Chapter 3

ISAR Image Focusing using Time-Frequency Transforms

Inverse synthetic aperture radar (ISAR) imaging is an effective way to acquire high-resolution images of targets of interest at long range and it is an irreplaceable tool in the task of non-cooperative target recognition (NCTR). ISAR has been investigated for target identification by the combat identification community for the past decade [1]. One of the main challenges in ISAR imaging is the unknown nature of the target’s motion. The commonly used technique for ISAR imaging is a Fourier-based image formation, which assumes time invariance of the Doppler frequency to resolve the image in the cross-range. However, in real-world ISAR imaging scenarios, when a target exhibits complex motion such as rotation, acceleration, or maneuvering, standard Fourier-based methods can cause range-Doppler image blurring and leave the image unrecognizable. Also, for long imaging times (i.e., long coherent integration time or for fast maneuvering targets), simple Fourier processing without motion compensation will lead to severe image blurring in cross-range. Unless a good motion compensation algorithm is implemented, severe blurring can result in the ISAR image formed by the Fourier transform [3,10,14,22]. Joint Time-Frequency representations can be used for focusing distorted images [10]. Certain transforms provide good resolution but are slow to calculate. Others are quick to calculate but give poor resolution. Some transforms provide a good balance of resolution and speed, but have interference cross terms [23].

The oldest and the most widely used time-frequency representation is the short-time Fourier transform. In order to improve its concentration, various quadratic representations have been introduced. The most prominent member of this class of representations is the Wigner-Ville distribution, which suffers
from cross-term interferences. Considerable efforts have been deployed to design time-frequency distributions which reduce the cross-terms while preserving many desirable properties of Wigner-Ville distribution [57]. One example of such time-frequency distribution is the smoothed pseudo Wigner-Ville distribution. The smoothed pseudo Wigner-Ville distribution has a separable kernel, with a time smoothing window and frequency smoothing window. These two windows are chosen to suppress spurious peaks in both frequency and time domains. The suppression of cross terms is improved by using shorter windows. This, however, results in an undesirable loss of resolution and smearing of the image. Kodera et. al proposed reassignment method for improved distribution concentration. It is based on the reassignment of distribution values in the time-frequency plane [26]. The reassigned smoothed pseudo Wigner-Ville time-frequency distribution allows us to solve the problem of spectral broadening and cross terms to have readable and localized distribution in the time-frequency plane [27]. In the recent years, time-frequency methods like S-method, Adaptive S-method and Hermite S-method have been proposed for focusing distorted radar images [14,30,32].

In our study, we compared the ability of Fourier transform (FT), short-time Fourier transform (STFT), smoothed pseudo Wigner-Ville distribution (SPWVD), reassigned smoothed pseudo Wigner-Ville (RSPWVD), S-method (SM), Adaptive S-method (ASM), and Hermite S-method (HSM), in focusing distorted ISAR images of rotating targets. The performance of the considered transforms is compared with respect to quality metrics like Image Contrast, Image Sharpness, Blur Metric, and the time taken by each transform in focusing distorted ISAR images.

This chapter is basically organized into four sections. In section 3.1, a brief mathematical definitions of the time-frequency methods, that are being evaluated, are given. In section 3.2, the comparison of the various time-frequency methods for focussing distorted radar images, is presented using simulation data. In section 3.3 and 3.4, experimental results and quantitative analysis of the time-frequency methods in focussing distorted radar images of rotating targets, are presented, respectively.
3.1 Time-Frequency methods for ISAR Imaging

In this section, a brief background information of the time-frequency methods that are being evaluated are given.

3.1.1 Short-Time Fourier Transform

One of the shortcomings of the Fourier Transform (FT) is that it does not give any information on the time at which a frequency component occurs. FT gives good results when it is used for analyzing stationary signals but fails to give desired results if non-stationary signals are analyzed using FT. Time-frequency methods are suitable for the analysis of non-stationary signals. One of the central objects in the theory of time-frequency analysis is short-time Fourier transform (STFT). STFT is a signal processing method used for analyzing non-stationary signals, whose frequency spectrum varies with time. The STFT contains simultaneous information both on time and frequency. However, it does not carry instantaneous information both on time and frequency due to the uncertainty principle. In essence, STFT extracts several frames of the signal to be analyzed with a window that moves with time. If the time window is sufficiently narrow, each frame extracted can be viewed as stationary so that FT can be used. With the window moving along the time axis, the relation between the variance of frequency and time can be identified.

The STFT is define as

\[ STFT(\tau, \nu) = \int_{-\infty}^{\infty} x(t)W(t-\tau)\exp(-j2\pi \nu t)dt \]  

(3.1)

Where \( x(t) \) is the signal to be analyzed and \( W(t) \), windowing function centered at \( t = \tau \). One of the drawbacks of the STFT is that it has a fixed resolution. The width of the windowing function relates to how the signal is represented. The window width determines whether there is good frequency resolution (frequency components close together can be separated) or good time resolution (the time at which frequencies change). A wide window gives better frequency resolution but poor time resolution. A narrower window gives good time resolution but poor frequency resolution.
3.1.2 Smoothed pseudo Wigner-Ville distribution

The WVD for a signal $x(t)$ at time $t$ is defined as \[57\]
\[
WVD_x(t, \nu) = \int_{-\infty}^{\infty} x^*(t - \frac{1}{2}\tau)x(t + \frac{1}{2}\tau)e^{-j2\pi\nu\tau}d\tau \tag{3.2}
\]
where $\tau$ is the time lag variable, $\nu$ is the frequency instant. Although WVD has many advantages, it suffers from cross-term interference in the case multi-component signals. In order to reduce the interference of cross-terms, smoothing kernels are used in the time and frequency domains. One of the consequences of this is that the smoothed pseudo Wigner-Ville distribution (SP-WVD) suppresses, to some extent, the cross-terms for multi-component signals. The SPWVD is defined as \[27\]
\[
SPWVD_x(t, \nu; g, h) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(\tau)g(t-\tau)x^*(t - \frac{1}{2}\tau)x(t + \frac{1}{2}\tau)e^{-j2\pi\nu\tau}d\tau \tag{3.3}
\]
where $g$ is the time smoothing window and $h$ is the frequency smoothing window. Compared with WVD, the SPWVD greatly suppresses the influence of the cross-terms of multi-component signal but at the expense of time-frequency concentration.

3.1.3 Reassigned smoothed pseudo Wigner-Ville distribution

The ‘method of reassignment’ is a technique for sharpening a time-frequency representation by mapping the data to time-frequency coordinates that are nearer to true region of support of the analyzed signal. This technique is also known as remapping, time-frequency reassignment and modified moving window method. Pioneering work on the method of reassignment was published by Kodera, Gendrin, and de Villedary under the name of ‘Modified Moving Window Method’ \[81\]. The RSPWVD is defined as \[27\]
\[
RSPWVD_x(t', \nu'; g, h) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} SPWVD_x(t, \nu; g, h)\delta(t'-t^n(x; t, \nu))\delta(\nu'-\nu^n(x; t, \nu))dt d\nu \tag{3.4}
\]
where $SPWVD_x(t, \nu; g, h)$ smoothed pseudo Wigner-Ville distribution, $t$ and $t'$ are time instants, $\nu$ and $\nu'$ are frequency instants, $g$ is the smoothing window, $h$ is the frequency smoothing window, $t^n$ is the assigned time instant at
which the center of gravity of energy contribution is located, and $\nu^n$ is the assigned frequency instant at which the center of gravity of energy contributions is located. $t^n$ and $\nu^n$ are defined as

$$t^n = t - \frac{SPWVD_x(t, \nu; \Gamma_g, h)}{2\pi[SPWVD_x(t, \nu; g, h)]}$$

(3.5)

where $\Gamma_g$ is the operator of multiplication and $\Gamma_g(t) = t \ast g(t)$.

$$\nu^n = \nu - \frac{SPWVD_x(t, \nu; g, D_h)}{2\pi[SPWVD_x(t, \nu; g, h)]}$$

(3.6)

where $D_h$ is the derivative operator which is defined as $D_h = \frac{d}{dt}[h(t)]$.

### 3.1.4 S-method

Recently S-method-based approach has been proposed for real-time motion compensation, image formation and image enhancement of moving targets in ISAR and SAR. Like the WVD, the S-method can produce concentrated representations of linear frequency changes and has the added advantage of being cross-term free or with significantly reduced cross-terms. In contrast to other reduced interference distributions, which are usually derived under the condition that the marginal properties are preserved (what inherently leads to auto-term degradation with respect to the WVD [28], the S-method is derived with the goal of preserving the same auto-terms as in the WVD), while avoiding cross-terms [29]. In other words, the S-method automatically compensates for quadratic and all even higher-order terms in phase induced by the target’s complex motion, leading to well-focused images. The radar image can be obtained by using the 2D FT as:

$$Q(m', n') = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} w(m)q(m, n) \exp(-j[2\pi mm'/M + 2\pi nn'/N]).$$

where $w(m)$ is a window function used to reduce spectral leakage effects in the FT domain.

The S-method (SM) can be defined for radar images as:

$$SM_1(m', n') = \sum_k \Pi(k)Q(m' + k, n')Q^*(m' - k, n'),$$

(3.7)

where $\Pi(k)$ is a window in the frequency domain. For fixed $n'$ this form is the 1D SM for fixed range cell. In similar manner, the radar image based on the
1D SM for fixed cross-range cell is:

\[ SM_2(m', n') = \sum_l \Pi(l)Q(m', n' + l)Q^*(m', n' - l). \tag{3.8} \]

Also, the 2D SM is:

\[ SM_3(m', n') = \sum_k \sum_l \Pi(k, l)Q(m' + k, n' + l)Q^*(m' - k, n' - l). \tag{3.9} \]

Commonly, the frequency window function is rectangular, and for 1D SM given with (3.7) exhibits:

\[ \Pi(k) = \begin{cases} 
1 & |k| \leq K \\
0 & \text{elsewhere.} 
\end{cases} \tag{3.10} \]

Then, the corresponding SM form can be calculated as:

\[ SM_1(m', n') = |Q(m', n')|^2 + 2 \Re \left\{ \sum_{k=1}^{K} Q(m' + k, n')Q^*(m' - k, n') \right\}. \tag{3.11} \]

For \( K = 0 \) we obtain the standard radar image (defocused but without spurious terms), while for large \( K \) the radar image approaches the WVD-based image (focused but with interfering cross-terms). Fortunately, for relatively small \( K \) the radar image could be significantly improved without introducing the interference.

### 3.1.5 Adaptive S-method

The adaptive 1D SM for radar images can be defined as:

\[ SM_1(m', n') = |Q(m', n')|^2 + 2 \Re \left\{ \sum_{k=1}^{K} Q(m' + k, n')Q^*(m' - k, n') \right\}. \tag{3.12} \]

The main problem here is the determination of \( K(m', n') \). \( K(m', n') \) can simply be obtained as a maximal value of \( k \) for which the term \( \Re \{ Q(m' + k, n')Q^*(m' - k, n') \} \) used for the SM calculation is greater than a specific threshold \( R(m', n') \), and where all \( \Re \{ Q(m' + k', n')Q^*(m' - k', n') \} \) for \( |k'| < |k| \) are greater than the threshold. This can be written as:

\[ K(m', n') = \arg \max_k \bigwedge_{k'=1}^{k} (\Re \{ Q(m' + k', n')Q^*(m' - k', n') \} \geq R(m', n')) \tag{3.13} \]

where \( \bigwedge_{k'=1}^{k} \) represents logical operation and applied to logical expressions from argument for various \( k' = 1, \ldots, k \).
The threshold value can be determined in various ways. The global threshold is proposed in [30] as:

\[ R = \varepsilon \max_{m', n'} |Q(m', n')|^2, \tag{3.14} \]

where \( \varepsilon \) can be adopted as a small value, for example \( \varepsilon \in [0.1\%, 5\%] \). This thresholding approach can be applied locally for regions of the radar image. In addition, well-described techniques from the digital image processing can be used in this application [42].

### 3.1.6 Hermite S-method

In order to improve performances of the Fourier transform and the standard S-method in focusing distorted radar images, the two-dimensional multi-window S-method is studied [32]. It can be considered as an extension of the method introduced in [14]. The multi-window S-method is based on the usage of two-dimensional Hermite functions [33–36]. These functions exhibit some desirable properties such as good time-frequency localization. A signal expansion into series of Hermite functions provides the insight to its Fourier transform, since they are the Eigen functions of the Fourier transform [33–36]. Therefore, when two-dimensional Hermite functions are combined with the advantages of the S-method, it provides an enhanced resolution in radar imaging. Since multi-window S-method uses only a few low order functions, the algorithm complexity will be slightly increased in comparison with the standard S-method [32]. Unlike some common time-frequency techniques, that split the ISAR image into a time-series of ISAR images, the multi-window S-method uses the whole set of data, in the same way as the standard S-method does [32]. It will start with an already obtained radar image, based on the multi-window Fourier transform, and will improve its concentration by additional matrix calculations. The definition of the Hermite S-method in range direction is given as:

\[ HSM_R(p, q) = \sum_{i=-L}^{L} \sum_{j=-L}^{L} P(i, j)HX^K(p + i, q)HX^{K^*}(p - i, q) \tag{3.15} \]

where

\[ HX^K(p, q) = \sum_{k=0}^{K-1} \sum_{l=0}^{K-1} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} d_{kl} \Psi_{kl}(m, n)x(m, n)e^{-j(2\pi mp/M + 2\pi nq/N)} \tag{3.16} \]

where \( d_{kl} \) are the weighting coefficients, while \( K \) is the number of functions used along the range and cross-range directions. The \( k \)-th order Hermite
function is defined as:

$$\Psi_k(t) = \frac{(-1)^k e^{t^2/2}}{\sqrt{2^k k! \sqrt{\pi}}} \frac{d^k(e^{-t^2})}{dt^k}$$ (3.17)

3.2 Simulation Results

In this section, we compare the ability of the considered time-frequency methods in focusing distorted ISAR images using simulated data.

3.2.1 MIG-25 data analysis

The model considered is a MIG-25 aircraft with 120 point-scatterers distributed along the edge of the 2-dimensional shape of the aircraft [10, 14]. The stepped frequency X-band radar operating at a center frequency of 9 GHz with bandwidth of 512 MHz was used for simulation. With a total of 64 stepped frequencies, it has a range resolution of 0.293 m. The coherent integration time (CIT) is 1.64 seconds and the pulse repetition frequency (PRF) is 20 kHz. A total of 64 range cells and 512 cross-range cells are used for the imaging. The aircraft is at a range of 3500 m and is rotating at 10 degrees/second, thus giving a cross-range resolution of about 0.058 m [22].

The ISAR image of MIG-25 obtained by FT is shown in Figure 3.1a. As we can see, the general shape of the aircraft can be made out, but it is not possible to locate the range/cross-range cell of individual point-scatterers as they are smeared in the cross-range dimension. Figure 3.1b shows the image obtained using STFT. It is observed that the image obtained using STFT is clearer than the image obtained using Fourier transform. Figure 3.1c shows the image obtained using SPWV distribution. The SPWV has reduced the cross-terms to some extent, but cross-range resolution of the image is affected. Figure 3.1d shows the image obtained using RSPWV distribution. It is evident that the image obtained using RSPWV distribution shows substantial improvement in the degree of smearing in the cross-range when compared to other presented distributions. The ISAR image obtained by the SM and ASM are shown in Figures 3.1e and 3.1f, respectively. The SM and ASM are able to remove most of the blurring caused by the quadratic and higher-order phase effects. The quality of the images obtained from SM and ASM can be further improved by HSM. Figure 3.1g shows the image obtained using HSM for $K = 3$ (three window functions are used in both range and cross-range directions).
Figure 3.1: ISAR image of MIG 25 obtained using a) FT, b) STFT, c) SP-WVD, d) RSPWVD, e)SM, f) ASM, and g) HSM.
**3.2.2 Boeing 727 data analysis**

The second model is a Boeing 727 aircraft [3]. The simulation uses a stepped frequency X-band radar operating at a center frequency of 9 GHz. With a total of 64 stepped frequencies, the waveform has a bandwidth of 150 MHz and a range resolution of 1 m. The PRF is 20 kHz and the CIT is 0.82 seconds. A total of 64 range cells and 256 cross-range cells are used in the imaging.

ISAR image of the Boeing 727 obtained using the FT based method is shown in Figure 3.2a. It can be seen that image is blurred, making it difficult to extract the target. Figures 3.2b, 3.2c, 3.2d shows the images obtained using STFT, SPWV, and RSPWV respectively. It is evident that RSPWV gives more focussed image than STFT and SPWV. Figures 3.2e and 3.2f show SM and ASM representation of the target. The SM and ASM were able to remove most of the blurring caused by the quadratic and higher-order phase effects. Compared to Figure 3.2a, both images show a significant decrease in the amount of smearing in the cross-range. Figure 3.2g shows the image obtained using HSM. It can be seen that HSM further refocuses the nose and tail point scatterers, thus giving a focused image of the target.

**3.3 Experimental Results**

In this section, the effectiveness of the considered time-frequency representations in focusing distorted ISAR images is compared by applying it to experimental radar data.

**3.3.1 Delta-Wing**

An ISAR experiment is set up to examine the distortion of ISAR images due to a time-varying rotational motion. A 2-dimensional delta-wing shaped target, the target motion simulator (TMS), is built for the ISAR distortion experiments [3]. The target has a length of 5 m on each of its three sides. Six trihedral reflectors are mounted on the TMS as scattering centers of the target; all the scatterers are located on the $x - y$ plane. They are designed to always face towards the radar as the TMS rotates. The TMS target is set up so that it may rotate in a plane perpendicular to the radar line of sight.

Radar data from the rotating delta-wing was collected using an X-band radar operating at a center frequency of 10.1 GHz with 300 MHz bandwidth and a range resolution of 0.5 m. The PRF is 2 kHz. Each HRR profile is
Figure 3.2: ISAR image of Boeing 727 obtained using a) FT, b) STFT, c) SPWVD, d) RSPWVD, e) SM, f) ASM, and g) HSM.
Figure 3.3: ISAR image of Delta-Wing obtained using a) FT, b) STFT, c) SPWVD, d) RSPWVD, e) SM, f) ASM, and g) HSM.
generated in 0.5 ms and each profile has 41 range bins. The total data set contains 60,000 HRR profiles. The delta-wing is at a range of 2 km and is rotating at 2 degrees/second. Figure 3.3 shows the ISAR images of Delta-Wing from 2048 pulses, which corresponds to a CIT of 1.024 seconds. Figure 3.3a shows the FT based representation of the target. It is evident that the image is severely blurred. Since significant phase errors due to non-uniform motion exist in the data, the FT based images are blurred in the cross-range dimension. Figures 3.3b, 3.3c, 3.3d show the images obtained using STFT, SPWV, and RSPWV, respectively. In the images formed by RSPWV, the cross-range smearing is significantly reduced, resulting in an improvement in the image quality. Also, RSPWV gives more focused image compared to other presented distributions. Figure 3.3e shows the SM representation of the target. In SM based image, the cross-range smearing is significantly reduced, resulting in a substantial improvement in the image quality. The ISAR image of the target obtained by using the ASM and HSM are shown in figure 3.3f and 3.3g, respectively. The ISAR images are more focused and the six reflectors are clearly visible.

3.4 Quantitative Analysis

Image Contrast (C), Blur Metric (BM) and Image Sharpness (IS) are used in order to compare the quality of the re-focused images. Image contrast is typically given by an expression such as [41]

\[
C = \sqrt{\frac{\sum_{n=1}^{N} |s_n|^4}{\sum_{n=1}^{N} |s_n|^2}}
\] (3.18)

Where \(|s_n|^2 \) is the image intensity of the \(n^{th}\) pixel. Image sharpness is defined as the intensity normalized sum of the absolute value of pixel photocurrent convolved with a spatial high-pass filter [43].

\[
IS = \frac{1}{E} \sum_i \sum_j |I_{i,j} * K|
\]

\[
K = \begin{bmatrix}
0 & -1 & 0 \\
-1 & 4 & -1 \\
0 & -1 & 0 
\end{bmatrix}
\] (3.19)
where \( I_{i,j} \) is the intensity at the \((i,j)\)th position and \( E = \sum_i \sum_j I_{i,j} \) is the total intensity of the image. Frederique Crete et al in [44] proposed a new No-Reference Perpetual Blur Metric to quantify the blur effect. The first step in calculating BM consists of computation of intensity variations between neighboring pixels of the input image. Applying a low-pass filter on the same image, the variations between the neighboring pixels are computed. Then the comparison between the intensity variations gives the estimate of the blur effect [44]. The quality metric values of C, IS and BM, for images presented in Figures 3.1, 3.2 and 3.3 are given in Tables 3.1, 3.2 and 3.3 respectively. High values of C and IS indicate the highest quality. The high value of BM indicates that the image is more blurred and low value of BM indicates that image is sharp. ISAR images obtained using Reassigned Smoothed Pseudo Wigner-Ville distribution are found to have highest Image Contrast and lowest Blur Metric values. ISAR images obtained using SM, ASM and HSM have relatively high C and low BM values when compared to FT, STFT and SPWV. It is evident that ISAR images obtained using SM, ASM and HSM are sharper and more focused compared to images obtained using FT, STFT and SPWV. But overall, images obtained using RSPWV have highest C and lowest BM values. Hence from the quality metric values in Tables 3.1, 3.2 and 3.3, we can quantitatively conclude that images obtained using RSPWV distribution are more focused compared to the other presented distributions.

<table>
<thead>
<tr>
<th>Time-Frequency method</th>
<th>Image contrast</th>
<th>Image sharpness</th>
<th>Blur metric</th>
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<tbody>
<tr>
<td>FT</td>
<td>0.129</td>
<td>4.572</td>
<td>0.743</td>
</tr>
<tr>
<td>STFT</td>
<td>0.241</td>
<td>4.902</td>
<td>0.625</td>
</tr>
<tr>
<td>SM</td>
<td>0.273</td>
<td>4.909</td>
<td>0.373</td>
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<tr>
<td>ASM</td>
<td>0.270</td>
<td>4.924</td>
<td>0.381</td>
</tr>
<tr>
<td>HSM</td>
<td>0.223</td>
<td>6.935</td>
<td>0.120</td>
</tr>
<tr>
<td>SPWVD</td>
<td>0.218</td>
<td>5.102</td>
<td>0.458</td>
</tr>
<tr>
<td>RSPWVD</td>
<td>0.373</td>
<td>5.275</td>
<td>0.080</td>
</tr>
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</table>

Table 3.1: Quantitative analysis: MIG data.

Table 3.4 shows measured execution time for the evaluated program codes. The program codes were executed in Matlab 7 using a Intel Xeon 2.40 GHz processor with 24 Gbyte of RAM. From Table 3.4, it is evident that in the case of MIG-25 simulated data, SM is around 2.3 times slower than standard Fourier transform and 1.6 times slower that STFT. But SM is 16 times faster than SPWV and around 54 times faster than RSPWV. It is observed that in
3.5 Conclusion

It is clearly observed that the images obtained using short-time Fourier transform are more clear than the images obtained using smoothed pseudo Wigner Ville distribution. In cases where resolution is not the main concern, STFT is a good focusing method for use. The reassigned smoothed pseudo Wigner-Ville distribution can be used to refocus distorted ISAR images. The advantage of reassigned smoothed Wigner-Ville distribution is that it drastically reduces the blur in cross-range direction and gives more focused images than all the standard Fourier transform, short-time Fourier transform, smoothed pseudo Wigner-Ville distribution, S-method, adaptive S-method and Hermite S-method. The only drawback of the reassigned smoothed

<table>
<thead>
<tr>
<th>Time-Frequency method</th>
<th>Image contrast</th>
<th>Image sharpness</th>
<th>Blur metric</th>
</tr>
</thead>
<tbody>
<tr>
<td>FT</td>
<td>0.163</td>
<td>5.505</td>
<td>0.397</td>
</tr>
<tr>
<td>STFT</td>
<td>0.312</td>
<td>5.277</td>
<td>0.252</td>
</tr>
<tr>
<td>SM</td>
<td>0.346</td>
<td>5.412</td>
<td>0.166</td>
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<tr>
<td>ASM</td>
<td>0.380</td>
<td>5.319</td>
<td>0.179</td>
</tr>
<tr>
<td>HSM</td>
<td>0.413</td>
<td>6.205</td>
<td>0.083</td>
</tr>
<tr>
<td>SPWVD</td>
<td>0.236</td>
<td>5.187</td>
<td>0.266</td>
</tr>
<tr>
<td>RSPWVD</td>
<td>0.455</td>
<td>5.3346</td>
<td>0.062</td>
</tr>
</tbody>
</table>

Table 3.2: Quantitative analysis: Boeing data.

<table>
<thead>
<tr>
<th>Time-Frequency method</th>
<th>Image contrast</th>
<th>Image sharpness</th>
<th>Blur metric</th>
</tr>
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<tbody>
<tr>
<td>FT</td>
<td>0.262</td>
<td>6.543</td>
<td>0.222</td>
</tr>
<tr>
<td>STFT</td>
<td>0.496</td>
<td>6.117</td>
<td>0.070</td>
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<tr>
<td>SM</td>
<td>0.586</td>
<td>7.286</td>
<td>0.038</td>
</tr>
<tr>
<td>ASM</td>
<td>0.568</td>
<td>7.308</td>
<td>0.037</td>
</tr>
<tr>
<td>HSM</td>
<td>0.399</td>
<td>7.429</td>
<td>0.030</td>
</tr>
<tr>
<td>SPWVD</td>
<td>0.506</td>
<td>6.710</td>
<td>0.084</td>
</tr>
<tr>
<td>RSPWVD</td>
<td>0.652</td>
<td>7.159</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Table 3.3: Quantitative analysis: Delta-Wing data.

the case of experimental data, SM is around 24 times faster than RSPWV. Results in the Table 3.4 show that RSPWV is computationally expensive, but it gives better focussed image than the remaining time-frequency methods considered in this study.

3.5 Conclusion
Table 3.4: Performance of TF methods (in seconds).

<table>
<thead>
<tr>
<th>Time-Frequency method</th>
<th>MIG</th>
<th>Boeing</th>
<th>Delta Wing</th>
</tr>
</thead>
<tbody>
<tr>
<td>FT</td>
<td>0.760</td>
<td>0.138</td>
<td>0.147</td>
</tr>
<tr>
<td>STFT</td>
<td>1.051</td>
<td>0.241</td>
<td>4.206</td>
</tr>
<tr>
<td>SM</td>
<td>1.747</td>
<td>0.314</td>
<td>6.836</td>
</tr>
<tr>
<td>ASM</td>
<td>2.726</td>
<td>0.458</td>
<td>10.355</td>
</tr>
<tr>
<td>HSM</td>
<td>3.890</td>
<td>1.828</td>
<td>12.406</td>
</tr>
<tr>
<td>SPWVD</td>
<td>27.750</td>
<td>7.164</td>
<td>50.310</td>
</tr>
<tr>
<td>RSPWVD</td>
<td>93.793</td>
<td>23.569</td>
<td>165.453</td>
</tr>
</tbody>
</table>

pseudo Wigner-Ville distribution is that it is computationally expensive. On the other hand, S-method performs better than the Fourier transform, short-time Fourier transform and smoothed pseudo Wigner-Ville distribution by significantly improving the images of fast maneuvering targets. Also, it is computationally very efficient. These advantages are the result of the S-method’s ability to automatically compensate for quadratic and all even higher-order terms in phase. Thus, targets with constant acceleration will undergo full motion compensation and their point-scatterers will be localized. Adaptive S-method and Hermite S-method gives slightly better results than standard S-method at the cost of increased computational complexity. S-method based distributions are suitable for real-time target identification in ISAR systems.