency signatures of the extracted oscillating signal in Figs. 4 (bottom) and 8 (bottom) (the wavelet and chirplet methods respectively). Furthermore, the autocorrelations of the extracted oscillating signal in Figs. 5 (bottom) and 9 (bottom) (wavelet and chirplet, respectively) both give the estimated rotation rate of 4.8 s, which is very close to the actual value of 4.7 s. It should be noted that when rotational components are only used, the SNR of the desired dominant period of the signal increases significantly compared to the original signal. It is apparent evident from the autocorrelation function, compare the plots of top and bottom of Fig. 5 and the top and bottom of Fig. 9.

### 4 Conclusion

Moving targets cause phase modulation of the SAR phase history corresponding to the target. A special case is rotating
point targets. This report examines the case of a rotating antenna within a SAR target scene collected by the APY-6 radar. Rotating targets cause sinusoidal phase modulation of the azimuth phase history of the SAR collection. The phase modulation may be seen equivalently as a time-varying Doppler frequency. Time-frequency methods are useful tools in the analysis of such signals as they use time integration while still allowing for non-stationary signals.

In this paper, we present two approaches for m-D analysis for the extraction of the m-D features of radar returned signals from targets in a SAR scene. Both approaches have two main processes; the first being the decomposition of the SAR return in order to extract m-D features, and the second being the time–frequency analysis in order to estimate motion parameters of the target in the SAR image. Two different approaches are used in the decomposition process. The first approach, a wavelet decomposition analysis, has been used in our previous m-D analysis work for an ISAR system, and it is presented here for the extraction of m-D features for a SAR system. The second approach, an alternative to the wavelet approach, is based on the adaptive chirplet decomposition combined with time–frequency analysis that is introduced for decomposing the time domain radar signal in SAR.

By applying the proposed procedures to data from a rotating antenna, the effectiveness of both these analysis techniques in a SAR target scenarios is confirmed. From the extracted m-D signatures, information about the target’s micro-motion dynamics, such as rotation rate, has been obtained.

As demonstrated, both wavelet-based and chirplet-based procedures are successful for the extraction of m-D features from SAR data. The wavelet-based method requires the decomposed signal’s constituent parts to be combined in a specific manner in order to extract m-D features. While this is not the case for the chirplet-based method, the chirplet-based method still requires its alpha and beta parameters to be specifically set in order to extract any m-D features. The chirplet-based method establishes itself to be a viable alternative. In general, however, it is shown that the results are much improved after the m-D extraction from the SAR target scenarios has taken place since only the vibration/rotational components are employed. The rotating frequency of the antenna was calculated using these two approaches and the results agree well with available ground truth. Such motion parameters may be useful for describing the target, or serve as inputs to automatic target recognition algorithms.

5 Acknowledgments

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6 References


Table 2 Estimated rotation rate of the antenna (s)

<table>
<thead>
<tr>
<th>Range cell</th>
<th>Wavelet decomposition</th>
<th>Adaptive chirplet decomposition</th>
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<tbody>
<tr>
<td>122</td>
<td>4.6</td>
<td>4.4</td>
</tr>
<tr>
<td>123</td>
<td>4.8</td>
<td>4.8</td>
</tr>
<tr>
<td>124</td>
<td>4.7</td>
<td>4.8</td>
</tr>
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Extracting micro-Doppler radar signatures from experimental helicopter data using adaptive Chirplet-based analysis

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Abstract— This paper highlights the extraction of micro-Doppler (m-D) features from radar signal returns of helicopter targets using adaptive chirplet-based method incorporated with time-frequency analysis. In order for the extraction of m-D features to be realised, the time domain radar signal is decomposed into stationary and non-stationary components based on the parameters of the chirplet bases, particularly chirp rate and variance. The components are then reconstructed by applying the inverse chirplet transform. After the separation of m-D features from the target’s original radar return, time-frequency analysis is then used to estimate the target’s motion parameters. The findings show that the results have higher precision after the m-D extraction rather than before it, since only the rotational components are employed. This proposed method of m-D extraction has been successfully applied to helicopter.

Index Terms— Micro-Doppler, Chirplet, Time-Frequency

I. INTRODUCTION

When the transmitted signal of a coherent radar system hits moving targets, the carrier frequency of the signal will be shifted, known as the Doppler effect. The Doppler frequency shift reflects the velocity of the moving target. Mechanical vibration or rotation of a target, or structures on the target, may induce additional frequency modulations on the returned radar signal, which generate sidebands about the target’s Doppler frequency, called the micro-Doppler effect [1-5]. Micro-Doppler radar signatures can provide unique information for recognition of targets of interest. It has been used to identify the natural resonant frequency of a tractor-trailer truck [6]. The m-D features of Jet Engine Modulation (JEM) lines in a Mi-24 Hind-D helicopter have also been used to estimate the turbine rotation rate and the number of turbine blades [7]. Research has also been conducted on radar Doppler signatures in the area of human gait analysis and experimental helicopter data [4-5,8].

In this paper, we demonstrate examples of micro-Doppler signatures of a target that can be used as radar signatures for target identification. Experimental trials were conducted to investigate and resolve the micro-Doppler radar signatures of a target using an X-band radar. The target in this experiment is a hovering helicopter. Joint time-frequency analysis is the basis of most of the existing methods used to extract m-D features [1-5]. The procedure of wavelet analysis incorporated with time-frequency analysis has been applied in our previous works for a ISAR system [4]. In this paper, we use adaptive chirplet decomposition along with the time-frequency distribution series for the extraction of m-D features.

II. ADAPTIVE CHIRPLET ANALYSIS

Recent studies show that radar signal can be properly modeled using the chirplet basis function. Many events, especially those found in nature, can be modeled as the superposition of short-lived chirp functions. The chirplet transform has been recently used in areas such as multi-component signal detection, instantaneous frequency estimation, time-varying filter design, and interference excision. In this work, the adaptive Gaussian chirplet decomposition is employed to extract m-D features. The Gaussian chirplet is defined as [2,9-10]

$$h_k(t) = \sqrt{\alpha_k} \exp\left\{-\frac{\alpha_k}{2}(t - t_k)^2 + \frac{\beta_k}{2}(t - t_k)^2\right\}$$

where \( \alpha_k > 0 \)

$$t_k, \omega_k, \beta_k \in \mathbb{R}$$

(1)

\( (t_k, \omega_k) \) indicates the time and frequency center of the linear chirp function, the variance \( \alpha_k \) controls the width of the chirp function, and the parameter \( \beta_k \) determines the rate of change of frequency. When the chirp rate \( \beta_k \) is equal to zero and the parameter \( \alpha_k \) approaches zero, the linear chirp function \( h_k(t) \) reduces to a sinusoidal signal. The adaptive spectrogram for the Gaussian chirplet is given as [9-10]

$$AS(t, \omega) = 2 \sum_k |B_k|^2 \exp\left\{-\frac{\alpha_k}{2}(t - t_k)^2 - \frac{1}{\alpha_k}(\omega - \omega_k - \beta_k t)^2\right\}$$

where \( B_k \) is the adaptive coefficient. The algorithm used here estimates the optimal elementary functions in the adaptive signal decomposition. Converting the optimization process to a traditional curve-fitting problem is the basic idea behind this algorithm. A more detailed description of the algorithm is given in [9-10].

The chirplet function has many striking characteristics: 1) The chirplet basis is a well understood four-parameter...
function, $\alpha_k$, $\beta_k$, $t_k$, and $\omega_k$, localized in the joint time-frequency plane. Only a moderate computation time is needed to search for the basis parameters; 2) Since both the amplitude modulation (AM) and frequency modulation (FM) are part of the basis function, the chirplet can more efficiently represent the reflected signal from a target with a rotating part. Only a few set of these bases are needed to approximate the time-frequency structure of the radar signal; and 3) More importantly, the radar returns from the target body and the rotating part can be more easily separated based on the parameters of the chirplet bases, particularly $\alpha_k$ and $\beta_k$. This is because signals from the main body and the rotating part are captured by chirplet bases with different parameters. We separate the body signal from the rotating part using threshold values of $\alpha_k$ and $\beta_k$. After the separation, we process the main body signal and the rotating part signal individually for better information extraction. This includes both the extraction of the geometrical features from the main body and the m-D features from the moving parts.

Following the extraction of m-D features from the radar returned signal using chirplet analysis, the time-frequency signatures of the m-D features can be utilized to visualize the oscillation and to extract motion parameters related to the target of interest. The reason for this is that the joint time-frequency analysis can provide time-varying Doppler frequency information. From the joint time-frequency representation of the m-D features, the motion parameters of a target such as the period and amplitude of the oscillation can be estimated. In this paper, we use time-frequency distribution series to obtain the time-frequency representation of the analyzed signal [10].

To further the analysis of the extracted m-D features, the autocorrelation of the m-D time sequence signals can be computed and investigated. The autocorrelation of time sequence signals can be used to estimate the rotation rate of micro-motions from a signal containing both rotational and stationary parts. It is probable that the estimated rate will be more accurate if this approach is used on data including only rotational components obtained from the chirplet decomposition. This is because the return from the stationary parts strongly overlaps with the return from the rotational parts in the Doppler frequency spectrum. Namely, signals from stationary parts and rotating parts superimpose on each other and thus are separable. The extracted m-D features are employed to estimate the target’s motion parameters.

### III. RESULTS

#### A. Experiment 1

High range resolution (HRR) profiles were collected using a stepped frequency waveform (SFWF) radar mode at X-band between 9.0 to 9.4 GHz; i.e., a synthetic bandwidth of 400 MHz; the frequency step size was 1 MHz. The test target was made up of four corner reflectors, three of which are stationary to provide a geometric reference and a contrast to the shape of the oscillating reflector in the HRR profiles. The test target was located at 2 km from the radar system. HRR scans were performed at 2 kHz PRF. This data set is used to extract the m-D features using adaptive chirplet-based method incorporated with time-frequency analysis. Figure 1a illustrates the time-frequency signature of the original signal. The adaptive chirplet-based is now used to separate the stationary and oscillating components. After the separation, time-frequency distribution series is used to obtain the time-frequency signature of the analyzed signal [10].

![Fig. 1](image)

Fig. 1. a) Time-frequency signature of the original signal; b) Time-frequency signature of the extracted body signal; and c) Time-frequency signature of the extracted oscillating signal.

#### B. Experiment 2

The rotational motion of rotor blades in a helicopter imparts a periodic modulation on radar returns. The rotation-induced Doppler shifts relative to the Doppler shift of the fuselage (or body) occupy unique locations in the frequency domain. Whenever a blade has specular reflection such as at the advancing or receding point of rotation, the particular blade transmits a short flash to the radar return. The rotation rate of the rotor is directly related to the time interval between these flashes. The duration of a flash is determined by the radar wavelength and by the length and rotation rate of the blades.
A flash resulting from a blade with a longer length and a radar with a shorter wavelength will have a shorter duration \([4]\).

The helicopter employed in the experiment is hovering above the ground at a height of approximately 60 m and at a range of 2.5 km from the radar. The main rotor is comprised of five blades and the tail rotor consists of six blades. The rotation rate of the main rotor blades is known to be 203 rpm for this helicopter. The rotation rate of the tail rotor blades is known to be 1030 rpm for this helicopter. The experiment was conducted using an X-band radar. Two trials were conducted, both with a time interval of 96.5 ms.

In order to demonstrate the procedure of m-D analysis, the results are now presented. First, the Fourier transform of the original radar returned data is computed and the image obtained is shown in Figure 2. As can be seen from the image, a main frequency bin with a large amplitude exists in the middle of the spectrum representing the helicopter’s body vibration. Surrounding this frequency bin, one can observe two other less prominent peaks representing the frequency of the main rotor and tail rotor rotation rate. These are the m-D features that are to be extracted. By applying the adaptive chirplet analysis as described above, the m-D features are obtained. The next step in the procedure is to make use of time-frequency analysis in order to depict the m-D oscillations and to estimate the target's motion parameters. The time-frequency signature of the original returned signal using time-frequency distribution series is given in Figure 3a. The stationary body is observed as a fairly constant signal at 0 Hz on the frequency axis. The micro-motion dynamics of the tail rotor are seen as small, quick flashes just below the constant stationary body. The micro-motions of the main rotor are not clearly visible as the three large flashes with the large period. The time-frequency signature of the extracted body signal and oscillating signal are given in Figures 3b and 3c. In Figure 3c, not only are the flashes made clearer, but the flashes are in fact stronger peaks than those observed in the time-frequency signature of the original signal in Figure 3a. The rotation rate of the main rotor blades is calculated from Figure 3c as follows. It is known that the main rotor of this helicopter has five blades. This is an important point as it means that the specular reflection at the advancing and receding point of rotation do not coincide with one another. Therefore, the resulting time-frequency plot will show alternating strong and weak flashes. This is indeed the case in Figure 3c. The period between the two strong flashes, i.e. the period between two blades at the advancing point of rotation, is 0.0591 s. Since there are five blades, this value is multiplied by five in order to obtain 0.2955 s, the length of time taken by a single blade to complete one full rotation. The number of rotations in one minute is given by \((60 \text{ s/min})/(0.2955 \text{ s/rotation}) = 203.05 \text{ rpm}\), which is in agreement with the actual value known to be 203 rpm. Similarly, the rotation rate of the tail rotor is measured using Figure 3c in a similar manner as it was computed for the main rotor. The rotation rate is calculated to be approximately 1031 rpm.

The rotation rate can also be estimated by taking the autocorrelation of the time sequence data of the extracted m-D features. The autocorrelation of the original signal is given in Figure 4a. It is evident that not much information can be easily extracted from this plot. The auto-correlation of the extracted body signal and auto-correlation of the extracted oscillating signal are given in Figures 4b and 4c. The distances between the peaks describe the period of the rotation, which
in turn allows one to estimate the rotation rate. Using Figure 4c, the rotation rate of the tail rotor blades is calculated to be 1031 rpm, which is consistent with the value from the time-frequency analysis. Both results agree with the actual value. Note that without m-D extraction, it is more difficult to estimate the rotation rate from the autocorrelation of the original data as shown in Figure 4a. The peaks are much less prominent due to higher interference. The signal-to-noise ratio (SNR) is significantly enhanced after m-D extraction as compared to the original data.

**IV. CONCLUSION**

This paper presents a procedure for m-D analysis in order to extract the m-D features of radar returned signals from targets. The method combines both adaptive chirplet-based and time-frequency analysis in order to extract the m-D features of radar target returns. This methodology has been applied to helicopter data. The results show that the proposed methodology is an effective tool for extracting m-D features. The rotation rates of the main rotor and tail rotor blades of the helicopter are successfully computed. The rotation rate of the helicopter is also estimated by taking the autocorrelation of the time sequence data. In general, it is shown that the results are much improved after the m-D extraction has taken place since only the rotational components are employed. The experimental results agree with the expected outcome.

**REFERENCES**


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Abstract—In this paper, we report the efficiency of the Fourier–Bessel transform (FBT) and time–frequency (TF)-based method in conjunction with the fractional Fourier transform (FrFT), for extracting micro-Doppler (m-D) radar signatures from the rotating targets. This approach comprises mainly of two processes, with the first being the decomposition of the radar return, in order to extract m-D features, and the second being the TF analysis to estimate motion parameters of the target. In order to extract m-D features from the radar signal returns, the time domain radar signal is decomposed into stationary and nonstationary components using the FBT in conjunction with the FrFT. The components are then reconstructed by applying the inverse Fourier–Bessel transform (IFBT). After the extraction of the m-D features from the target’s original radar return, TF analysis is used to estimate the target’s motion parameters. This proposed method is also an effective tool for detecting maneuvering air targets in strong sea clutter and is also applied to both simulated data and real-world experimental data.

Index Terms—Fourier–Bessel transform (FBT), fractional Fourier transform (FrFT), short-time Fourier transform (STFT), time–frequency (TF) analysis, Wigner–Ville distribution (WVD).

I. INTRODUCTION

The most widely used time–frequency (TF) transforms are the short-time Fourier transform (STFT) and Wigner–Ville distribution (WVD). In STFT, time and frequency resolutions are limited by the size of the window function used in calculating STFT. For monocomponent signals, WVD gives the best time and frequency resolutions without any cross-terms. However, in the case of multicomponent signals, the occurrence of cross-terms degrades the readability of the TF representation and limits the usefulness of WVD.

In order to achieve cross-term-free WVD, Pachori and Sircar in [1] and [2] used the Fourier–Bessel transform (FBT) to decompose the multicomponent signal and then applied WVD to each component separately to analyze its TF distribution. An improvement of this method has been proposed in [3]. This approach is applied to the multicomponent signal whose signal components overlap only in the time domain. This is applied to the simulated data. However, if the components of a multicomponent signal overlap in both time and frequency domains, then it is not possible to separate the signal components using the method in [1]–[3]. However, in real-time applications, several scenarios are related to a multicomponent signal whose signal components overlap in both time and frequency domains. Therefore, FBT and WVD alone cannot be used in real-time applications. This paper presents a new approach, which is based on the FBT in conjunction with the fractional Fourier transform (FrFT) to decompose the nonstationary signal whose component frequencies overlap in both time and frequency domains. The WVD is then applied to each component separately to analyze its TF distribution. This approach is an advancement to the method used by Pachori and Sircar [1], [2] and has now several real-time applications. We have successfully demonstrated the proposed approach to experimental data sets.

This paper is basically organized into six sections. In Section II, a brief introduction to TF analysis, particularly STFT and WVD, is presented. Sections III and IV deal with the mathematical formulation of FBT and FrFT. In Section V, the application of the Fourier–Bessel and TF (FB–TF) method, in removing the interference terms that occur when a multicomponent signal is analyzed using WVD, is presented. Section VI demonstrates the effectiveness of the proposed method in extracting micro-Doppler (m-D) features of the rotating targets and also in the reduction of sea clutter, thus by enhancing the target detection.

II. TF ANALYSIS

TF techniques are broadly classified into two categories: linear transforms and quadratic (or bilinear) transforms.

A. Linear TF Transforms

All those TF representations that obey the principle of superposition can be classified under the linear TF transforms. Some of the linear TF transforms are STFT, continuous wavelet transform, and the adaptive TF transforms. STFT is the most widely used TF technique among the linear TF transformations.
The basic principle behind STFT is segmenting the signal into narrow time intervals using a window function and taking the Fourier transform (FT) of each segment. STFT has limited TF resolution which is determined by the size of the window used. The uncertainty principle prohibits the usage of arbitrarily small duration and small bandwidth windows. A fundamental resolution tradeoff exists: A smaller window has a higher time resolution but a lower frequency resolution, whereas a larger window has a higher frequency resolution but a lower time resolution. Hence, STFT is not capable of analyzing transient signals that contain high- and low-frequency components simultaneously.

### B. Quadratic TF Transforms

Cohen, in 1966, showed that all the existing bilinear TF distributions could be written in a generalized TF form. In addition, this general form can be used to facilitate the design of new TF transforms [4]. The prominent members of Cohen’s class include WVD, pseudo Wigner–Ville distribution, Choi–Williams distribution, cone-shaped distribution, and adaptive kernel representation [5]. The WVD was originally developed in the area of quantum mechanics by Wigner [6] and then introduced for signal analysis by Ville [7]. Compared to STFT, WVD has much better time and frequency resolution, but the main drawback of the WVD is the cross-term interference. This interference phenomenon shows frequency components that do not exist in reality and considerably affect the interpretation of the TF plane. Cross-terms are oscillatory in nature and are located midway between the two components [4]. The presence of cross-terms severely limits the practical applications of WVD. Various modified versions of WVD have been developed to reduce cross-terms. These techniques include distributions from Cohen’s class by Cohen [4], nonlinear filtering of WVD by Arce and Hasan [8], S-method by Stankovic [9], and polynomial WVD by Boashash and O’Shea [10]. The application of the FBT to obtain a cross-term-free WVD distribution is explained in Section V.

### III. Fourier–Bessel Transform

The FBT decomposes a signal into a weighted sum of an infinite number of Bessel functions of the zeroth order. Mathematically, the FBT $F(\rho)$ of a function $f(r)$ is represented as [11]

\begin{equation}
F(\rho) = 2\pi \int_{0}^{\infty} f(r) J_0(2\pi \rho r) \, dr \tag{1}
\end{equation}

\begin{equation}
f(r) = 2\pi \int_{0}^{\infty} F(\rho) J_0(2\pi \rho r) \, d\rho \tag{2}
\end{equation}

where $J_0(2\pi \rho r)$ are the zeroth-order Bessel functions and $\rho$ is the transform variable.

FBT is also known as the Hankel transform. As the FT over an infinite interval is related to the Fourier series over a finite interval, therefore, the FBT over an infinite interval is related to the Fourier–Bessel (FB) series over a finite interval. The FB series expansion of a signal $x(t)$, in the interval $(0, T)$, is given as [1]

\begin{equation}
x(t) = \sum_{r=1}^{M} A_r J_0 \left( \frac{\alpha_r}{T} t \right), \quad 0 < t < T. \tag{3}
\end{equation}

FB coefficients $A_r$ are computed by using following equation:

\begin{equation}
A_r = \frac{2}{T^2} \int_{0}^{T} t x(t) J_0 \left( \frac{2\pi \alpha_r}{T} t \right) \, dt \tag{4}
\end{equation}

where $\alpha_r$, $r = 1, 2, 3, \ldots M$, denotes the ascending order positive roots of $J_0(t) = 0$. Since the Bessel function supports a finite bandwidth around a center frequency, the spectrum of the signal can be represented better using FB expansion. As the Bessel functions form the orthogonal basis and decay over the time, nonstationary signals can be better represented using FB expansion [12]. It turns out to be a one-to-one relation between the frequency content of the signal and the order of the FB expansion, where the coefficients attain maximum amplitude [13]. As the center frequency of the signal is increased, it is observed that the order of the FB coefficients is increased. Similarly, there is a relationship between the bandwidth of the signal and the range of FB coefficients. In particular, the range of FB coefficients increases with the increase in the bandwidth of the signal [14]. Since both amplitude modulation and frequency modulation are part of the Bessel basis function, the FB expansion can represent the reflected signal from a rotating target more efficiently.

### IV. FrFT

FrFT is the generalization of the classical FT. The applications of FrFT can be found in signal processing, communications, signal restoration, noise removal, and many other science disciplines. It is a powerful tool used for the analysis of time-varying signals. The FrFT is a linear operator that corresponds to the rotation of the signal through an angle, i.e., the representation of the signal along the axis $u$, making an angle $\alpha$ with the time axis. The $\alpha$th-order FrFT of the function $f(u)$ is defined as [15]

\begin{equation}
f_{\alpha}(u) = \int f(u') K_{\alpha}(u, u') \, du' \tag{5}
\end{equation}

\begin{equation}
K_{\alpha}(u, u') = A_{\phi} \exp \left[ i \pi (\cot \phi u^2) - 2 \csc uu' + \cot \phi u'^2 \right] \tag{6}
\end{equation}

where

\begin{equation}
\phi = \frac{\alpha \pi}{2} \tag{7}
\end{equation}

\begin{equation}
A_{\phi} = \sqrt{1 - i \cot \phi}. \tag{8}
\end{equation}

For $\alpha = 1$, we find that $\phi = \pi/2$, $A_{\phi} = 1$, and

\begin{equation}
f_1(u) = \int_{-\infty}^{\infty} \exp(-i2\phi uu') f(u') \, du'. \tag{9}
\end{equation}
Fig. 1. (a) STFT of the multicomponent signal. (b) WVD of the multicomponent signal.

Fig. 2. FB coefficients of the multicomponent signal.

For $a = 0$, FrFT reduces into the identity operation. For $a = 1$, FrFT is equal to the standard FT of $f(u)$. For $a = -1$, FrFT becomes an inverse FT. FrFT can transform a signal either in time or in frequency domain into a domain between time and frequency. FrFT depends on the parameter $a$ and can be interpreted as rotation by an angle $\phi$ in the TF plane. The FrFT of a signal can also be interpreted as a decomposition of the signal in terms of chirps [16].

V. Simulation Results

In this section, we demonstrate the application of the proposed method by removing the cross-terms in the WVD representation of a multicomponent signal. Consider a discrete time domain signal $s[n]$ which is the sum of the three linear chirps given by

$$s[n] = \sum_{i=1}^{3} A_i \exp \left(2\pi f_i n T + \frac{1}{2} \beta_i (n T)^2 \right) \tag{10}$$

where $A_i$ denotes the amplitudes of the constituent signals, $f_i$ denotes the fundamental frequencies, $\beta_i$ denotes the chirp rates, and $T$ is the sampling interval. Fig. 1(a) and (b) shows the STFT and WVD representations of the multicomponent signal in (10).

From Fig. 1(a), it is evident that the STFT representation of the signal is free from cross-terms but its time and frequency resolutions are poor. As expected, WVD gives good time and frequency resolution but is corrupted with the occurrence of cross-terms. In order to remove these cross-terms, the signal is analyzed using FBT. FB coefficients are calculated using (4). Fig. 2 shows the FB coefficients of the multicomponent signal. By taking the most significant order of the FB coefficients, the multicomponent signal can be decomposed into its individual components. Table I shows the order of the significant FB coefficients that are selected for each chirp signal. Individual components are reconstructed by applying inverse Fourier-Bessel transform (IFBT) using the selected FB coefficients. Fig. 3(a)–(c) shows the WVD representation of each component of the multicomponent signal. Fig. 3(d) shows the plot obtained by adding WVD representations of the three linearly frequency modulated (LFM) signals together.

The results in Fig. 3 show that the occurrence of cross-terms in WVD can be eliminated if the multicomponent signal is decomposed into its individual components, by expanding the signal using FB series and applying WVD to the constituent signals separately. Using FBT, we can separate the components of the multicomponent signals if their frequencies do not overlap in the frequency domain. However, if their frequencies overlap in the time and/or frequency domain, it is not possible to separate them using FBT. By using FrFT and FBT, we can separate the components of the multicomponent signal whose frequencies overlap in the time and/or frequency domain.

Fig. 4(a) shows the STFT representation of the two LFM signals whose frequencies overlap in the frequency domain. The TF characteristics of the signal were rotated by 36° in the clockwise direction by using FrFT such that their frequency components do not overlap in the frequency domain. Fig. 4(b) shows the STFT representation of the signal after rotation. Now, using the FBT, the two frequency components of the multicomponent signal were separated. Fig. 4(c) and (d) shows the separated components of the signal. After the separation of the components, the TF characteristics of the signal were rotated by 36° in the counterclockwise direction using FrFT. Fig. 4(e) and (f) shows the separated LFM components. It should be emphasized here that this approach works well for any number of chirps with different angles.

VI. Experimental Results

In this section, we demonstrate the application and effectiveness of the FB-TF method, with three different types of radar data obtained in various scenarios.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>SELECTED FOURIER–BESSEL COEFFICIENTS FOR THE CHIRP SIGNALS</th>
</tr>
</thead>
<tbody>
<tr>
<td>signal</td>
<td>Required FB Coefficients</td>
</tr>
<tr>
<td>chirp 1</td>
<td>(1-45)</td>
</tr>
<tr>
<td>chirp 2</td>
<td>(108-150)</td>
</tr>
<tr>
<td>chirp 3</td>
<td>(151-230)</td>
</tr>
</tbody>
</table>
A. Rotating Reflectors

Experimental trials were conducted to investigate and determine the m-D radar signatures of targets using an X-band radar. The target used for this experimental trial was a spinning blade with corner reflectors attached. These corner reflectors were designed in such a way that they reflect electromagnetic radiation with a minimal loss. These controlled experiments can simulate the rotating type of objects, generally found in an indoor environment such as a rotating fan and in an outdoor environment such as a rotating antenna or rotors. Controlled experiments allow us to set the desired rotation rate of the target, to cross check and assess the results. The target employed in this experiment was at a range of 300 m from the radar. STFT representation is utilized in order to depict the m-D oscillation [17].

Fig. 5(a) shows the STFT representation of the signal obtained from one rotating corner reflector, facing the radar. From the TF signature, we can observe that the m-D of the rotating corner reflector is a time-varying frequency spectrum. The second weaker oscillation represents the reflection from the counterweight that was used to stabilize the corner reflector during the operation. It also contains a constant frequency component which is due to the reflection from the stationary body of the corner reflector. FBT was utilized in order to separate the stationary component from the rotating component. Fig. 5(b) shows the TF signature of the extracted oscillating signal. Fig. 5(c) shows the TF signature of the extracted body signal. The rotation rate of the corner reflector is directly related to the time interval of the oscillations. The estimated rotation rate of the corner reflector was about 60 r/min. Rotation rates estimated by the TF analysis agree with the actual values.

B. Rotating Antenna in SAR

Radar returns were collected from the rotating antenna using APY-6 radar in a synthetic aperture radar (SAR) scenario. Using these data sets, we extracted the m-D features relating to a rotating antenna. The m-D features for such rotating targets may be seen as a sinusoidal phase modulation of the SAR azimuth phase history. The phase modulation may equivalently be seen as a time-varying Doppler frequency [21]. The Fourier transform of the original time series is shown in Fig. 6. The rotating antenna is located close to the zero Doppler and cannot be detected using the FT method. The original time series was decomposed using FBT, and rotating and stationary components of the signal were captured by different orders of FB coefficients. Stationary signal and oscillating signals were reconstructed by applying IFBT on the selected coefficients.

Fig. 7(a) illustrates the TF signature of the original signal, and Fig. 7(b) illustrates the TF signature of the extracted oscillating signal, whereas Fig. 7(c) illustrates the TF signature of the extracted body signal. Using the TF plot, the rotation rate of the antenna is estimated by measuring the time interval between the peaks. The period is the time interval between peaks [21]. As an example, in Fig. 7(b), there are three peaks. The time intervals between peaks 1 and 2, 2 and 3, and 1 and 3 were measured [18]. The average value was then used to estimate the rotation rate. The estimated rotation rate is 4.8 s, which is very close to the actual value of 4.7 s.

C. Maneuvering Air Target in Sea Clutter

The signals used in the following analysis were collected from the experimental aircraft. It was performing maneuvers
Fig. 7. (a) TF signature of the original signal. (b) TF signature of the extracted oscillating signal. (c) TF signature of the extracted body signal.

Fig. 8. (a) FT of signal 1—Nonaccelerating target far from Bragg’s lines. (b) FT of signal 2—Accelerating target far from Bragg’s lines. (c) FT of signal 3—Target very close to Bragg’s lines. (d) FT of signal 4—Target crossing the sea clutter.

and being tracked by a high-frequency surface wave radar. Since the sea clutter is more stronger than the target signal, detecting a target in the presence of the sea clutter is a challenging problem. For efficient detection and extraction of the target features, the target signal has to be separated from the sea clutter and should be analyzed using TF analysis [5]. One way to separate the target signal from the sea clutter is to use digital filtering techniques in the frequency domain.

1) Filtering in Frequency Domain: The Fourier spectra of the four signals are shown in Fig. 8(a)–(d), respectively. We observe that the target signal is buried in the background that consists of sea clutter and noise (thermal and atmospheric). Here, the sea clutter is due to the Bragg scattering from the surface of the ocean [5]. The Fourier spectra in Fig. 8 contained two large spectral lines around the zero Doppler, which correspond to the sea clutter components. Fig. 8(c) clearly illustrates that, when the target is accelerating close to zero frequency or sea clutter, the FT method fails to provide optimum detection performance [5].

Since the sea clutter appeared around zero Doppler, it can be removed using digital filtering techniques in the frequency domain. Fig. 9 shows the band-rejection filter that was used to filter the sea clutter. Fig. 10(a), (c), and (e) shows the STFT plots of the three signals, respectively. Fig. 10(b), (d), and (f) shows the results of separating the target from the sea clutter using the band-rejection filter.

The aforementioned results show that the target signal and the sea clutter can be separated using filtering techniques in the frequency domain. However, these filtering techniques fail to separate the target signal from the sea clutter when the target signal crosses the sea clutter. Fig. 10(g) shows the STFT representation of the target signal crossing the sea clutter, and Fig. 10(h) shows the STFT representation of the target signal, after the sea clutter is removed using the band-rejection filter. The aforementioned results show that it is not possible to separate the target signal and sea clutter if the target is crossing the sea clutter. In the next section, we proposed a method to separate the target signal and sea clutter even when the target signal crosses the sea clutter.

2) Filtering Using FB-TF Method: Radar returns were analyzed using FBT, and FB coefficients were calculated using (4). Fig. 11 shows the plot of FB coefficients of signal 1. We can observe that the returns from the sea clutter were captured by the lower order FB coefficients and the target signal was captured...
Fig. 11. FB coefficients of signal 1.

TABLE II
SELECTED FOURIER–BESSLER COEFFICIENTS FOR THE SEA-CLOUTER AND TARGET

<table>
<thead>
<tr>
<th>Signals</th>
<th>Sea Clutter Coefficients</th>
<th>Target Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal 1</td>
<td>(1-25)</td>
<td>(120-138)</td>
</tr>
<tr>
<td>Signal 2</td>
<td>(1-25)</td>
<td>(44-84)</td>
</tr>
<tr>
<td>Signal 3</td>
<td>(1-25)</td>
<td>(141-199)</td>
</tr>
</tbody>
</table>

by the higher order FB coefficients of the Fourier–Bessel basis functions. Since the target signal and sea clutter are captured by different orders of FB coefficients, we can easily separate the target from the sea clutter. Table II contains selected FB coefficients for the sea clutter and target for three signals.

The target signal was reconstructed by applying IFBT on the selected FB coefficients of the target. After the target signal is separated from the sea clutter, TF representations like STFT and WVD were used to extract more information from it. Plots in Fig. 12 show the results of STFT and FB-STFT methods for signals 1, 2, and 3. By using FBT, we can separate the target signal from the sea clutter more efficiently even when the target signal is very close to sea clutter. In the case of the target signal crossing the sea clutter, as shown in Fig. 13, it is possible to separate them using FrFT and FBT. By using FrFT, the TF signature of the signal is rotated in counterclockwise direction through an angle \( \theta \) such that the target signal is aligned perpendicular to the frequency axis at around zero Doppler. Now, the signal is analyzed using FBT, and the target signal is separated by selecting the higher order FB coefficients corresponding to the target signal. The TF signature of the target signal is reconstructed by applying IFBT on the selected FB coefficients. Now, the TF signature of the target signal is rotated in the clockwise direction through an angle \( \theta \) to obtain the separated target signal. Fig. 13(b) and (c) shows the FB-STFT and FB-WVD representations of the signal after the target is separated from sea clutter.

VII. CONCLUSION

By applying the proposed method to simulated and several experimental data sets, the effectiveness of this FB-TF technique is confirmed. This method combines both FBT and TF analysis to extract the m-D features of the radar returns. By applying the proposed method to the rotating antenna data and to the rotating corner reflector data, the potential of the proposed method is ascertained. From the extracted m-D signatures, the information about the target’s micromotion dynamics such as the rotation rate is obtained. The experimental results agree with the expected outcome. FB-TF proves to be a useful tool in the reduction of the sea clutter and the target enhancement. Using the FB-TF method, we could separate the target from the strong sea clutter. In the case of the target signal crossing the sea clutter, the target signal was separated from the sea clutter using the FrFT and FBT. Results demonstrate that the proposed method could be used as a potential tool for detecting and enhancing low observable maneuvering, accelerating air targets in the littoral environments.
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Separation of Nonstationary Signals using Fourier Bessel, Fractional Fourier and Time-Frequency analysis

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Abstract: This paper presents new approach for detecting a maneuvering target from strong sea clutter environment, which is based on a combination of Fourier Bessel transform, fractional Fourier transform and time-frequency based method. In order to detect a maneuvering target from radar signal returns, the time domain radar signal is decomposed into stationary and non-stationary components using Fourier Bessel transform. The components are then reconstructed by applying both the fractional Fourier transform and the inverse Fourier Bessel transform. This proposed approach is applied to both simulated data and measured data. Results demonstrate the effectiveness of the proposed method for detecting maneuvering target in a heavy sea clutter.

Keywords: Fourier Bessel transform, Time-Frequency analysis, Fractional Fourier transform, short-time Fourier Transform, Wigner Ville Distribution.

1. INTRODUCTION

Most of the signals that we encounter in real world are non-stationary in nature. Fourier transform fails to produce desired results if the signals are non-stationary. Time-frequency transform methods are used to analyze non-stationary signals. Most widely used time-frequency transforms are short-time Fourier Transform (STFT) and Wigner Ville distribution (WVD). In STFT, time and frequency resolutions are limited by the size of window function used in calculating STFT. For mono-component signals, WVD gives best time and frequency resolutions without any cross terms. However, in the case of multi-component signals, the occurrence of interference terms degrades the readability of the time-frequency representation and limits the usefulness of WVD. Pachori et al. in [1-2] used FBT to decompose the multi-component signal whose frequency components overlap and crosses in both the time and/or frequency domain. This can be achieved by applying Fourier Bessel transform (FBT) to the multi-component non-stationary signal and choosing the Fourier Bessel (FB) coefficients corresponding to each signal. Component signals can be reconstructed by applying the inverse Fourier-Bessel transform (IFBT) on the selected FB coefficients. After separation of the signals, the time-frequency analysis is then used to estimate the instantaneous frequency of each signal.

II. FOURIER BESSEL TRANSFORM – FRACTIONAL FOURIER TRANSFORM

The FBT decomposes a signal in to a weighted sum of an infinite number of Bessel functions of first kind and of nth order. FBT is also known as Hankel transform. As the FT over an infinite interval is related to the Fourier series over a finite interval, so the FBT over an infinite interval is related to the FB series over a finite interval. FB series expansion of a signal \( \Phi(t) \) in the interval \((0,1)\) is given as:

\[
\Phi(t) = \sum_{k=0}^{\infty} A_k J_n(\lambda_k t)
\]

FB coefficients, \( A_k \) are computed by using following equation.

\[
A_k = \frac{2}{(j_{n+1}(\lambda_k))} \int_0^1 t \Phi(t) J_n(t\lambda_k) dt
\]

As the Bessel functions form orthogonal basis and decay over the time, non-stationary signals can be better represented using FB expansion [1-2]. Since Bessel function supports a finite bandwidth around a center frequency, the spectrum of the signal can be represented better using FB expansion. It turns out to be a one-to-one relation between frequency content of the signal and the order of the FB expansion, where the coefficients attain maximum amplitude [1-2]. As the center frequency of the signal is varied, it is observed that the order of the FB coefficients is increased. Similarly there is a relationship between the bandwidth of the signal and the range of FB.
coefficients. In particular, the range of FB coefficients increases with the increase in the bandwidth of the signal [1-2].

The Fractional Fourier transform (FRFT) is a time-frequency distribution and an extension of the classical Fourier transform. The applications of FRFT can be found in time-frequency analysis, signal processing, communications, and signal restoration and in many other science disciplines. It can be used to filter noise, provided the noise does not overlap with the desired signal in the time-frequency domain. FRFT can be interpreted as a rotation by an angle $\alpha$ in the time-frequency plane or decomposition of the signal in terms of chirps.

III. SIMULATION RESULTS

Using FBT, we can separate the components of the non-stationary signal, if their frequencies do not overlap in the frequency domain. But if their spectra overlap in both time and frequency domain, it is not possible to separate them using FBT alone. By using FRFT and FBT, we can separate the components of the non-stationary signal whose frequencies overlap simultaneously in the time and frequency domain.

Figure 1a shows the STFT representation of the three LFM signals whose frequencies overlap in both time and frequency domain. In order to separate the components, time-frequency characteristics of the signal was rotated by $18^\circ$ in the counterclockwise direction using FRFT, such that their frequency components have no overlap in the frequency domain. Figure 1b shows the STFT representation of the signal after rotation. Now FB coefficients are calculated using the FBT. Figure 2 shows the plot of the obtained FB coefficients. The three components of the non-stationary signal were separated by applying IFFT on the selected FB coefficients. Figures 3b, 3c, and 3d show STFT representation of the separated components of the signal. After the separation of the components, time-frequency characteristics of the separated components were rotated by $18^\circ$ in the clockwise direction using FRFT. Figures 4b, 4c and 4d show the extracted components from the non-stationary signal in Figure 4a. It should be emphasized here that this approach works well for any number of LFMs with different angles.

![Fig. 1. a) STFT of the three LFM signal b) STFT of the rotated three LFM signal](image)

![Fig. 2. FB coefficients of the rotated three LFM signal](image)

<table>
<thead>
<tr>
<th>signal</th>
<th>FB Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>component 1</td>
<td>(1-92)</td>
</tr>
<tr>
<td>component 2</td>
<td>(93-171)</td>
</tr>
<tr>
<td>component 3</td>
<td>(172-256)</td>
</tr>
</tbody>
</table>
Fig. 3. a) STFT of rotated 3 LFM signal  b) STFT of first LFM signal  c) STFT of second LFM signal  d) STFT of third LFM signal

Fig. 4. a) STFT of three LFM signal  b) STFT of first LFM signal  c) STFT of second LFM signal  d) STFT of third LFM signal
IV. EXPERIMENTAL RESULTS

Decomposing Time-Varying Radar Signals in a Strong Sea-Clutter: Radar signals were obtained experimentally by detecting the maneuvering aircraft, King-Air 200, using high frequency surface wave radar with a 10 element linear receiving antenna array. The set of 69 complex valued radar signals were used in the analysis. HFSWR was operating at 5.672 MHz and scans were performed at a pulse repetition frequency of 9.17762 Hz. Each trial corresponds to a block of 256 pulses. Therefore, the CIT (coherent integration time) of each signal was 27.89 sec [5, 6]. As shown in Figure 5, the King-Air performed two figure-of-eight maneuvers. Each figure-of-eight maneuver consisted of two circles with an approximate diameter of 10 km [8].

Radar returns were analyzed using FBT and FB coefficients were calculated using (2). Figure 6 shows the plot of FB coefficients of the signal 1. We can observe that radar returns from the sea clutter were captured by the lower order FB coefficients and the target signal was captured by the higher order FB coefficients. Since the target signal and sea clutter are captured by different orders of FB coefficients, we can easily separate the target from the sea clutter. Target signal was reconstructed by applying IFBT on the selected FB coefficients of the target. After the target signal is separated from the sea clutter, time-frequency representations like STFT and WVD were used to extract more information from it. Figures 7a and 7b show the results of STFT representation of the signals 1 and 2. By using FBT, we can separate the target signal from the sea clutter more efficiently even when the target signal is very close to the sea clutter. Figures 7c and 7e show the FB-STFT and FB-WVD representations of the separated target component from signal 1.

In the case of target signal crosses the sea clutter, as shown in Figure 7b, it is possible to separate them using FRFT and FBT. By using FRFT, time-frequency signature of the signal is rotated in counter clockwise direction through an angle $\alpha$ such that the target signal is aligned perpendicular to the frequency axis at around zero Doppler. Now the signal is analyzed using FBT and the target signal is separated by selecting the higher order FB coefficients corresponding to the target signal. Time-frequency signature of the target signal is reconstructed by applying IFBT on the selected FB coefficients. Now the time-frequency signature of the target signal is rotated in the clockwise direction through an angle $\alpha$ to obtain the separated target signal. Figures 7d and 7f show the FB-STFT and FB-WVD representation of the signal after the target signal is separated from sea clutter.

CONCLUSION

This paper presents FB-TF based approach in conjunction with FRFT for the separation of non-stationary signals whose spectra overlap and crossing in time and/or frequency domain. The FB-TF method is used to extract signal components from non-stationary signal if the components of the signal overlap in time domain. If the spectra of the signal components overlap in both time and frequency domain, then FBT in conjunction with FRFT is used for the separation of the spectral components. FB-TF proves to be a useful tool in the reduction of the sea clutter and target enhancement. Results demonstrate that the proposed method could be used as a potential tool for detecting and enhancing the low observable maneuvering, accelerating air targets in the littoral environments.
ACKNOWLEDGMENT

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Intelligent target recognition using micro-Doppler radar signatures

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ABSTRACT

We present an effective quadratic time-frequency S-method based approach in conjunction with the Viterbi algorithm to extract m-D features. The effectiveness of the S-method in extracting m-D features is demonstrated through the application to indoor and outdoor experimental data sets such as rotating fan and human gait. The Viterbi algorithm for the instantaneous frequency estimation is used to enhance the weak human micro-Doppler features in relatively high noise environments. As such, this paper contributes additional experimental micro-Doppler data and analysis, which should help in developing a better picture of the human gait micro-Doppler research and its applications to indoor and outdoor imaging and automatic gait recognition systems.

Keywords: Micro-Doppler, Fourier Transform, S-method, Viterbi, Non-stationary

1. INTRODUCTION

The ongoing war on terror is quickly becoming asymmetric in nature wherein the old rules of conflict are no longer applicable. The adversary is usually dispersed and attacks come from non-conventional theaters, such as the urban and littoral zones where detection of the well-concealed adversary is impossible by conventional means. Adversaries or criminals hide within and behind building walls and dense foliage making it impossible to detect them by optical means. The only technology that has a reasonable chance of success is microwave radar, which has the ability to penetrate building wall materials, such as concrete, brick, plaster, cloth, and dense foliage.

There is continuing interest in being able to detect illegal activity behind opaque walls. Radar signals reflected from individual humans contain biometric information related to periodic contraction of a heart, blood vessels, lungs, other fluctuations of the skin in the process of breathing and heart beating. These processes are cyclical and range in frequency from 0.8 to 2.5 Hz for heartbeat and 0.2-0.5 Hz for breathing. Such micro-Doppler (m-D) signals are concealed within other naturally varying signals making detection extremely difficult.

Obtaining radar signatures of humans is another important application of m-D. The human walking gait is a complex motion behavior that comprises different movements of individual body parts. Recently, the development of automatic radar gait recognition technology has grown. Because radar gait recognition technology is so new, researchers are assessing its uniqueness and methods by which it can be evaluated. Various computer vision and ultrasound techniques have been developed to measure gait parameters. However, real-time automatic gait recognition radar systems have recently been recognized as advantageous solutions for detecting, classifying and identifying human targets from stand-off distances under conditions of multipath, clutter, and foliage obstruction in all light and weather conditions. Radar has certain advantages over electro-optical (EO) systems and video
cameras in that it can penetrate clothes, does not require light, and operates in fog and other low-visibility weather conditions. Several research labs and universities have been involved in radar-based gait recognition technology for the past years; nevertheless, more fundamental research is still needed in this area. The radar sends out a signal and then measures the echo that contains rich information about the various parts of the moving human body. Body parts cause different shifts since they are moving with various velocities. In this case, the micro-Doppler refers to Doppler scattering returns produced by non-rigid human body motion. Micro-Doppler gives rise to many detailed radar image features in addition to those associated with bulk human body motion. For example, a walking man with swinging arms may induce frequency modulation (FM) of the returned signal and generate sidebands about the body Doppler. It is reasonable to expect that the m-D features representing the micro-motions such as swinging arms of a human can be extracted from the returned signal, much in the same way as properties are extracted from radar returns of targets undergoing only translational motion. Since different humans can have different micro-motions, every human would have its own "m-D signature", making it possible to distinguish and identify humans under consideration based on the additional information provided by the m-D features. Hence, an effective method is needed for extracting m-D features in order to fully exploit the additional and unique information they provide.

Traditional techniques, such as Fourier analysis or the sliding window FT (short time Fourier transform - STFT), lack the required resolution for extracting and processing these unique m-D features. Therefore, high-resolution linear and quadratic time-frequency (TF) analysis techniques are recently employed for extracting m-D features.\textsuperscript{8-14} Several papers have been written about the ways to deal with the m-D effect. Wavelet analysis of helicopter and human data, along with the TF representation based imaging system, is presented in reference.\textsuperscript{10} Details on the m-D effect physics, with some typical examples, are given in reference.\textsuperscript{11} A method for the separation of the m-D effect from the radar image, based on the chirplet transform, is proposed in reference.\textsuperscript{12} Both wavelet-based and chirplet-based procedures are used in reference\textsuperscript{13} to extract m-D features such as the rotating frequency of an antenna in SAR data. Recently, two techniques for the separation of a target’s rigid body from m-D parts have been proposed in reference.\textsuperscript{14} The first approach is based on order statistics of the spectrogram samples. The second approach is based on the Radon transform processing of obtained radar signals. The analysis of the TF representations application in radar target identification is presented in reference.\textsuperscript{15} The reduced interference distributions from the Cohen class are applied as a tool for the target identification. A technique for the m-D effect estimation from the reflected signal, based on the TF signatures and decomposition of basis functions, is presented in reference.\textsuperscript{16} This technique can be used for m-D effect signals that can be represented as sinusoidal FM signals. The goal of this paper is to find an improved method, which gives enhanced resolution and suppresses most of the cross-terms in relatively high noise environments.

In this paper, we apply the S-method (SM) based approach, which appears to have great promise for recognizing the m-D features such as a rotating fan, and human gait in indoor and outdoor environments. Section 2 briefly provides an introduction to the basic mathematical description of the m-D phenomenon. Section 3 introduces the SM based approach from basic principles and demonstrates its application for extracting m-D features. The Viterbi algorithm for IF estimation is reviewed in Section 4. Results are presented in Section 5 and show that m-D features can be accurately extracted using the SM. Conclusions and recommendations for future studies are given in Section 6.

2. BASIC DESCRIPTION OF THE MICRO-DOPPLER PHENOMENON

The basic mathematical description of the m-D phenomenon induced by vibrational motions is discussed in this section. Rotation can be seen as a special case of vibration. In coherent radar, variations in range cause a phase change in the returned signal from a target. A half-wavelength change in range can cause a 360-degree phase change. It is conceivable that the vibration of a reflecting surface may be measured with the phase change. Thus, the Doppler frequency shift that represents the change of phase function with time can be used to detect vibrations or rotations of structures in a target.\textsuperscript{8} The mathematics of the m-D effect can be derived by introducing vibration or rotation (micro-motion) to conventional Doppler analysis. A target can be represented as a set of point scatterers, which are the primary reflecting points on the target. The point scattering model can simplify the analysis while preserving the m-D induced by micro-motions. In our case, there exists a vibrating
point scatterer in a returned radar signal. The received Doppler from a target as a function of time is modeled by the following equation

\[ s(t) = A \exp[j(2\pi f_0 t + \phi(t))], \quad (1) \]

where \( A \) is the reflectivity of the vibrating point scatterer and \( f_0 \) is the carrier frequency of the transmitted signal. The \( \phi(t) \) is the time-varying phase change of the vibrating scatterer. Assuming that the vibrating scatterer is set to a radian frequency oscillation of \( \omega_v \), the time-varying phase is

\[ \phi(t) = \beta \sin(\omega_v t), \quad (2) \]

where \( \beta = 4\pi D_v / \lambda \), \( D_v \) is amplitude of the vibration and \( \lambda \) is the wavelength of the transmitted signal. Substituting (2) into (1) yields

\[ s(t) = A \exp[j(2\pi f_0 t + \beta \sin(\omega_v t))]. \]

The phase term function in (3) is time varying, the instantaneous frequency \( f_D(t) \) (IF), i.e. the m-D frequency induced by the vibration of the scatterer, may be expressed as

\[ f_D(t) = \frac{1}{2\pi} \frac{d\phi}{dt} = \frac{1}{2\pi} \beta \omega_v \cos(\omega_v t) = \frac{2}{\lambda} D_v \omega_v \cos(\omega_v t). \quad (3) \]

It should be noted that the maximum m-D frequency change is \((2/\lambda)D_v \omega_v\), which is used to estimate the displacement of a vibrating scatterer. The m-D induced by vibration is a sinusoidal function of time at the vibrating frequency \( \omega_v \). Usually, when the vibrating modulation is small, it is difficult to detect the vibration in the frequency domain. Thus, a method that is able to separate the radar return induced by the target body from that induced by its vibrating structure might help to isolate the vibrating spectrum from other contributions. This vibration-induced m-D signature is an important feature for identifying targets of interest. An example is the human walking gait.

Modulation induced by rotating structures can also be regarded as a unique signature of a target. This m-D signature is an important feature for identifying targets of interest (e.g., helicopters, ships or aircraft with rotating antennas). When there is a rotating scatterer on a target, the phase term in (2) may be expanded as follows

\[ \phi(t) = \beta \sin(\Omega t + \theta_0), \quad (4) \]

where \( \Omega \) is the rotation rate and \( \theta_0 \) is the rotating angle of the scatterer on the rotating structure at \( t = 0 \), called initial rotating angle. Therefore, the received Doppler from one rotating scatterer may be expressed by (1), which is an expansion of a vibrating structure. A detailed mathematical description of m-D modulations induced by several typical basic micro-motions is derived in references,\(^{8,11}\) which is beyond the scope of this paper.

3. S-METHOD

The SM\(^{18}\) is derived from the relationship between the STFT and the Wigner distribution (WD), which reads:

\[ WD(t, \omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} STFT(t, \omega + \theta)STFT^*(t, \omega - \theta) d\theta. \quad (5) \]

A discrete version of the previous relation reads:

\[ WD(n, k) = \sum_{i=-N/2}^{N/2} STFT(n, k + i)STFT^*(n, k - i) = |STFT(n, k)|^2 + 2\text{Re} \left\{ \sum_{i=1}^{N/2} STFT(n, k + i)STFT^*(n, k - i) \right\}. \quad (6) \]
The mathematical formulation of the SM in the discrete form is:

\[ \text{SM}(n, k) = \sum_{i=-N/2}^{N/2} P(i) \text{STFT}(n, k + i) \text{STFT}^*(n, k - i) \]

where \( P(i) = 1 \) for \( |i| \leq L \) and \( P(i) = 0 \) for other values of \( i \). The SM with \( L \) terms can be written in the form:

\[ \text{SM}_L(n, k) = \sum_{i=-L}^{L} \text{STFT}(n, k + i) \text{STFT}^*(n, k - i). \]

(7)

In particular, for \( L = 0 \), the SM is identical to the spectrogram

\[ \text{SM}_0(n, k) = |\text{STFT}(n, k)|^2 = \text{STFT}(n, k) \text{STFT}^*(n, k), \]

while for \( L = N \), the SM is identical to the WD. It should be noted that SM with \( L \) terms is obtained by adding one more term to the SM with \( L - 1 \) terms:

\[ \text{SM}_L(n, k) = \text{SM}_{L-1}(n, k) + 2 \text{Re}\{ \text{STFT}(n, k + L) \text{STFT}^*(n, k - L) \}. \]

The SM will produce the same auto terms as the WD if we take \( L \) such that \( (2L + 1) \) is equal to the auto terms width in the discrete domain (i.e., to the number of samples within the auto term). In practice it means fewer terms are needed, for example \( L \in [3, 10] \), since most of the auto-term energy is located around its maximal value. The precise mathematical proof that the the SM produces the WD of each component separately, in those regions of the TF plane where the components do not overlap, is given in reference. 18

4. M-D FEATURE EXTRACTION: THE VITERBI ALGORITHM

The m-D signatures associated with human motion are multicomponent FM signals. The extraction of human motion features can be performed through component separation. The analysis of components is commonly performed through IF estimation since it contains important features of human motion such as velocity. The TF representation-based IF estimators can be divided into two groups: parametric and nonparametric. In the case of parametric estimations, we can assume that m-D features are sinusoidal FM signals and for the extraction of parameters we can apply some techniques that are close to the maximum likelihood estimation or projection (Radon transform) based technique. 14 This can be applied only in the case that the number of component parameters is relatively small or in the case when we have an exact model of motion. As will be seen later in the paper, some experimental human motions cannot be modeled as a simple sum of sinusoidal FM components. For general motion, we can assume that the non-parametric IF estimation technique, should be applied. The simplest non-parametric IF estimation technique is based on the position of the TF maxima:

\[ \hat{k}(n) = \arg \max_k \text{TF}(n, k). \]

(8)

This technique is simple and well-known, but is suitable for monocomponent signals or multicomponent signals that are non-overlapping in the TF plane. In the latter case, we should perform the separation of signals from distinct regions, or in the case when components significantly differ in magnitude, we can perform the estimation of the dominant component. After that, we can calculate the new TF representation with the dominant component removed (i.e., peeled off):

\[ \text{TF}'(n, k) = \begin{cases} 
0 & |k - \hat{k}(n)| \leq \Delta \\
\text{TF}(n, k) & \text{elsewhere,} 
\end{cases} \]

(9)

where \( (2\Delta + 1) \) is the width of the neighborhood of the dominant component removed from the TF representation. The IF estimation of the next component can be performed by using the position of the maxima of \( \text{TF}'(n, k) \). The algorithm for peeling components and IF estimation should be performed for each component present. However,
for multicomponent signals intersecting in the TF plane, we need some more sophisticated analysis procedure. Since m-D features of the human motion have interesting components in the TF plane, both parametric techniques or simple techniques based on the position of maxima are not appropriate for analysis and feature extraction. This motivates use of the Viterbi algorithm for IF estimation proposed in reference\(^\text{19}\) in this research.

The Viterbi algorithm is a common technique for the estimation of hidden states in the signal. The Viterbi algorithm is essentially a shortest length path algorithm that has found numerous applications in coding theory,\(^\text{22}\) image processing,\(^\text{23}\) etc. The IF estimator based on the Viterbi algorithm and TF representations recently proposed in references.\(^{19,20}\) This estimator has shown excellent accuracy for high noise environments. In development of this estimator, two assumptions are adopted:

1. If the TF maximum at the considered instant is not at the IF point, there is a high probability that the IF is at a point having one of the largest TF values (for example second, third, but not as far as, for example, the hundredth position);

2. The IF variation between two consecutive points is not extremely large.

Then the IF estimate can be written as the line minimizing \(k(n)\), the corresponding sum of the path penalty functions \(p(k(n);n_1,n_2)\):

\[
\hat{\omega}(n) = \arg \min_{k(n) \in K} \left\{ \sum_{n=n_1}^{n_2} g(k(n), k(n+1)) + \sum_{n=n_1}^{n_2} f(TF(n,k(n))) \right\} 
\]

(10)

where \(p(k(n);n_1,n_2)\) is a sum of the penalty functions \(g(x,y)\) and \(f(x)\), along the line \(k(n)\), from the instant \(n_1\) to \(n_2\). The penalty function \(f(x)\) corresponds to the first assumption, while the penalty function \(g(x,y)\) corresponds to the second assumption. The function \(g(x,y) = g(|x-y|)\) is nonincreasing, with respect to the absolute difference between \(x\) and \(y\) (between the IF values in the consecutive points \(x = k(n)\) and \(y = k(n-1)\)), while \(f(x)\) is a nondecreasing function of \(x = TF(n,k(n))\). In this way, the larger values of the TF are more important candidates for the position of the IF at the considered instant. For a considered \(n\), the function \(f(x)\) can be formed as follows. The TF values, \(TF(n,k)\), for particular instant:

\[
TF(n,k_1) \geq TF(n,k_2) \geq \ldots \geq TF(n,k_j) \geq \ldots \geq WD(n,k_M)
\]

(12)

where \(j = 1,2,\ldots,M\), is the position within this sequence. Then, the function \(f(x)\) is formed as:

\[
f(TF(n,k_j)) = j - 1.
\]

(13)

For \(g(x,y) = \text{const}\), the IF estimation (10) is reduced to the position of the TF maxima, i.e., the function \(f(x)\) completely determines the minimum of (10). In this paper, we use a linear form of \(g(x,y)\), for the difference between two points greater than an assumed threshold \(\Delta\):

\[
g(x,y) = \begin{cases} 
0 & \text{if } |x-y| \leq \Delta \\
c(|x-y| - \Delta) & \text{if } |x-y| > \Delta.
\end{cases}
\]

(14)

A reasonable choice for \(\Delta\) would be the maximal expected value of the IF variation between consecutive points. It means that there is no additional penalty due to this function for small IF variation (within \(\Delta\) points, for two consecutive instants). In the realization we obtained good results by taking \(\Delta\), which corresponds to a few neighboring points (for example, values around \(\Delta = 3\). For \(\Delta \to \infty\), the estimation given by (10) will reduce to the estimation based on the TF maxima. Note that this is one possible form of the penalty functions \(f(x)\) and \(g(x,y)\).

The reasoning for the selection of path penalty functions is described in detail in reference.\(^\text{19}\) The proposed algorithm can be realized in a recursive manner with reasonable calculation complexity.

As far as we know, the proposed algorithm outperforms all other non-parametric IF estimators based on the TF representation for a high noise environment. For multicomponent signals we should perform estimation
component by component, peeling off detected components. However, the Viterbi algorithm has the ability to connect components even in the case that some of its parts are removed. This algorithm can be expanded by the third penalty function related to the amplitude variation of the signal components. This algorithm extension is proposed in reference\textsuperscript{19} where its accuracy in tracking intersecting components has been demonstrated.

5. RESULTS

In this section we demonstrate the application and effectiveness of the S-method and Viterbi algorithm as m-D feature extraction technique with four different types of experimental radar data obtained in different indoor and outdoor scenarios.

5.1. Rotation-induced micro-Doppler

5.1.1. Rotating corner reflector

Experimental trials were conducted to investigate and determine the m-D radar signatures of targets using an X-band radar. The target used for this experimental trial was a spinning blade with corner reflectors attached that were designed to reflect electromagnetic radiation with minimal loss. These controlled experiments can simulate the rotating types of objects, generally found in an indoor environment such as a rotating fan and, in an outdoor environment such as a rotating antenna or rotors. Controlled experiments allow us to set the desired rotation rate and then permit us to cross check and assess the results.

A picture of the target is shown in Figure 1. The blade was set up to simulate real data that might be collected from a similar target such as a rotating antenna or rotating fan or any other rotation of structures on a target. The experiment was conducted with the radar operating at 9.2 GHz. The pulse repetition frequency (PRF) was 1 kHz. The target employed in this experiment was at a range of 300 m from the radar. The distance between the two reflectors is 38 inches. The corner length of the reflector is 10 inches and the side length of the reflector is 12 inches. The SM is utilized in order to depict the m-D oscillation. The result in Figure 2a was obtained using one rotating corner reflector facing the radar. Details of the figure clearly show the sinusoidal motion of the corner reflector. The second weaker oscillation represents the reflection from the counter weight that was used to stabilize the corner reflector during operation. From the TF signature we can see that the m-D of the rotating corner reflector is a time-varying frequency spectrum. The rotation rate of the corner reflector is directly related to the time interval of the oscillations. From the additional time information, the rotation rate of the corner reflector is estimated at about 60 rpm. Figure 2b shows the result when the blade is rotating with two corner reflectors. In this case, the rotation rate of the corner reflector was 40 rpm. Rotation rates estimated by the TF analysis agree with the actual values. These results demonstrate that the SM can be used to extract m-D features and estimate motion parameters. In a future study we will quantitatively compare the SM with other traditional m-D analysis techniques.
Figure 2. S-method experimental data: (a) m-D effects from one rotating corner reflectors facing the radar; (b) m-D effects from two rotating corner reflectors facing the radar. Note that 3650 time samples correspond to 5.2 seconds of data.

5.1.2. Rotating fan

Experimental trials were conducted to investigate and determine the m-D radar signatures of objects that could be found in indoor radar imaging. The object in these experiments is rotating fan data supplied to us by Prof. Moeness Amin, Villanova University. The rotational motion of blades in a fan imparts a periodic modulation on radar returns. The rotation-induced Doppler shifts relative to the Doppler of the body occupy unique locations in the frequency domain. Whenever a blade has specular reflection such as at the advancing or receding point of rotation, the particular blade transmits a short flash or periodic modulation to the radar return. The rotation rate of the blade is directly related to the time interval between these flashes. The duration of a flash is determined by the radar wavelength and by the length and rotation rate of the blades. A flash resulting from a blade with a longer length and radius with a shorter wavelength will have a shorter duration.

The fan in this experiment is rotating at a height of approximately 2 m and at a range of 3 m from the radar. The fan has 4 metallic blades. The rotation rate of the blades is known to be 1050 rpm for this data. The experiment was conducted with the radar operating at frequency of 903 MHz. The sampling frequency is 5000 Hz.

The original rotating fan data behaves like a very low pass filter in the frequency domain and makes it quite difficult to extract any information. In order to obtain the inner m-D features, we perform down-sampling on the signal by a factor of 4 and then use a relatively narrow window of 16 samples. Figure 3a shows the S-method of the down-sampled signal with N=16 samples and L=2. When L=3 is used for the S-method, Figure 3b shows further concentration improvement. Figure 3b also exhibits that the S-method can accurately describe the signal’s features showing the sinusoidal behavior, not just the amplitude. The image in Figure 3c shows a zoomed version of the time interval between 0.13 and 0.25 seconds. This figure distinctly depicts the sinusoidal...
oscillation of the fan blade. From Figure 3c, the period of oscillation is 0.1/7.0 seconds. Since there are four blades, the period per blade is (0.1*4)/7. The number of rotations in one minute is given by (7*60)/(0.1*4) =1050 rpm/blade, which is in agreement with the actual value known to be 1050 rpm.

5.2. Vibration-induced micro-Doppler

Gait recognition by radar focuses on the gait cycle formed by the movements of a person’s various body parts over time. Radar echoes contain rich information about the various parts of the moving body. Various body parts have different shifts since they are moving with various velocities. For example, a walking man with swinging arms may induce frequency modulation of the returned signal and generate side-bands about the body Doppler. There are often multiple physical movements taking place simultaneously and the interaction of these produces the particular characteristics of a “gait”. Radar experiments were conducted and the proposed SM based approach was used to analyze the Doppler shift of the body, the m-D signature of the swinging arms, and to estimate gait parameters such as swinging rate.

In this experiment we considered human data. The main part of a human body causes the strongest component as can be seen from the SM of the considered signal, Figure 4a. For the analysis of other components, caused by swinging arms and moving legs, we need to remove the main body component. The classical IF estimator based on the position of the TF representation maxima is used in the first stage of the algorithm:

$$\hat{k}(n) = \arg \max_k SM(n, k).$$  \hfill (15)
This IF estimate corresponds to the strongest component, in this case the main body component. Figure 4b depicts the IF estimation superposed onto the same TFR graph. Then, classical peeling techniques can be employed to remove the main body component from the rest of the TF representation:

$$SM'(n, k) = \begin{cases} 0 & |k - \hat{k}(n)| \leq \Delta \\ SM(n, k) & \text{elsewhere,} \end{cases}$$

where $2\Delta + 1$ is number of samples around the IF of the main component removed from the SM. The TF representation with ‘the peeled-off’ main body component is depicted in Figure 4c. Now other weaker components can be clearly seen. These components correspond to moving legs and arms. Components with smaller amplitude correspond to moving arms while components with larger amplitude (larger frequency variations) correspond to moving legs. It can be easily noticed that the moving leg produces sinusoidal FM signal during the relative motion of the leg, but in the next step, this component has a slowly varying component and the second leg produces a ‘semi-period’ of the sinusoidally FM signal. This clearly shows that, in this case, it is not possible to use the Radon transform or a parametric estimator for extraction of the component.

In this case we cannot claim that one component is dominant over others since their magnitudes are similar and they are crossing in the TF plane. The algorithm based on the position of the TF representation maxima

Figure 4. Time-frequency representations and IF estimation for human data: (a) Time-frequency representation of m-D of a walking man signal; (b) Instantaneous frequency estimation of the main body component; (c) Time-frequency representation of m-D of a walking man signal, after removing the main body component; (d) Instantaneous frequency estimation of the weak component by using the Viterbi algorithm.
would give a line that is a combination of the IFs of different components. Then a more sophisticated algorithm for the IF estimation and segmentation of these components is required. A simple scheme such as (15) cannot produce accurate results for separation of these components. Here, the Viterbi algorithm for IF estimation is used. This algorithm is able to track the components in the TF plane that are intersected even for relatively high noise environment. Figure 4d depicts the IF estimation of the next component associated with the leg motion based on the Viterbi algorithm. It can be seen that the component is recognized in a proper manner except for the small part around the intersection of this component, which is caused by both the second leg component and the main body motion component. After peeling of this component, the next component can be determined in a similar manner. However, complicated signatures such as those produced by human motion require a very detailed analysis of the Viterbi algorithm parameters with possible alternative techniques for selecting the path penalty functions. This topic remains for future research.

6. CONCLUSION

In order to extract and enhance the m-D features, the S-method based approach and the Viterbi algorithm are proposed to analyze and resolve radar m-D signatures of targets. It is shown that the S-method based approach provides an effective method of achieving improved resolution, highly concentrated and readable representation without the auto-term distortion and cross-term artifacts. This method in conjunction with the Viterbi algorithm are suitable for m-D data when multiple scatterers are present, and noise and artifact reduction are essential for target identification applications. The effectiveness of the S-method in conjunction with Viterbi algorithm in extracting m-D features is demonstrated through the application to experimental indoor rotating fan data and outdoor moving human data.

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Analysis of micro-Doppler radar signatures in SAR using S-method-based approach

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Abstract—In many cases, a target or a structure on a target may have micro-motions, such as vibrations or rotations. Micro-motions of structures on a target may introduce frequency modulation on the returned radar signal and generate sidebands on the Doppler frequency shift of the target’s body. The modulation due to micro-motion is called the micro-Doppler (m-D) phenomenon. In this paper, we present an adaptive time-frequency S-method-based approach to extract m-D features. For target recognition applications, mainly those in military surveillance and reconnaissance operations, micro-Doppler features have to be extracted quickly so that they can be used for real-time target identification. The adaptive S-method is computationally simple, requiring only slight modifications to the existing Fourier transform-based algorithm. The effectiveness of the adaptive S-method in extracting m-D features has been successfully applied to different experimental data sets.

Index Terms—Micro-Doppler, S-method, Time-Frequency

I. INTRODUCTION

The exploitation of micro-Doppler signatures provides additional target recognition capabilities that are complementary to existing recognition methods. It frequently occurs that a target or some structure on the target is vibrating or rotating in addition to the target’s translational motion. These vibrations and rotations are referred to as micro-motion dynamics. The frequency modulation due to micro-motion dynamics is called the micro-Doppler (m-D) phenomenon. Joint time-frequency analysis is the basis of most of the existing methods used to extract m-D features [1-6]. Another viable approach to extracting m-D features is wavelet analysis [5]. The oldest and the most widely used time-frequency representation is the short-time Fourier transform (STFT). In order to improve its concentration, various quadratic representations have been introduced. The most prominent member of this class of representations is the Wigner distribution (WD), which, however suffers from cross-term interferences.

The time-frequency representation referred to as the S-method (SM) has a property that, under certain assumptions, its value for multicomponent signals is equal (or nearly so) to the sum of the Wigner distributions of individual signal components [7-8]. It has been also used as a model in the implementation of time-scale representations, time-varying spectra estimation, and detection and realization of higher order representations. The S-method is also numerically very simple and requires just a few more operations than the standard FT-based algorithm. In this paper, an adaptive S-method-based approach is introduced for the purpose of extracting m-D features, which appears to have great promise to recognize the m-D features such as a rotating antenna in a SAR (synthetic aperture radar) target scenario and a rotating corner reflector in an outdoor environment.

II. ADAPTIVE S-METHOD

The S-method is derived from the relationship between the STFT and the Wigner distribution, which reads [7]:

\[
WD(t, \omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} STFT(t, \omega + \theta)STFT^*(t, \omega - \theta) d\theta.
\]  

A discrete version of the previous relation reads:

\[
WD(n, k) = \sum_{i=-N/2}^{N/2} STFT(n, k + i)STFT^*(n, k - i) = |STFT(n, k)|^2 + 2Re \left\{ \sum_{i=1}^{N/2} STFT(n, k + i)STFT^*(n, k - i) \right\}.
\]  

The mathematical formulation of the SM in the discrete form is:

\[
SM(n, k) = \sum_{i=-N/2}^{N/2} P(i)STFT(n, k + i)STFT^*(n, k - i)
\]

where \(P(i) = 1\) for \(|i| \leq L\) and \(P(i) = 0\) for other values of \(i\). The SM with \(L\) terms can be written in the form:

\[
SM_L(n, k) = \sum_{i=-L}^{L} STFT(n, k + i)STFT^*(n, k - i).
\]

In particular, for \(L = 0\), the SM is identical to the spectrogram \(SM_0(n, k) = |STFT(n, k)|^2 = STFT(n, k)STFT^*(n, k)\), while for \(L = N\), the SM is identical to the WD. It should be noted that SM with \(L\) terms is obtained by adding one more term to the SM with \(L - 1\) terms:

\[
SM_L(n, k) = SM_{L-1}(n, k) + 2Re\{STFT(n, k + L)STFT^*(n, k - L)\}.
\]
Fig. 1. The original SAR image.

The SM will produce the same auto terms as the WD if we take $L$ such that $(2L+1)$ is equal to the auto terms width in the discrete domain (i.e., to the number of samples within the auto term). In practice it means fewer terms are needed, for example $L \in [3, 10]$, since most of the auto-term energy is located around its maximal value. The precise mathematical proof that the SM produces the WD of each component separately, in those regions of the TF plane where the components do not overlap, is given in reference [7].

The adaptive S-method is presented in [8]. The main idea is to limit summation in Equation (2) to the auto-terms only. In this way we can avoid situation when $STFT(n, k + i)$ belongs to one component and $STFT^*(n, k - i)$ belongs to another component. One approach is to stop summation in Equation (2) when the zero value is detected, but the true zero values in the Fourier transform are very sensitive to the noise and even to numerical errors. Another approach is to use the reference level $R_L$. The summation stops when value of either $|E(k + i)|$ or $|E^*(k - i)|$ falls below the threshold $R_L$. The good choice of $R_L$ is a few percentages of the maximal value of the $|E(k)|$. Then, the cross-terms will be completely avoided in the cases of non-overlapping auto-terms.

III. RESULTS

In this section, we demonstrate the application and effectiveness of the adaptive S-method as m-D feature extraction technique with two different types of experimental radar data obtained in different scenarios.

A. Rotating antenna in SAR

We extract the m-D features relating to a rotating antenna in a SAR target scenario. The m-D for such rotating target may be seen as a sinusoidal phase modulation of the SAR azimuth phase history. The phase modulation may equivalently be seen as a time-varying Doppler frequency.

Fig. 2. The Fourier transform of the original time series.

Fig. 3. Time-frequency signatures of the original time series at range cell 123 a) STFT and b) WD.
The original SAR image is shown in Figure 1. The Doppler smearing due to the rotating parts is often well localized in a finite number of range cells. It is reasonable to process the Doppler signal for each range cell independently. Since the ground truth of the target is already known, the data at the range cell 123 was analyzed using the adaptive S-method. The Fourier transform of the original time series at range cell 123 is shown in Figure 2. Figure 3a illustrates the time-frequency signature using STFT and Figure 3b illustrates the time-frequency signature using WD at range cell 123. Figure 4 illustrates the time-frequency signature using the adaptive S-method for L=3 and L=5. Results show that the adaptive S-method produces concentrated time-frequency signatures compare to STFT and WD. Using the time-frequency plot, the rotation rate of the antenna is estimated by measuring the time interval between peaks. The period is the time interval between peaks. As an example, in Figure 4, there are 3 peaks. The time interval between peak 1 and peak 2, between peak 2 and peak 3, and between peak 1 and peak 3 are measured. The average value is then used to estimate the rotation rate. The estimated rotation rate is 4.8 seconds, which is very close to the actual value of 4.7 seconds.

B. Rotating corner reflector

Experimental trials were conducted to investigate and determine the m-D radar signatures of targets using an X-band radar. The target used for this experimental trial was a spinning blade with corner reflectors attached that were designed to reflect electromagnetic radiation with minimal loss. These controlled experiments can simulate the rotating types of objects, generally found in an indoor environment such as a rotating fan and, in an outdoor environment such as a rotating antenna or rotors. Controlled experiments allow us to set the desired rotation rate and then permit us to cross check and assess the results.

A picture of the target is shown in Figure 5. The experiment was conducted with the radar operating at 9.2 GHz. The pulse repetition frequency (PRF) was 1 kHz. The target employed in this experiment was at a range of 300 m from the radar. The distance between the two reflectors is 38 inches. The corner length of the reflector is 10 inches and the side length of the reflector is 12 inches. The adaptive SM is utilized in order to depict the m-D oscillation. The result in Figure 6 was obtained using one rotating corner reflector facing the radar. Details of the figure clearly show the sinusoidal motion of the
corner reflector. The second weaker oscillation represents the reflection from the counter weight that was used to stabilize the corner reflector during operation. From the TF signature we can see that the m-D of the rotating corner reflector is a time-varying frequency spectrum. The rotation rate of the corner reflector is directly related to the time interval of the oscillations. From the additional time information, the rotation rate of the corner reflector is estimated at about 60 rpm. Figure 7 shows the result when the blade is rotating with three corner reflectors. In this case, the rotation rate of the corner reflector was 60 rpm. Rotation rates estimated by the TF analysis agree with the actual values.

These results demonstrate that the S-method can be used to extract the m-D signatures since these signals are commonly represented as sinusoidal (FM) frequency modulated signals. In addition, the technique is quite simple since the common STFT evaluator is improved with the addition of small post-processing blocks. Furthermore, the SM does not require oversampling as other bilinear representations. All these favorable properties have motivated us to apply the SM in radar signal processing and in m-D analysis.

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IV. CONCLUSION

This paper presents an adaptive S-method-based approach for the extraction of micro-Doppler features. By applying the proposed adaptive S-method-based approach to experimental data, the effectiveness of this analysis technique is confirmed. From the extracted m-D signatures, information about the target’s micro-motion dynamics, such as rotation rate and period of oscillation, can be obtained. This method is also computationally simple, requiring only slight modifications to the existing Fourier transform based algorithm. In a future study we will quantitatively compare the S-method with other traditional m-D analysis techniques.

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Micro-Doppler Analysis of Rotating Targets using Adaptive S-method

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Abstract

Experimental trials were conducted to investigate and determine the m-D radar signatures of rotating targets using an X-band radar. In this paper, we present an adaptive S-method based approach to extract m-D features. We show that adaptive S-method is computationally simple, requiring only slight modifications to the existing Fourier transform-based algorithm. The effectiveness of the adaptive S-method in extracting m-D features is demonstrated through the experimental data sets. As such, this paper contributes additional experimental micro-Doppler analysis, which should help in developing a better picture of the micro-Doppler research and its applications to indoor and outdoor imaging and automatic gait recognition systems.

Keywords: Micro-Doppler, Adaptive S-method, Time-Frequency

I INTRODUCTION

When the transmitted signal of a coherent radar system hits moving targets, the carrier frequency of the signal will be shifted, known as the Doppler effect. The Doppler frequency shift reflects the velocity of the moving target. Mechanical vibration or rotation of a target, or structures on the target, may induce additional frequency modulations on the returned radar signal, which generate sidebands about the target’s Doppler frequency. This phenomenon is called the micro-Doppler effect [1-5]. Micro-Doppler radar signatures can provide unique information for recognition of targets of interest. For target recognition applications, mainly those in military surveillance and reconnaissance operations, micro-Doppler features have to be extracted quickly so that they can be used for real-time target identification.

In this paper, we demonstrate examples of micro-Doppler signatures of a target that can be used as radar signatures for target identification. Experimental trials were conducted to investigate and resolve the micro-Doppler radar signatures of a target using an X-band radar. The target used for this experimental trial was a spinning blade with corner reflectors attached that were designed to reflect electromagnetic radiation with minimal loss. These controlled experiments can simulate the rotating types of objects, generally found in an indoor environment, for example, a rotating fan and outdoor environment, for example, a rotating antenna or rotors. Controlled experiments allow us to set the desired rotation rate and then permit us to cross check and assess the results. Joint time-frequency analysis is the basis of most of the existing methods used to extract m-D features [1-5]. In this paper, we use adaptive S-method for the extraction of m-D features.

II Adaptive S-method

The mathematical formulation of the S-method(SM) in the discrete form is

\[ SM(n,k) = \sum_{i=-N/2}^{N/2} P(i) \text{STFT}(n,k+i) \text{STFT}^*(n,k-i) \]  (1)

where \( P(i) = 1 \) for \( |i| \leq K \) and \( P(i) = 0 \) for other values of \( i \). The SM for with \( K \) terms can be written in the form:

\[ SM(l,n,k) = \sum_{i=-L}^{L} \text{STFT}(n,k+i) \text{STFT}^*(n,k-i) \]  (2)

In particular, for \( L=0 \) the SM is identical to the spectrogram

\[ SM_{0}(n,k) = |\text{STFT}(n,k)|^2 = \text{STFT}(n,k)\text{STFT}^*(n,k) \]  (3)

while for \( L=N \), the SM is identical to the WVD. The adaptive SM is originally developed for improving the TF representation in [6]

\[ SM_l(n,k) = |\text{STFT}(n,k)|^2 + 2\text{Re} \left\{ \sum_{i=-l}^{l} \text{STFT}(n,k+i) \text{STFT}^*(n,k-i) \right\} \]  (4)

The challenge is here determination of \( L(n,k') \). \( L(n',k') \) can obtained as maximal value of \( l \) for which the term \( \text{Re}\{\text{STFT}(n',k'+i)\text{STFT}^*(n',k'-i)\} \) used for the SM calculation is greater than a specific threshold \( R(n',k') \) where all \( \text{Re}\{\text{STFT}(n',k'+i)\text{STFT}^*(n',k'-i)\} \) for \( |I'| < |I| \) are greater

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than the threshold. This can be written as

\[ L(n', k') = \arg \max_{k'} \mathcal{A} \left( \text{Re} \{ \text{STFT}(n', k' + i') \cdot \text{STFT}^*(n', k' - i') \} \right) \]

\[ \geq R(n', k') \]  \hspace{1cm} (5)

Where \( \mathcal{A} \) represents logical operation AND applied to logical operations from argument for various \( l' = 1, \ldots, l \). The threshold value can be determined in various ways. The global threshold is proposed in [6] as

\[ R = \varepsilon \max_{n', k'} \left| \text{STFT}(n', k') \right|^2 \]  \hspace{1cm} (6)

The threshold can also be calculated using Otsu algorithm[7]. The magnitude \( |\text{STFT}(n,k)| \) is taken as image pixels intensity needed for the algorithm.

Step 1. Estimate initial value for threshold. Here, initial value for threshold is set to the half of the pixels intensity maximum:

\[ \rho = \frac{1}{2} \max_{n', k'} \left| \text{STFT}(n', k') \right|^2 \]  \hspace{1cm} (7)

Step 2. Calculate two sums S1 and S2 where S1 is a sum of intensity values of the pixels whose intensity is larger than the current threshold \( \rho \):

\[ S1 = \sum_{n', k'} \left| \text{STFT}(n', k') \right| \quad (\forall(n', k'), \text{STFT}(n', k') > \rho) \]  \hspace{1cm} (8)

while S2 is a sum of intensity values of the pixels whose intensity is smaller than the current threshold \( \rho \):

\[ S2 = \sum_{n', k'} \left| \text{STFT}(n', k') \right| \quad (\forall(n', k'), \text{STFT}(n', k') < \rho) \]  \hspace{1cm} (9)

Step 3. Calculate two new thresholds, \( \rho_1 \) and \( \rho_2 \), as average values of the obtained sums:

\[ \rho_1 = \frac{S1}{N1}, \quad \rho_2 = \frac{S2}{N2} \]  \hspace{1cm} (10)

where N1 and N2 are number of elements summed in eq(8) and eq(9), respectively.

Step 4. Compute a new threshold value

\[ \rho = \frac{1}{2} (\rho_1 + \rho_2) \]  \hspace{1cm} (11)

Step 5. Repeat Steps 2 through 4 until the difference in \( \rho \) in successive iterations is smaller than a predefined parameter, or for a specified number of iteration.

III. RESULTS

In this section, we demonstrate the application and effectiveness of the adaptive S-method as m-D feature extraction technique with two different types of experimental radar data obtained in different scenarios

A. Corner reflectors

High range resolution (HRR) profiles were collected using a stepped frequency waveform (SFWF) radar mode at X-band between 9.0 to 9.4 GHz, i.e., a synthetic bandwidth of 400 MHz; the frequency step size was 1 MHz. The test target was made up of four corner reflectors, three of which are stationary to provide a geometric reference and contrast to the shape of the oscillating reflector in the HRR profiles. Out of three stationary reflectors, two were placed closer to each other than the third stationary reflector. The test target was located at 2 km from the radar system. HRR scans were performed at 2 kHz PRF. This data set is used to extract the m-D features using different time-frequency analysis. Figure 1a illustrates the time-frequency signature of the signal which is obtained using Spectrogram. As expected time-frequency representation of spectrogram is cross term free but its time and frequency resolution is not good. Figures 1b and 1c illustrate the time frequency signatures of the same signal using Wigner Ville(WV) and S-method (L=2) time frequency representations respectively. Even though WV gives best resolution, it is corrupted with cross terms. On the other hand S method with L value 2 gives better time frequency resolution than spectrogram. But still there are some cross terms between the two stationary reflectors which are close to each other. Figure 1d shows Adaptive S-method time-frequency representation of the signal. It can be seen that cross terms are removed between two close stationary reflectors. Results demonstrate that the adaptive S-method is an effective tool to extract m-D features.

B. Rotating antenna in SAR

We extract the m-D features relating to a rotating antenna in a SAR target scenario. The m-D for such rotating target may be seen as a sinusoidal phase modulation of the SAR azimuth phase history. The phase modulation may equivalently be seen as a time-varying Doppler frequency. The Doppler smearing due to the rotating parts is often well localized in a finite number of range cells. It is reasonable to process the Doppler signal for each range cell independently. Since the ground truth of the target is already known, the data at the range cell 123 was analyzed using the adaptive S-method. Figure 2a illustrates the time-frequency signature using STFT and Figure 2b illustrates the time-frequency signature using WD at range cell 123. Figure 2c illustrates the time-frequency signature using the S-method for L=4. Even though S method gives better resolution compared to spectrogram but still suffered from cross terms between close scatterers. Figure 2d shows that result from proposed adaptive S-method. Clearly it produces concentrated time-frequency signatures compare to STFT and WD. Using the time-frequency plot, the rotation rate of the antenna is estimated by measuring the time interval between peaks. The period is the time interval between peaks. As an example, in Figure 2d, there are 3 peaks. The time interval between peak 1 and peak 2, between peak 2 and peak 3, and between peak 1 and peak 3 are measured. The average value is then used to estimate the rotation rate. The estimated rotation rate is 4.8 seconds, which is very close to the actual value of 4.7 seconds.
Figure 1. Time-frequency signatures of the original time series a) STFT and b) WD. c) S-method d) adaptive S-method

Figure 2. Time-frequency signatures of the original time series at range cell123 a) STFT and b) WD. c) S-method d) adaptive S-method
IV. CONCLUSION

This paper presents an adaptive S-method-based approach for the extraction of micro-Doppler features. By applying the proposed adaptive S-method-based approach to experimental data, the effectiveness of this analysis technique is confirmed. From the extracted m-D signatures, information about the target’s micro-motion dynamics, such as rotation rate and period of oscillation, can be obtained. This method is also computationally simple, requiring only slight modifications to the existing Fourier transform based algorithm.

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Analysis of micro-Doppler Radar Signatures of Rotating Targets Using Gabor-Wigner Transform

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Abstract — Micro-Doppler signatures provide unique information about properties of the target. These micro-Doppler features can be used for real time target recognition in military applications and surveillance operations. In this paper, we present Gabor-Wigner transform for extracting micro-Doppler features from the radar returns. The effectiveness of the Gabor-Wigner transform in extracting micro-Doppler features has been compared with short-time Fourier transform, Wigner Distribution and S-method. The efficiency of the Gabor-Wigner transform in micro-Doppler feature extraction is demonstrated by applying it to different experimental data sets.

Keywords— Time-Frequency analysis, micro-Doppler effect, Short-Time Fourier Transform, Wigner Distribution, S-method, Gabor-Wigner Transform

I. INTRODUCTION

In many cases, a target or structure on a target may have rotations or vibrations. These micro-motions of the target may induce frequency modulation on the radar returned signal. This may lead to generation of side bands about the center of the Doppler shifted carrier frequency. The frequency modulation due to target’s rotating or vibrating motion is called micro-Doppler (m-D) effect [1]. Joint time-frequency methods can be used to extract m-D features [2]-[14]. The short-time Fourier transform (STFT) is the most widely used time-frequency representation. In STFT, the time and frequency resolution depends on the size of the window function used in calculating STFT. In order to improve time-frequency concentration, various quadratic time-frequency methods have been proposed. Wigner distribution (WD) is a well-known time-frequency method among the quadratic time-frequency methods, however it suffers from cross-term interferences. Stankovic et.al proposed S-method for improved distribution concentration in the time-frequency plane. S-method (SM) has a property that, under certain assumptions, its value for multicomponent signals is nearly equal to the sum of the Wigner distributions of individual signal components [3]. The S-method is also numerically very simple and requires just a few more operations than the standard FT-based algorithm [4]. In 2007, S.C Pei et.al proposed Gabor-Wigner transform for the suppression of cross-terms in the output of WD function [5]. In this paper, we use Gabor-Wigner transform for the extraction of m-D features.

II. GABOR-WIGNER TRANSFORM

STFT is the simplest time-frequency representation introduced for time localization of the frequency contents of a signal by using a suitable window. Basic principle behind STFT is segmenting the signal into narrow time intervals using a window function and taking Fourier transform of each segment.

\[
STFT(t, \omega) = \int_{-\infty}^{\infty} x(\tau) y(t-\tau) \exp(-j\omega \tau) d\tau
\]

(1)

Where x(t) is the signal to be analyzed and y(t-\tau), windowing function centered at t = \tau, STFT has limited time frequency resolution which is determined by the size of the window used. The STFT with Gaussian window is called a Gabor transform [5]. Mathematically Gabor transform (GT) of x(t) is defined as:

\[
GT(t, \omega) = \sqrt{\frac{1}{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(\tau-t)^2}{2}} e^{-j\omega \tau} x(\tau) d\tau
\]

(2)

The WD was originally developed in the area of quantum mechanics by Wigner [6] and then introduced for signal analysis by Ville [7]. It is defined as:
modulation of the SAR azimuth phase history. The phase modulation may equivalently be seen as a time-varying Doppler frequency[10]. The Doppler smearing due to the rotating parts is often well localized in a finite number of range cells. It is reasonable to process the Doppler signal for each range cell independently. Since the ground truth of the target is already known, the data at the range cell 123 was analyzed using the GW transform. Figure 1a illustrates the time-frequency signature using STFT and Figure 1b illustrates the time-frequency signature using WD at range cell 123. Figure 1c illustrates the time-frequency signature using the S-method for L=4. Even though S method gives better resolution compared to STFT but still suffered from cross terms between close scatterers. Figure 1d shows that result from the proposed GW transform. Clearly it produces concentrated time-frequency signatures compare to STFT and WD. Using the time-frequency plot, the rotation rate of the antenna is estimated by measuring the time interval between peaks. The period is the time interval between peaks. As an example, in Figure 1d, there are 3 peaks. The time interval between peak 1 and peak 2, between peak 2 and peak 3, and between peak 1 and peak 3 are measured. The average value is then used to estimate the rotation rate. The estimated rotation rate is 4.8 seconds, which is very close to the actual value of 4.7 seconds.

B. Corner reflectors

High range resolution (HRR) profiles were collected using a stepped frequency waveform (SFWF) radar mode at X-band between 9.0 to 9.4 GHz, i.e., a synthetic bandwidth of 400 MHz; the frequency step size was 1 MHz. The test target was made up of four corner reflectors, three of which are stationary to provide a geometric reference and a contrast to the shape of the oscillating reflector in the HRR profiles. Out of three stationary reflectors, two were placed closer to each other than the third stationary reflector. The test target was located at 2km from the radar system. HRR scans were performed at 2kHz PRF. This data set is used to extract the m-D features using different time-frequency analysis. Figure 2a illustrates the time-frequency signature of the signal which is obtained using STFT. As expected time-frequency representation of STFT is cross term free but its time and frequency resolution is not good. Figures 2b and 2c illustrate the time frequency signatures of the same signal using WD and S-method time frequency representations respectively. Even though WD gives best resolution, it is corrupted with cross terms. On the other hand S method with L value 2 gives better time frequency resolution than STFT. But still there are some cross terms between the two stationary reflectors which

\[
WVD(t, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(t+\tau/2)x^*(t+\tau/2)e^{-j\omega \tau} d\tau 
\]  

(3)

Compared to STFT, WD has much better time and frequency resolution. But the main drawback of the WD is the cross-term interference. This interference phenomenon shows frequency components that do not exist in reality and considerably affect the interpretation of the time frequency plane. Cross-terms are oscillatory in nature and are located midway between the two components [8]. Presence of cross-terms severely limits the practical applications of WD. S. C. Pei and J. J. Ding proposed Gabor-Wigner (GW) transform that is defined as a multiplication of STFT with WD, can significantly remove the cross-terms without affecting the quality of auto-terms much[5]. Since cross-terms do not appear in GT, the time-frequency distribution of the GT can be used as a filter, to filter out the cross-terms in the output of the WD. There are many different combinations to define the GW transform. Here four different definitions are given.

\[
GW(t, \omega) = GT(t, \omega)W D(t, \omega) 
\]  

(4)

\[
GW(t, \omega) = \min \{ |GT(t, \omega)|^2, |W D(t, \omega)| \} 
\]  

(5)

\[
GW(t, \omega) = W D(t, \omega)\{ |GT(t, \omega)| > 0.25 \} 
\]  

(6)

\[
GW(t, \omega) = GT^{2.6}(t, \omega)W D^{0.6}(t, \omega) 
\]  

(7)

GW transform combines the advantages of GT and WD by eliminating cross-terms while maintaining clarity as good as WD. However GW transform fails to provide useful results once the cross terms are superimposed on the auto components of WD in a multi component signal [9].

III. RESULTS

In this section, we demonstrate the application and effectiveness of GW transform as m-D feature extraction technique with two different types of experimental radar data obtained in different scenarios.

A. Rotating antenna in SAR

We extract the m-D features relating to a rotating antenna in a SAR target scenario. The m-D for such rotating target may be seen as a sinusoidal phase
are close to each other. Figure 2d shows GW transform time-frequency representation of the signal. It can be seen that cross terms are removed between two close stationary reflectors. Results demonstrate that the GW transform is an effective tool to extract m-D features

IV. PERFORMANCE ANALYSIS

The performance of the time-frequency representations (TFRs) can be compared based on their readability, resolution, cross-terms suppression and energy concentration. Cross-terms suppression and energy concentration of a time-frequency representation is evaluated by visual inspection or on the basis of quantitative measures like entropy measures and ratio of norms [15].

Renyi entropy [16] of time-frequency representation, \( \phi(t, \omega) \) is defined as:

\[
\text{Entropy}_{\text{Renyi}} = \frac{1}{1-\alpha} \log_2 \left( \sum \sum \phi^\alpha (t, \omega) \right)
\]

(8)

where \( \alpha \) is the order of entropy. In the case of TFRs, low entropy stands for high concentration of auto-terms, whereas high entropy means lower energy concentration of auto-terms. Ratio of norms (RN) is calculated by dividing fourth power norm of a TFR, \( \phi(t, \omega) \) by its second power norm [17]. Mathematically

\[
RN = \frac{\sum \sum |\phi^\alpha (t, \omega)|^4}{\left( \sum \sum |\phi^\alpha (t, \omega)|^2 \right)^2}
\]

(9)

Higher value of RN implies that signal auto-components are highly concentrated [17]. From table I and II, it is evident that GW transform has minimum value of entropy and maximum value for RN. Therefore the proposed GW transform shows good energy concentration property compared to other TFRs.

Fig 1. Time frequency signature of Original time series at range cell123
(a) STFT (b)WD (c) S-method (d) GW Transform
V. CONCLUSION

This paper presents Gabor-Wigner transform approach for the extraction of micro-Doppler features of radar returned signals from targets. The method combines the advantages of short-time Fourier transform and Wigner Distribution in order to extract the m-D features of radar target returns. By applying the proposed Gabor-Wigner transform to experimental data, the effectiveness of this analysis technique is confirmed. From the extracted m-D signatures, information about the target’s micro-motion dynamics, such as rotation rate and period of oscillation, can be obtained.

VI. ACKNOWLEDGEMENTS

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Gabor-Wigner Transform for Micro-Doppler Analysis

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Abstract:

Micro-Doppler signatures provide unique information about properties of the target. These micro-Doppler features can be used for real time target recognition in military applications and surveillance operations. In this paper, we present Gabor-Wigner transform for extracting micro-Doppler features from the radar returns. The effectiveness of the Gabor-Wigner transform in extracting micro-Doppler features has been compared with short-time Fourier transform, Wigner Distribution. The efficiency of the Gabor-Wigner transform in micro-Doppler feature extraction is demonstrated by applying it to different experimental data sets.

Keywords: Time-Frequency analysis, micro-Doppler effect, Short-Time Fourier Transform, Wigner Distribution, Gabor-Wigner Transform.

I INTRODUCTION

In many cases, a target or structure on a target may have rotations or vibrations. These micro-motions of the target may induce frequency modulation on the radar returned signal. This may lead to generation of side bands about the center of the Doppler shifted carrier frequency. The frequency modulation due to target’s rotating or vibrating motion is called micro-Doppler (m-D) effect [1]. Joint time-frequency methods can be used to extract m-D features [2]-[14]. The short-time Fourier transform (STFT) is the most widely used time-frequency representation. In STFT, the time and frequency resolution depends on the size of the window function used in calculating STFT. In order to improve time-frequency concentration, various quadratic time-frequency methods have been proposed. Wigner distribution (WD) is a well-known time-frequency method among the quadratic time-frequency methods, however it suffers from cross-term interferences. Recently, a new approach based on Gabor-Wigner transform is proposed, which efficiently suppresses the cross-terms in the output of WD function [3]. In this paper, we use Gabor-Wigner transform for the extraction of m-D features.

II. GABOR-WIGNER TRANSFORM

STFT is the simplest time-frequency representation introduced for time localization of the frequency contents of a signal by using a suitable window. Basic principle behind STFT is segmenting the signal into narrow time intervals using a window function and taking Fourier transform of each segment.

\[ STFT(t, \omega) = \int_{-\infty}^{\infty} x(\tau) \gamma(t-\tau) \exp(-j\omega \tau) d\tau \quad (1) \]

Where \( x(t) \) is the signal to be analyzed and \( \gamma(t-\tau) \) is a windowing function centered at \( t = \tau \). STFT has limited time-frequency resolution which is determined by the size of the window used. The STFT with Gaussian window is called a Gabor transform [3]. Mathematically Gabor transform (GT) of \( x(t) \) is defined as:

\[ GT(t, \omega) = \int_{-\infty}^{\infty} x(\tau) \gamma(t-\tau) e^{-j\omega \tau} d\tau \quad (2) \]

The WD was originally developed in the area of quantum mechanics by Wigner [4] and then introduced for signal analysis by Ville [5]. It is defined as:

\[ WD(t, \omega) = \int_{-\infty}^{\infty} x(t+\frac{\tau}{2}) x^*(t+\frac{\tau}{2}) e^{-j\omega \tau} d\tau \quad (3) \]

Compared to STFT, WD has much better time and frequency resolution. But the main drawback of the WD is the cross-term interference. This interference phenomenon shows frequency components that do not exist in reality and considerably affect the interpretation of the time-frequency plane. Cross-terms are oscillatory in nature and are located midway between the two components [6]. Presence of cross-terms severely limits the practical applications of WD. Pei and Ding proposed Gabor-Wigner (GW) transform that is defined as a multiplication of STFT with WD, which can significantly remove the cross-terms without affecting the quality of auto-terms [3]. Since cross-terms do not appear in GT, the time-frequency distribution of the GT can be used as a filter, to filter out the cross-terms in the output of the WD.

There are many different combinations to define the GW transform. Here four different definitions are given [3].

\[ GW(t, \omega) = GT(t, \omega) WD(t, \omega) \quad (4) \]

\[ GW(t, \omega) = \min\{|GT(t, \omega)|^2, |WD(t, \omega)|\} \quad (5) \]

\[ GW(t, \omega) = WD(t, \omega) \{|GT(t, \omega)| > 0.25\} \quad (6) \]
GW transform combines the advantages of GT and WD by eliminating cross-terms while maintaining the clarity as good as WD. However GW transform fails to provide useful results once the cross terms are superimposed on the auto components of WD in a multi component signal [7].

III. RESULTS

In this section, we demonstrate the application and effectiveness of GW transform for extracting m-D features. Two different types of experimental radar data sets from different scenarios are used in this demonstration.

A. Rotating corner reflector

Experimental trials were conducted to investigate and determine the m-D radar signatures of targets using an X-band radar. The target used for this experimental trial was a spinning blade with corner reflectors attached that were designed to reflect electromagnetic radiation with minimal loss. These controlled experiments can simulate the rotating types of objects, generally found in an indoor environment such as a rotating fan and, in an outdoor environment such as a rotating antenna or rotors. Controlled experiments allow us to set the desired rotation rate and then permit us to cross check and assess the results.

A picture of the target is shown in Figure 1. The experiment was conducted with the radar operating at 9.2 GHz. The pulse repetition frequency (PRF) was 1 kHz. The target employed in this experiment was at a range of 300 m from the radar. The distance between the two reflectors is 38 inches. The corner length of the reflector is 10 inches and the side length of the reflector is 12 inches. The result in Figure 2 was obtained using one rotating corner reflector facing the radar. Details of the figure clearly shows the sinusoidal motion of the corner reflector. The second weaker oscillation represents the reflection from the counter weight that was used to stabilize the corner reflector during operation. Figure 2a illustrates the time-frequency signature using STFT and Figure 2b illustrates the time-frequency signature using WD. Figure 2c shows that result from the proposed GW transform. Clearly it produces concentrated time-frequency signatures compare to STFT and WD. From the time-frequency signature we can see that the m-D of the rotating corner reflector is a time-varying frequency spectrum. The rotation rate of the corner reflector is directly related to the time interval of the oscillations. From the additional time information, the rotation rate of the corner reflector is estimated at about 60 rpm. Figure 3 shows the result when the blade is rotating with two corner reflectors. In this case, the rotation rate of the corner reflector was 60 rpm. Rotation rates estimated by the time-frequency analysis agree with the actual values.

Figure 1: Picture of the target simulator experimental apparatus.

Figure 2: m-D effect from one rotating corner reflector facing the radar: a) GT b) WD c) GW.
B. Performance Analysis

The performance of the time-frequency representations (TFRs) can be compared based on their readability, resolution, cross-terms suppression and energy concentration. Cross-terms suppression and energy concentration of a time-frequency representation is evaluated by visual inspection or on the basis of quantitative measures like entropy measures and ratio of norms [13].

Renyi entropy [14] of time-frequency representation, \( \phi(t, \omega) \) is defined as:

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Entropy_{\text{Renyi}} = \frac{1}{1-\alpha} \log \left( \sum_{t, \omega} \phi^\alpha(t, \omega) \right)
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(8)

where \( \alpha \) is the order of entropy. In the case of TFRs, low entropy stands for high concentration of auto-terms, where high entropy means lower energy concentration of auto-terms. Ratio of norms (RN) is calculated by dividing fourth power norm of a TFR, \( \phi(t, \omega) \) by its second power norm [15]. Mathematically, the RN is given by:

\[
RN = \frac{\sum_{t, \omega} \left| \phi^\alpha(t, \omega) \right|^2}{\left( \sum_{t, \omega} \left| \phi(t, \omega) \right|^2 \right)^{\frac{1}{2}}}
\]

(9)

Higher value of RN implies that signal auto-components are highly concentrated [15]. From Tables 1 and 2, it is evident that GW transform has minimum value of entropy and maximum value for RN. Therefore the proposed GW transform shows good energy concentration property compared to STFT and WD.

<table>
<thead>
<tr>
<th>Signal</th>
<th>STFT</th>
<th>WD</th>
<th>GW</th>
</tr>
</thead>
<tbody>
<tr>
<td>One Corner Reflector</td>
<td>39.045</td>
<td>37.179</td>
<td>31.034</td>
</tr>
<tr>
<td>Two Corner Reflectors</td>
<td>35.690</td>
<td>34.386</td>
<td>24.523</td>
</tr>
</tbody>
</table>

Table 1. Comparison of GW transform with other time-frequency transforms based on Entropy.

<table>
<thead>
<tr>
<th>Signal</th>
<th>STFT</th>
<th>WD</th>
<th>GW</th>
</tr>
</thead>
<tbody>
<tr>
<td>One Corner Reflector</td>
<td>0.667</td>
<td>1.013</td>
<td>2.578</td>
</tr>
<tr>
<td>Two Corner Reflectors</td>
<td>1.019</td>
<td>1.234</td>
<td>5.021</td>
</tr>
</tbody>
</table>

Table 2. Comparison of GW transform with other time-frequency transforms based on Ratio of norms.

CONCLUSION

This paper presents Gabor-Wigner transform approach for the extraction of micro-Doppler features of radar returned signals from targets. The method combines the advantages of short-time Fourier transform and Wigner Distribution in order to extract the m-D features of radar target returns. By applying the proposed Gabor-Wigner transform to the experimental data, the effectiveness of this analysis technique is confirmed. From the extracted m-D signatures, information about the target’s micro-motion dynamics, such as rotation rate and period of oscillation, can be obtained.

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