Chapter 2

Multi-Attribute Decision-making: A Survey

2.1 Introduction

In today's complex world decision making has become more and tougher and can barely be solved by considering a single attribute or which can also be termed as criterion for a certain problem. So there comes the utility and the hallmark of MCDM methodologies in multi-objective problems where comparisons as well as ranking and selection can be done between the multiple attributes and multiple alternatives with the initial help of the decision makers. Decision-making can be treated as the cognitive process where choosing the best option among the alternatives is logical. It consists of a set of criteria and alternatives. Each criteria has a weighted value that can be obtained from decision-maker or expert group. After evaluating the weighted value of different criteria, the decision-making can be made. Depending on the type of problem, MCDM model contains various elements and the following picture depicts the most widely found elements-

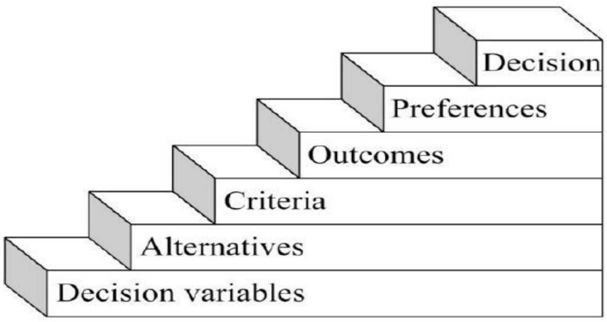

Figure 2.1 MCDM Model's elements

There are several other classes of MCDM which can be termed as multi-attribute decision making (MADM) and multi-objective decision making (MODM). In the multi-criteria problem, It is complex to determine the best optimal choice among the alternatives when several criteria are involved. A problem can be solved in different ways. One of the ways is to select the best alternative from a group of alternatives (where “best” can be treated as “the most preferred alternative” of a decision maker) and another way is to select from a small set of good alternatives [1]. Choosing the best solution is definitely a complex task where the problem consists different criteria.

Sometimes such information, which is provided to decision-maker, might be incomplete or imprecise. Human thought can create the impression of vagueness. So in this situation problem solving is quite difficult. To overcome this problem Fuzzy set theory is introduced along with the MCDM, which is able to solve the uncertain situations. That is called Fuzzy Multicriteria Decision Making (FMCDM). In 1965, Zadeh proposed the fuzzy set theory to support uncertainty associated with vagueness or impression, and thus relevant to human cerebration [2]. A fuzzy MCDM model consists of several criteria, alternatives and weight of each criteria, which can be represented in the term of linguistic values and expressed by fuzzy numbers with the help of a committee of decision-makers. Most of FMCDM problems, final ratings of alternatives are still in fuzzy numbers. De-fuzzification is required to convert from fuzzy value to crisp value for decision-making.

There are several types of MCDM and FMCDM methods available that are used to solve the decision-making problems and this survey is mainly based on understanding the MCDM and how to solve this problem by providing the various MCDM methods. As the application area of MCDM method is very large, there are lots of work that have been proposed in MCDM domain and different type of MCDM methods is applied for selection of the optimal choice in the different field. Each MCDM method has its own characteristics and uniqueness. Two or more methods can also be combined to produce a hybridization approach which can be used for solving complex decision-making problems. Some of the application areas of MCDM & FMCDM methods are location planning [3], Supply chain management [4][5], E commerce[6], Software Industry[7], Financial[8], Airlines[9] etc. There is some example of they bridapproaches in MCDM, such as AHP and Fuzzy TOPSIS [10], Fuzzy AHP and DEA [11] etc.

2.2 Characteristics of MCDM

MCDM analysis has some unique characteristics such as the presence of multiple and conflicting criteria, different units of measurement among the criteria, and the presence of quite different alternatives. It is an attempt to review the various MCDM methods and need was felt of further advanced methods for empirical validation and testing of the various available approaches for the extension of MCDM into group decision-making situations for the treatment of uncertainty.

The weighted sum model (WSM) is the earliest and probably the most widely used method. The weighted product model (WPM) can be considered as a modification of the WSM, and has been proposed in order to overcome some of its weakness [1]. The analytic hierarchy process (AHP), as proposed by Saaty is a later development and it has recently become popular. Recently modification to the AHP is considered to be more consistent than the original approach. Some other widely used methods are the ELECTRE and the TOPSIS methods.

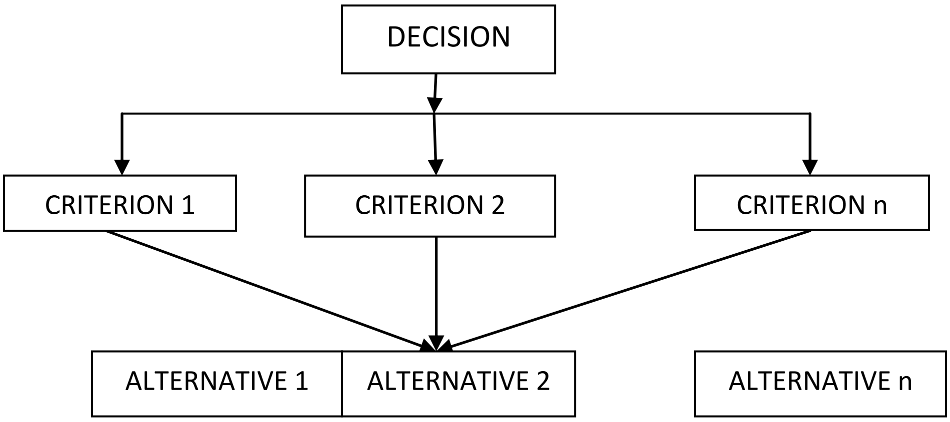

Figure 2.2Multi-criteria decision making (MCDM) Tree

2.3 Comparison matrix

In applied problems, decision makers give subjective judgements on the relative importance of criteria and set of alternatives with respect to each criterion. MCDM problem can be expressed as

$$\mathbf{D} = \begin{matrix} & C_1 & C_2 & \cdots & C_n \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} & \left[\begin{matrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{matrix}\right] \end{matrix}$$

$$\mathrm{W} = [w_1 \quad w_2 \quad \cdots \quad w_n]$$

D is a comparison matrix. Where A_1, A_2, ... Am is the set of possible alternatives from which decision maker have to select the best alternative. C_1, C_2, ...,C_n is the set of criteria. Based on these criteria performance of alternatives are measure d.x_{ij} has representedthe rating of alternatives which is obtained from the comparison between alternative A_i and each criteria C_j. The weight of each criteria C_j is expressed by W_j.

2.4Methods of MCDM

Attributes can be classified into two different types, qualitative and quantitative. These above mentioned MCDM methods can be applied to both data types. The main goal of these methods is to find the best solution and selecting the best alternative. The hierarchical structure of various MCDM methods is showed in figure 2.3. These methods have been discussed in the following section as follows-

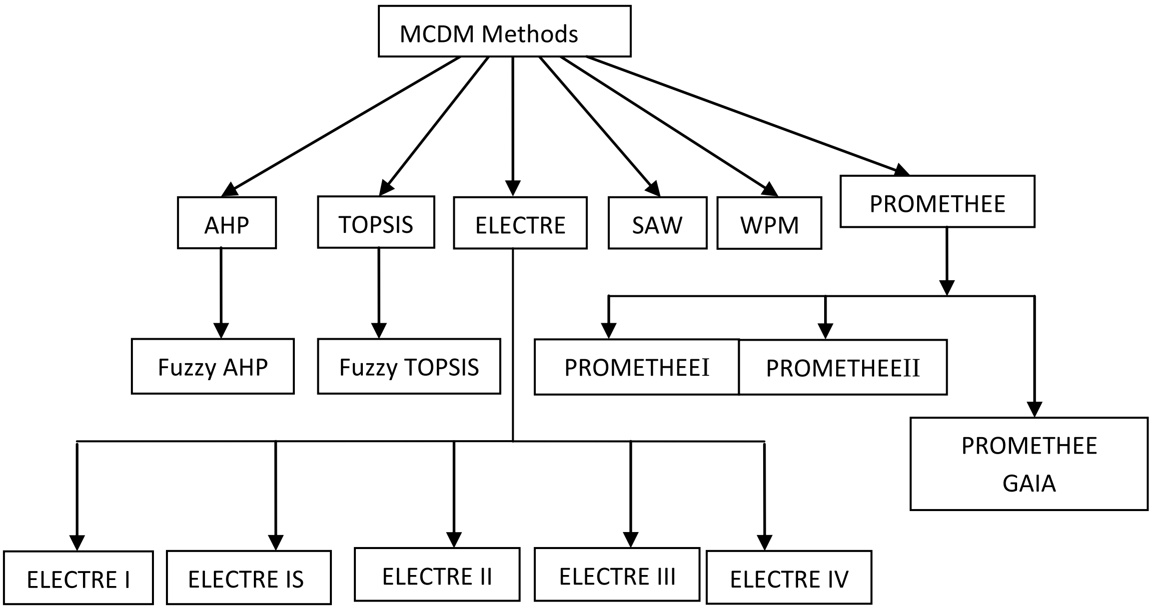

Fig 2.3. Hierarchical representation of MCDM Methods

2.4.1Analytic Hierarchy Process (AHP)

AHP was originally developed by Thomas L. Saaty (1980). It is mainly developed for dealing with the complex decision-making problem which has several types of conflicting criteria and alternatives. This method reduces the workload of decision-makers. The goal of AHP is to evaluate the final ranking that is obtained from the pair-wise comparison of both alternatives and criteria [16]. AHP is a simple method because it doesn't require constructing complex expert system with the knowledge of decision-maker enclosed in it. Computations made by the AHP are always supported by the decision-maker.

In AHP, every individual evaluation is very simple that can be easily deduced by a user, but when the number of criteria and alternatives increase, then it requires a large number of evaluations. In fact, the number of pair-wise comparisons grows immensely with the number of criteria and options. For an example, suppose a problem consist of 10 criteria and 4 alternatives. So a number of comparisons required to build a weight vector become$(4 \times 3)/2 = 6$ and the number of pair-wise comparisons required to build the score matrix becomes$4 \times (10 \times 9)/2 = 180$.

Steps of AHP

The major steps of AHP can be implemented as follows.

Step 1: Make pair-wise comparisons between the objects and construct the comparison matrix.

All the pair-wise comparisons are expressed by a Comparison matrix. Each object has a score, which is provided by the decision-maker that can be calculated based on the comparison scale. Upper triangular matrix is filled up by actual values of judgements and the lower triangular matrix is filled up by reciprocal values. Suppose A is a $m \times m$ comparison matrix, where $'m'$ is the number of criteria. Each entry a_{ij} of the matrix A represents the importance of the i^{th} criterion corresponds to the j^{th} criterion. Each pair of a_{ij} and a_{ji} are satisfying the following constraint,

$$a_{ij} . a_{ji} = 1 \quad (2.1)$$

Step 2: Construct the Normalized matrix and Weighted Normalized matrix.

After building the comparison matrix, it is required to be normalized by making the sum of each column equal to 1. Suppose A_1 is normalized matrix and each entry of that matrix, $\bar{a}_{ij}$ is calculated as

$$\bar{a}_{ij} = \frac{a_{ij}}{\sum_{k=1}^{m} a_{kj}} \quad (2.2)$$

Finally, the Criteria Weight Vector $'w'$ is calculated by averaging the entries on each row of A_1 i.e.

$$w_i = \frac{\sum_{k=1}^{m} \bar{a}_{ik}}{m} \quad (2.3)$$

Step 3: Computation of the Option Score matrix:

The Option Score matrix, B is a $m \times n$ real matrix, where $'m'$ is a set of criteria and $'n'$ is a set of alternatives. Each entry b_{ij} of Brepresents the score of ith option with respect to jth criteria. For each criteria a pair-wise comparison matrix $B^i$is built $(i = 1, \ldots, m)$.$B^i$is a $n \times n$ real matrix, where n is the number of alternatives. The same procedure which is described above is applied to each B^i. After evaluating each B^i, finally score matrix S is obtained.

Step 4. Ranking the Options

After computing the weight vector w and score matrix$S$, global score of vector v is obtains by multiplying $S$and .

$$v = S.w \qquad (2.4)$$

Each entry v_i of v is represented by the global score which is obtained after applying AHP. Finally the largest entry of v is considered the best option and the option ranking is completed by ordering the global scores in diminishing order.

Step 5: Checking the consistency

During computing the pair-wise comparison, inconsistency may occur. So it is always important to check the consistency during the pair-wise comparison. The step of checking consistency as follows

i. Calculate the Principle Eigenvalue(λ_{max}), which is obtained from the summation of product between each element of Eigen vector(EV) and the sum of column of the decision matrix.

ii. Calculate the Consistency Index(CI) as follows,

$$CI = \frac{\lambda_{max} - n}{n-1} \qquad (2.5)$$

Where $n$is a number of objects.

iii. Calculate the consistency ratio (CR), which is obtained from the following equation,

$$CR = \frac{CI}{RI} \qquad (2.6)$$

Where RI stands for Random Consistency Index.

If the value of Consistency Ratio is smaller or equal to 10%, the value of present inconsistency is acceptable. If the Consistency Ratio is greater than 10%, it is necessary to revise the subjective judgments in the decision matrix in order to get a renewed value of inconsistency which will be put to test again.

2.4.2Simple Additive Weighting(SAW)

Multi-attribute decision-making (MADM) is one type of MCDM problem. MADM models are selector models and used for evaluating, ranking and selecting the most appropriate alternative among the set of alternatives [12]. It is a simple approach to finding afinal score of alternatives. SAW (Fish burn, 1967) consists of mainly two steps, the first evaluation of the-the final score of each alternative is done and then finally they are ranked. The method is described is as follows

$$P_i = \sum_{j=1}^{k} w_j . r_{ij} \; ; \; i = 1,2, \ldots\ldots, m \quad (2.7)$$

Where r_{ij} is the normalized value of decision matrix, that can be calculated as follows,

for profit attribute

$$r_{ij} = {d_{ij}}/{d_j^{Max}} ; d_j^{Max} = \max_{1 \le i \le m} d_{ij} \quad ; j = 1,2, \ldots\ldots, k \qquad (2.8)$$

for cost attribute

$$r_{ij} = {d_j^{Min}}/{d_{ij}} ; \; d_j^{Min} = \min_{1 \le i \le m} d_{ij} \quad ; j = 1,2, \ldots\ldots, k \qquad (2.9)$$

2.4.3 Weighted Product Method(WPM)

WPM is similar to the SAW method. The only difference between SAW and WPM is, instead of performing summation operation to calculate the rank in case of SAW, here in case of WPM (Miller and Starr, 1969) multiplication operation is performed to calculate the rank. In WPM, the procedure to calculate the normalized value of an alternative is also same as that of SAW method.

2.4.4 Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)

It is an another brilliant methodology for solving MCDM problems, developed by Hwang and Yoon in 1981 with further developments by Yoon in 1987 and Hwang, Lai and Liu in 1993 respectively. The principle of the TOPSIS is to select the alternative that is closest the positive ideal solution and farthest from the negative ideal solution [21]. The positive ideal solution$A^+$is formed as a composite of the best performance values exhibited. The negative ideal, A^-, is the composite of the worst performance values. The process of the TOPSIS method is carried out as follows

Step1: Calculate the normalized decision matrix $R = [r_{ij}]_{mxn}$ using the alternatives m and criteria n. The normalized value r_{ij} is calculated by the following equation

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i-1}^{m} x_{ij}^2}}, i = 1, 2, \ldots, m and j = 1, 2, \ldots, n. \quad (2.10)$$

Step2: Calculate the weighted normalized decision matrix $V = [v_{ij}]$. The weighted normalized value v_{ij} is calculated as follows:

$$v_{ij} = (r_{ij})(w_j), i = 1, 2, \ldots, m and j = 1, 2, \ldots, m \quad (2.11)$$

where w_j is the weight of the j^{th} attribute and $\sum_{j-1}^{n} w_j = 1$

Step 3: Determine the positive ideal solution (PIS) A^+ and negative ideal solution (NIS) A^-

$$A^+ = \left\{\left(\max_i v_{ij} \middle| j \in J\right), \left(\min_i v_{ij} \middle| j \in J'\right), i = 1,2, \ldots, m\right\} = \{v_1^+, v_2^+, \ldots, v_n^+\} \quad (2.12)$$

$$A^- = \left\{\left(\min_i v_{ij} \middle| j \in J\right), \left(\max_i v_{ij} \middle| j \in J'\right), i = 1,2, \ldots, m\right\} = \{v_1^-, v_2^-, \ldots, v_3^-\} \quad (2.13)$$

Where J is a set of benefit attributes and J' is a set of cost attributes.

Step 4: Calculate the separation measures using them-dimensional Euclidean distance.

The separation measures of each alternative from the positive ideal solution are as follows

$$S_i^+ = \sqrt{\sum_{j-1}^{n} \left(v_{ij} - v_j^+\right)^2}, \quad i=1,2,\ldots,m \quad (2.14)$$

The separation measure of each alternative from the positive ideal solution is as follows

$$S_i^- = \sqrt{\sum_{j-1}^{n} \left(v_{ij} - v_j^-\right)^2}, \quad i=1,2,\ldots,m \quad (2.15)$$

Step 5: Calculate the relative closeness to the ideal solution. The relative closeness of the alternative A_i with respect to A^+ is defined asfollows:

$$C_i = \frac{S_i^-}{S_i^+ + S_i^-} \quad i=1,2,\ldots,m; 0 \le C_i \le 1 \quad (2.16)$$

Step 6: Rank the alternatives in descending order with respect to C_i.

2.4.5 Elimination and Choice Translation Reality (ELECTRE)

ELECTRE was initiated by the Benayoun, Roy, and Sussmann in 1966. Several versions of ELECTRE method were proposed like ELECTRE I, ELECTRA IS, ELECTRE II, ELECTRE III, ELECTRE IV and ELECTRE TRI. This method is efficient and effective for the MCDM. The basic principle of the ELECTRE method is based on the concept of outranking by using pairwise comparisons among alternatives under each criteria. There are two steps of ELECTRE method

1. Building the outranking relation
2. Exploitation of the outranking relation

ELECTRE method is used to discard some alternatives from the problem which are not acceptable. After discarding unacceptable alternatives another MCDA is used to select the best one. The main advantages of the ELECTRE method are that using this method before applying another MCDA with a restricted set of alternatives it saves much time. ELECTRE method varies from one to another version according to the type of the decision-making problem, degree of complexity, information quality. In ELECTRE method there are two sets of parameters- i) the importance coefficient, ii) veto thresholds.

The simple ELECTRE provides the basic understanding concept and followed by the extensions of ELECTRE I, ELECTRA IS, ELECTRE II, ELECTRE III, ELECTRE IV, and ELECTRE TRI for the purpose of introducing veto thresholds concept and pseudo criteria which are the fundamental applications of ELECTRE method for MCDM.

2.4.6 Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE)

PROMETHEE was first introduced by professor Jean-Pierre Barns in 1982. It is based on the principle of the outranking method by using the mutual comparison of each alternative pair for each criteria. There are two steps included-

Step 1 : Assigning a preference function

In this step, starting point is evaluation matrixes that represent the performance of each alternative under each criteria. By using the data of the evaluation matrix compares the alternatives pair-wise under each criteria. The results are represented by a function called the preference function. The preference function ranges from 0 to 1, where 0 means no difference and 1 means a big difference between the pair.

Step 2: Estimating the out ranking degree of the options

Calculate the global preference matrix by multiplying the preferences with the weights of the criteria and adding the single value. In global preference matrix, the sum of the row represents the strength of an alternative (dominance) and the sum of the column represent how much an alternative is dominated by the other ones (sub-dominance). Calculate the rank of the alternatives by subtracting the sub-dominance value from the dominance value.

PROMETHEE method does not provide the weights of the criteria. The decision-makers provide the weights of the criteria and the preference function. Various PROMETHEE method is introduced like PROMETHEE I, PROMETHEE II, PROMETHEE GAIA.

Table 2.1 MCDM methods with its advantages and disadvantages

Sl. No	**MCDM Methods**	**Description**	**Advantages**	**Disadvantages**
1	Analytic hierarchy process (AHP)	It provides pair-wise comparison of several alternatives for several criteria.	1. Straight forward, flexible and favourable. 2. Always checks inconsistency. 3. The problem is built into a hierarchical structure that helps in finding the goal. 4.It gives a clear idea about the importance of each criterion. 5. Performa pair-wise comparison between the attributes.	1.To find the goal it needs a large number of pairwise comparisons. 2.Ranking evaluation is in an irregular manner. 3. Inconsistency obligatory by 1 to 9 scale. 4. Subjective evaluation. 5. Not efficient for large set of criteria.
2.	Technique for Order of Preference by Similarity to Ideal Solution(TOPSIS)	Choose the alternative which is near to positive ideal solution and farthest from negative ideal solution	1. Decision making is simple using both cost and profit criteria. 2. Evaluate the rank of each alternative. 3.Easily programmable and simple computation process 4. Good computational efficiency.	1. Only independent criteria are allowed. 2. Normalization is required for criteria evaluation. 3. Criteria are monotonically decreasing or Increasing in nature.

3.	Simple Additive Weighting(SAW)	Provides pairwise comparison of several alternatives for several criteria. and calculate a score for each alternative. It based on the weighted average.	1. It is simple technique and most often used in MCMD. 2. Consistency is measured.	It is only efficient when criteria evaluation is Maximized.
4.	Weighted Product model(WPM)	Perform a comparison between alternatives by the weights and ratio of each criterion.	1. Can remove any unit of measure. 2. It is used relative values.	No support for calculating weights.
5.	Data Envelopment Analysis (DAE) (Afshari, 2010)	DEA is used to find The efficiency of the combination of multi inputs and multi-outputs of the problem	1. Multiple inputs and outputs can be handled. 2. TheRelation between inputs and outputs are not necessary. 3. Comparisons are directly against peers 4.Inputs and outputs can have very different units	1.Measurement error can cause significant problems 2.Absolute efficiency Cannot be measured. 3. Statistical tests are not applicable. 4. Large problems can be demanding.
6.	ELECTRE	It builds the outranking relation then explore the relation. This method discards some alternative which is not acceptable.	1.Outranking is used	1.Time consuming 2. It is a complex decision-making method and requires lot of primary data.
.7.	PROMETHEE	Choose the best Alternative by using the mutual comparison of each alternative pair for each criteria. In this method, preference function is used which is provided by the Decision maker for represents the Performance of each criteria of each alternative.	1.Group level decision making is supported	1.Does not provides any guideline of weighting information of criteria but assume that the decision makers are able to provide the weights of the criteria Properly. 2. The way in which the preference ranking information is processed is complicated and hard to explain for the non specialist.

2.5FUZZY set theory in MCDM problem

In MCDM problem, constructing pair-wise comparison between the objects is dealing with the judgement of decision-maker. Sometimes information provided to the decision-maker is incomplete or imprecise and some problem dealing with the uncertainties and vagueness. Human thought or perception cannot be judged by the form of exact numerical value. To support this problem, the fuzzy set theory was introduced into decision-making domain where the decision maker can give their opinion in the form of the linguistic term rather than exact numerical value.

2.5.1FUZZY set theory

The fuzzy sets are represented by linguistic terms that build one or more linguistic variables, i.e. the linguistic variables have their possible states defined in a universe of discourse , represented by these linguistic terms[13].

A fuzzy set 'C 'can be represented as,

$$C = \{(x, \mu_c(x)) \mid x \epsilon X\}$$

Where $\mu_c(x)$ is called the Membership function (MF) for the fuzzy set $C. X$ is reoffered to as Universe of Discourse's is represented as linguistic values. Each element of X has membership grade between 0 and1.

Fuzzy set and its MF can be represented as different ways, such as Triangular, Trapezoidal, Sigmoid, Gaussian etc. In this study, we have used triangular MF

Fuzzy set theory can be applied to different types MCDM methods for supporting the uncertainties and vagueness. It is compact with the various types of MCDM methods and it helps to increase the performance of this methods. Methods of FMCDM are Fuzzy Analytical Hierarchical Process (FAHP), Fuzzy Technique for Order of Preference by Similarity to Ideal Solution (FTOPSIS), Fuzzy Simple Additive Weighting (FSAW), and Fuzzy Weighted Product Method (FWPM) etc.

2.6Applications of FMCDM methods

In day-to-day life, FMCDM methods are used in various fields. It reduces the complexity of decision-making problem and helps to provide flexible decision-making. Some of FMCDM methods such as FAHP, FSAW, and FWAP have the capability of consistency checking. It removes the inconsistency while making the judgement by decision-maker. Some of FMCDM methods and its application are discussed.

Some application area of FAHP is described in Table.2.2 i.e. a suitable bridge construction [14], Evaluation of the Best Technical Institutions [15], Contractor Selection [16], and Evaluating Tourism Islands [17].

Sub criteria[14] Durability, Suitability Damage cost, Construction cost, Traffic conflict, Site condition, Constructability, Weather condition, Landscape, Geometry, Environmental preservation.

Sub criteria[15]: Hostel, Transport/canteen/Internet, Power backup, Security, Teacher/Student ratio, Qualification/Experience of Faculty, Faculty retention, Admission, Academic Result, Placement, Classroom, Laboratory, Library, Syllabus coverage, Tutorial/ remedial Use of Advance Teaching Aid, Alumni, Co-curricular activity, Cultural activity, seminar/ Workshop.

Table2.2. Application of Fuzzy AHP

Author & year	**Title of Article**	**Variable, Parameter**		**Methodology**	**Finding (Best Alternatives)**
		Criteria	**Alternatives**		
Pan, 2008	Fuzzy AHP approach for selecting the suitable bridge Construction method [14]	1.Quality 2.Cost 3 .Safety 4.Duration 5.Shape	3 alternatives method 1.Full-span Precast & Launching Method 2.Advance Shoring Method 3.Incremental Launching Method	Fuzzy AHP	Advancing Shoring Method is the most appropriate alternative
Chatterjee&. Mukherjee, 2010	Study of Fuzzy-AHP Model To Search The Criterion In The Evaluation Of The Best Technical Institutions: A Case Study [15]	1.Campus Infrastructure. 2,Faculty. 3. Student 4. Academic Ambience 6.Teaching Learning Process 7.Supplementary Process	3 alternatives of college. 1.BCREC 2.BCET 3.DIATM	Fuzzy AHP	Find the Best Technical Institutions. BCREC is the select as best Technical Institution
Alias, Maizura , Noor,Selamat , Saman & Abdullah, 2011	Contractor Selection using Fuzzy Comparison Judgement[16]	1.financial:C1 2.performance:C2 3.Staff:C3 4.Equipment:C4	4 alternatives of contractor 1.A1 2.A2 3.A3 4.A4	Fuzzy AHP (FAHP)	Select the best contractor A1>A3>A2>A4 Contractor A1 is the best preferred choices by decision maker

Maizura Noor, Amalin a, Sabri, Hitam, Ali &smail, 2012	Fuzzy Analytic Hierarchy Process (FAHP) Approach For Evaluating Tourism Islands in Terengganu, Malaysia [17]	1.Attraction:D1 2.Environment:D2 3. Accommodation:D3 4.Transportation:D4 5.Restaurant:D5 6.Other Facilities:D6 7.Activity:D7 8.Entertainment:D8 9.Residents Attitudes:D9 10.Souvenir:D10	3 Domain experts	Fuzzy AHP (FAHP)	Find the best criteria of social attributes performance for tourism island. Attraction is the most important criteria for selection island Evaluation.

Sub criteria [16]: Asset: C1-2, Liability: C1-2, Current: C2-1, Previous: C2-2, Experience: C31, Qualification: C3-2.

Sub-criteria [17]: Unspoiled Nature, Unspoiled Forest, Colourful Fish, Beautiful Scenery, Traditional Fishermen Village, Marvellous Coral Reef, Nice Beaches, Waterfall. The first three important criteria are an attraction, environment, and accommodation. The first three important sub-dimensions are unspoiled nature, beautiful scenery, and marvellous coral reef.

Some implementation areas of FTOPSIS method are described in Table3. These fields consist Manufacturing System [18], Threat Synthetic Evaluation in Multi-Target Tracing System [19], Supplier Selection [13][20], Location planning [3][21][22], Stock Marketing[23] etc.

Table2.3. Application of Fuzzy TOPSIS

Author & year	**Title of Article**	**Variable, Parameter**		**Methodology**	**Finding (Best Alternatives)**
		Criteria	**Alternatives**		
Karsak, 2000	Fuzzy MCDM procedure for evaluating Flexible Manufacturing System(FMS) alternatives[18]	1.Capital and operating Cost 2.Required floor space 3.Product flexibility 4.volume flexibility 5.Quality Improvement 6.Work In Progress(WIP)	8FSM alternatives. 1.FMS_1 2.FMS_2 3.FMS_3 4.FMS_4 5.FMS_5 6.FMS_6 7.FMS_7 8.FMS_8	Fuzzy TOPSIS (FTOPSIS)	FMS_3>FMS_8>FMS_2 FMS_4>FMS_7> FMS_5 > FMS_1 > FMS_6 FMS_3 is the best FSM alternatives
Wang, Huan, Qin, Yan &Bai, 2007	Research on FTOPSIS Model of Threat Synthetic Evaluation in Multi-target Tracing System[19]	1. Change ratio of target velocity: D_1 2. Change ration of target radial velocity: D_2 3. Change ration of target navigational angel: D_3 4.Target orientation mobile velocity:D_4 5. Absolute value of target velocity between estimation	5 alternatives of target 1.Target1 2.Target2 3.Target3 4.Target4 5.Target5	Fuzzy TOPSIS (FTOPSIS)	To find which target Underwater Unmanned vehicle (UUV) should attack first. Target1> Target3> Target5> Target4> Target2 UVV should attack Target1 first

		and anticipation: D_5 6. Absolute value of navigational the angle between target and UUV: D_6 7. Absolute value of depth between target and UUV: D_7 8. Absolute value of distance between target and UUV: D_8 9. Probability to be naval vessels: D_9			
Sevkli, Zaim, Turkyılmaz&Satır, 2010	An Application of Fuzzy TOPSIS Method for Supplier Selection[13]	1. Delivery performance 2. Quality performance 3.Price/Cost 4.Financial strength 5.Management and organizational strength	3 Supplier alternatives 1.A 2.B 3.C	Fuzzy TOPSIS	Select the best provide forging parts for Propeller shaft for the light and heavy commercial vehicles C>A>B C is selected as best supplier
Awasthi, Chauhan &Goyal, 2010	A multi-criteria Decision-making approach for location planning for urban distribution centres under uncertainty[3]	1.Accessibility(C1) 2.Security (C2) 3.Connectivity to multimodal transport (C3) 4.Costs (C4) 5.Environmental impact (C5) 6.Proximity to customers (C6) 7.Proximity to suppliers (C7) 8.Resource availability (C8) 9.Conformance to Sustainable freight regulations (C9) 10.Possibility of expansion (C10) 11.Quality of service (C11)	3 Location alternatives 1.A1 is situated outside the city close to a highway while locations. 2.A2 is situated inside the city on the outskirts inside the city close to highways and to the customer locations 3. A3 is situated in the city centre far from highways	Fuzzy TOPSIS (FTOPSIS)	Selection of potential locations for urban distribution centres A1>A3>A2 A1 is select as the the best location for urban distribution centres
Madi& Tap,2011	Fuzzy TOPSIS Method in the Selection of Investment Boards by Incorporating Operational Risks[23]	1. Market Valuation(RM billion):C1 2) Stock Trading Volume (million units), :C2 3) Stock Trading Value(RM million):C3	3 alternatives of Investment Boards on Bursa Malaysia 1.The Main Board: A1 2. The Second Board:A2 3.The MESDAQ Market:A3	Fuzzy TOPSIS (FTOPSIS)	Select a preferable investment boards by incorporating operational risks . Main Board is the best suitable choice MESDAQ is the second choice and Second Board is the last choice.

Boran, 2011	An integrated Intuitionist fuzzy Multi-criteria decision making method for facility location selection[21]	1.Expansion possibility: C1 2. Availability of acquirement Material:C2 3.Community considerations:C3, 4.Distance to market:C4 5. labour cost:C5	4 Alternatives of the candidate. 1.A_1 2.A_2 3.A_3 4.A_4	Fuzzy preference relation, Fuzzy TOPSIS(FTOPSIS)	select The best location for building a new plant A_2 has been selected as the bestlocation .
Ashrafzadeh, 2012	Application of fuzzy TOPSIS method for the selection of Warehouse Location: A Case Study[22]	1.Labor costs 2.Transportation costs 3.Handling costs 4.Land cost 5.Skilled labour 6Availability of labour force.7.Land availability 8.Climate 9.Existence of modes of transportation 10.Telecommunication systems.11.Quality and reliability of modes of transportation 12. Quality and reliability of utilities 13. Proximity to Customers 14. Proximity to Suppliers or producers 15. Lead times and Responsiveness	5 Alternatives Locations 1. Isfahan:A_1 2. Arak:A_2 3. Rasht:A_3 4. Urmia:A_4 5. Tabriz:A_5	Fuzzy TOPSIS	Selecting the best location for new warehouse $A_1 > A_2$ $A_{5>}$ $A_4 > A_3$ Isfahan(A_1) select as best location for new warehouse
Yayla , Yildiz& Özbek, 2012	Fuzzy TOPSIS Method in Supplier Selection and Application in the Garment Industry[20]	1.Quality 2.Delivery Time 3.Cost 4.Flexibility 5.Geographic Location	3 Alternatives of supplier 1.Supplier1:A1 1.Supplier2:A2 1.Supplier3:A3	Fuzzy TOPSIS (FTOPSIS)	Select the best supplier A1> A3 > A2 Supplier 1(A1) as best supplier

Various types of application field of Fuzzy SAW method are described in table4. Some applications of this method are Personnel Selection problem[24], Optimal Robots and Manipulators Selection[25], Project Manager Selection[26] etc.

Table2.4. Application of SAW &Fuzzy SAW

Author & year	**Title of Article**	**Variable, Parameter**		**Methodology**	**Finding (Best Alternatives)**
		Criteria	**Alternatives**		
Afshari, Mojahed& Yusuff, 2010	Simple Additive Weighting approach to Personnel Selection problem[24]	1. Ability to work in different business units:C1 2. Past experience: C2 3. Team player:C3 4. Fluency in a Foreign language:C4 5. Strategic Thinking:C5 6. Oral communication skills:C6 7. Computer Skills:C7	5 Personal alternatives 1.P1 2. P2 3.P3 4.P4 5.P5	Simple Additive Weighting (SAW)	Select the best personnel who have passed examination in a Telecom company P3>P2>P5>P1>P4 P3 is select as best personnel
Bai& Wang,2010	Applying Fuzzy Multi-Criteria Decision Making for Optimal Robots and Manipulators Selection[25]	1. Axes:C1 2. Payload (kg):C2 3.Repeatability (mm):C3 4.Accuracy (mm):C4 5:System cost (US$):C5 6:Weight (kg):C6 7: Max Motion Speed (rad/s):C7 8.Mounting method (average, good, super):C8 9.Power dissipation (kW):C9 10.H-Reach (mm):C10 11 V-Reach(mm):C11 12.Installation space (m3):C12	20 .Alternatives of Robot A,B,C,D,E,F,G ,H,I,J,K,L,M,N ,O,P,QR,S,T	Fuzzy simple additive weighting (FSAW)	Select the optimal robot system from a large group of robot candidates. The top 10 optimal robot is: D > C > P > J > B > E > O > N > I > Q.
Afshari, Yusuff& Derayatifar ,2012	Project Manager Selection by Using Fuzzy Simple Additive Weighting Method[26]	1.Basic Requirements 2.Project Management Skills 3.Management Skills 4.Interpersonal Skills	3 Project manager alternatives .(candidate) 1.P1 2.P2 3.P3	Fuzzy Simple Additive Weighting(FSAW)	Selecting project manager in MAPNA Company P2>P3>P1 Candidate P2 Select as best project manager

Sub criteria[26]: Past experience, Education, Communication skills, Computer skills, Time Management, Cost Management, Resource Management, Quality Management, Planning, Organizing, Controlling, Problem solving, Decision making, Team development.

2.7Multi-Criteria Group Decision-Making (MCGDM)

In the multi-criteria environment, sometimes it is quite difficult for single decision-maker to give his/her appraisal for a different domain such as banking, stock market etc. One decision maker can't give sufficient information due to insufficient knowledge or experience. This problem can be solved by the group decision-making (GDM), where a certain group of decision-makers is present and they can give their judgements on some problem. Sometimes problem contains uncertainties and vagueness; therefore the judgements of decision makers go in the form of the linguistic term rather than exact numerical values [27]. In the multi-criteriaenvironment, the GDM is called Multi-Criteria Group Decision-making (MCGDM).

MADM is one type of MCDM problem. It is dealing with the selection problem, where the numbers of alternatives are chosen supported on a set of attributes. It is a discrete method and dealing with the finite number of alternatives. Table5 describe some application area of MCGDM.

Table 2.5: Application of Multi-Criteria Group Decision-making (MCGDM)

Author & year	**Title of Article**	**Variable, Parameter**		**Methodology**	**Finding (Best Alternatives)**
		Criteria	**Alternatives**		
Saghafian &Hejazi, 2005	Multi-criteria Group Decision Making Using A Modified Fuzzy TOPSIS Procedure [28]	1.Publications and researches (C1) 2. Teaching skills (C2) 3.Practical experiences in industries and corporations (C3) 4.Past experiences in teaching (C4) (5) Teaching discipline (C5)	Name of three eligible candidates 1.A1 2.A2 3.A3	Multi-Criteria Group Decision Making(MCGDM), Fuzzy TOPSIS (FTOPSIS)	Finding the best candidate for teaching in a University A2>A3>A1 A2 is the best candidate
Wang ,Chen & Chen ,2007	Group Fuzzy Multi-criteria Decision Making in Supplier Evaluation[29]	1. profitability of supplier (C1) 2. Relationship closeness (C2) 3. Technological capability (C3) 4. Conformance quality (C4) 5. Conflict resolution (C5)	5 suitable Material Supplier 1.A1 2.A2 3.A3 4.A4 5.A5	Group Fuzzy Multi-criteria Decision Making, Fuzzy TOPSIS (FTOPSIS)	Select a suitable material supplier for purchasing martial ofanew product. A2>A3>A1>A4>A5 A2 is the best alternatives

Wang & SKao, 2009	A fuzzy multi-criteria group decision-making model for the financial performance evaluation of airlines[30]	1.Debt to total assets ratio 2.Working capital to total assets ratio 3.Quick ratio 4.Cash flow ratio 5.Working capital to current assets ratio.6.Accounts payable turnover 7. accounts receivable Turnover 8. Fixed assets turnover 9. Net income(loss) turnover 10.Gross profit ratio. 11.Operation profit ratio 12.Net income ratio	3 companies 1.A_1 2.A_2 3.A_3	Fuzzy multi-criteria group decision-making (FMCGDM), fuzzy TOPSIS (FTOPSIS)	$A_2 > A_1 > A_3$ A_2 has best beneficial performance.
Jiang & Liu,2013	A Multi-Criteria Group Decision-Making Model for Performance Evaluation of Commercial Banks [27]	1.Financial measurements:y_1 2.Customers:y_2 3.Internal business process:y_3 4. Learning and growth:y_4	Four commercial banks 1.x_1 1.x_2 1.x_3 1.x_4	Multi-Criteria Group Decision Making (MCGDM), Balanced scorecard (BSC),linguistic 2-tuples	Select the best commercial bank $x_4 > x_{1>}$ $x >_2 x_3$ $x_4$is selected asbest commercial bank.
Wimatsari, Putra, Buana,2013	Multi-Attribute Decision Making Scholarship Selection Using A Modified Fuzzy TOPSIS[28]	1. GPA (Grade Point Average):C1 2. Quotient of income parents by the number of dependents:C2 3. The Usage of Electrical Power:C4 4. Student Activities:C5	8 Students where Achievement Scholarship is 5 students and Underprivileged the scholarship is 3 students 1.001 2.002 3.003 4.004 5.005 6.006 7.007 8.008	Fuzzy Multi Attribute Decision Making (FMADM), Fuzzy TOPSIS (FTOPSIS)	Student selection for achievement scholarship and Underprivileged scholarship. The five candidates who have five highest the score was selected as the recipient of a scholarship achievement and rank is 006>005> 001>008> 003 3 Candidates who achieve Underprivileged scholarship and rank is 004>002>007

Two or more method can be combined in MCDM domain for evaluating the best result. So hybridization of methods is possible for solving a decision-making problem. Some application area of hybridization method is discussed in table6. Sustainable city logistics planning [10] problem is solved by the combined method of AHP and fuzzy TOPSIS is a beautiful example of hybridization between MCDM methods.

Table2.6: Application of Combinational and others FMCDM methods

Author & year	**Title of Article**	**Variable, Parameter**		**Methodology**	**Finding (Best Alternatives)**
		Criteria	**Alternatives**		
Chang & .Tseng, 2008	Fuzzy TOPSIS Decision Method for Configuration Management[32]	1.Cost:x_1 2.Speed:X_2 3.Strength:x_3 4.Lubrication system:x_4 5.Coolant pump system:x_5	16 configuration alternatives A_1 to A_{16}	Fuzzy TOPSIS (FTOPSIS) , Fuzzy quality function deployment (QFD)	Select the best configuration alternative of CNC lathe machine. A_4 is chosen as best alternative
Zhuofu, Wei-min & Jun-Su; Bin,2008	Improved multi-attribute fuzzy comprehensive evaluation in project delivery decision-making[33]	1.Project Characteristics 2.Owners' Needs & Preferences 3.Project Circumstances.	Comprehensive evaluation value 1.Traditional method (DBB) 2. Design-build method (DB) 3. construction management at risk method (CM-at-Risk)	Entropy method, Fuzzy comprehensio n evaluation	Choose the proper project delivery system for a large-scale water supply project CM at-Risk>DB> DBB CM-at-Risk is chosen as best project delivery method.
Apak&Va yvay, 2009	Evaluating an intelligent business a system with a fuzzy multi-criteria approach[34]	1. M1 decision Management System 2. M2 Intelligent text mining 3.M3 risk Management	3 Intelligent Business System (IBS) alternatives 1.A1 2. A2 3.A3	Fuzzy AHP, Fuzzy TOPSIS	Evaluating a proper IBS of IT department A3>A1>A2 A2 select as best IBS.
Santos, 2010	Fuzzy Systems for Multicriteria Decision Making[35]	1.Attributes Revenue:C1 2. Percentage of bills late more than 30 days:C2 3.Regularity of payment bills:C3 4.Total weight carried:C4 5.Amount of invoice by customer:C5 6.Amount of Transport invoice:C6	10 customers Alternatives A_n n=1 To 10.	Fuzzy Rule-Based Systems Fuzzy TOPSIS (FTOPSIS),Fu zzy Flexible TOPSIS (FFTOPSIS)	CRM (Customer Relationship Management) systems in a transport company. A2 is select asthe best customer.
Awasthi &Chauhan , 2011	A hybrid approach integrating Affinity Diagram, AHP and	1.Technical 2.Social 3.Economic 4.Environment	4 sustainable city logistics initiative 1.Vehicle sizing restrictions:A1 2.Congestionchargi	AHP & Fuzzy TOPSIS	Select the best sustainable city logistics initiative A4 > A2 > A1 > A3.

	fuzzy TOPSIS for sustainable city logistics planning[10]		ng schemes:A2 3.Urban distribution centre:A3 4.Access Timing Restrictions:A4		A4 (Timing Restrictions) is select as the best sustainable city logistics initiative.
Nagar, 2011	Development of Fuzzy Multi Criteria Decision Making Method for Selection of Optimum Maintenance Alternative[36]	1. Purchasing cost:C1 2. Establishment cost (machine – floor requirements, etc.):C2 3.Operating cost:C3 4.Reliability:C4 5.Operational flexibility:C5 6.Productivity:C6 7. Risks (safety):C7 8. Supplier's Environmental behaviors:C8	5 Maintenance alternatives 1. Predictive maintenance:.A_1 2. Breakdown maintenance:A_2 3. Routine maintenance:A_3 4.Preventive Maintenance:A_4 5.Corrective maintenance:A_5	Multiple-Criteria Decision Making(MCDM),Fuzzy sets	selecting the most appropriate maintenance approach for Air caster. $A_1 > A_2 > A_4 > A_3 > A_5$. $A_1$is select as the best maintenance alternative for Air caster.
Hicdurmaz, 2012	A Fuzzy Multi Criteria Decision Making Approach to Software Life Cycle Model Selection[7]	1.People 2.Process 3.Technical	4 type of Software Life Cycle Model (SLCM) 1. Waterfall Model 2. V Model 3. Spiral Model 4. Evolutionary Prototyping	Fuzzy AHP, Fuzzy TOPSIS	selection of appropriate software life cycle model (SLCM) of software development process. Evolutionary Prototyping > V Model > Spiral > Waterfall. Evolutionary Proto-Typing model select as best software life cycle model of software development process

Evaluation attributes[33]: Project scale(A1), Project complexity(A2), Depth of the design document(A3), Degree of involvement after contract award(A4), Cost control(A5), Schedule control(A6), Risk allocation(A7), Circumstance of local construction market(A8), Law and local regulation(A9).

Sub criteria[34]: Optimization model(C1), Time series analysis(C2), Structured text analysis(C3), Numeric data analysis(C4,), Forecasting model(C5), Clustering(C6), Classification(C7), Profiling(C8), Hyper linking(C9), System(C10), Prediction(C11).

Ranking evolution[35] of each alternative after applying 3 different method 1.Fuzzy Rule Based System: A2> A1> A6> A8 > A7 >A3 A4 >A10 >A5 >A9, 2. Fuzzy TOPSIS A2> A1> A3> A6 > A7 >A8 A5 >A4 >A10 >A9, 3. Fuzzy F-TOPSISA2> A1> A6> A5 > A3 >A10 A7 >A4 >A8 >A9

Sub criteria[10]: Logistical efficiency (C1), Mobility(C2), Accessibility(C3),Service quality(C4), Loading factor(C5), Customer coverage(C6), Freeing of public space(C7), Energy conservation(C8), Trip effectiveness(C9), Revenues(C10), Volume of freight handled(C11), Accidents(C12), Costs:(C13), Congestion (C14), Air pollution(C15), Noise(C16).

Sub criteria[7]: Ease of management, User involvement and Feedback, Cost, Complexity, Critically, Flexibility, Reusability, Doc. ,and software quality, Testing and integration, Focus on design and architecture, Formal reviews, Requirement stability.

2.8Summary of Findings

MCDM are gaining importance as potential tools for analysing complex real problems due to their inherent ability to judge different alternatives (Choice, strategy, policy, scenario can also be used synonymously) on various criteria for a possible selection of the best/suitable alternative. These alternatives may be further explored in-depth for their final implementation.

MCDM has certainly become one of the supreme techniques in decision-making field. Methods of MCDM are designed perfectly to choose the best option for a complex decision-making problem based on criteria evaluation and ranking the criteria. Though it is very much difficult, to sum up, all the different techniques in MCDM world but our main motto behind is to give an initial outline to a novice researcher in this area and to show the various application domains of MCDM methods such as FAHP, FTOPSIS, and FSAW are discussed.

We observed wide variations of application domain on which Fuzzy MCDM techniques were applied such as Manufacturing system, Supply chain management, Location planning, Stock marketing, Construction, Evaluation Of The Best Technical Institutions, Contractor Selection, Evaluating Tourism Islands, Robotics, E-commerce, Software Industries, Project

Manager Selection, Quality Management. Domain-wise Applications of MCDM methods are represented in Table.2.7.

Table2.7: Domain-wise Applications of MCDM methods

Sl no	Commercial	Industrial	Environmental estimation	Performance rating
1	To find preferable investment boards by incorporating operational risks .	To select the optimal method for bridge construction.	Find the best criteria of social attributes performance for tourism island.	To search criteria in the evolution of the best Technical institution.
2	To find the optimal robot system from a large group of robot candidates.	Contractor selection, in selecting the best contractor who is able to provide the best service.	To find the best potential locations for urban distribution centres.	To find which target Underwater Unmanned vehicle (UUV) should attack first.
3	To evaluate the financial performance of different airlines companies.	To find the best Flexible Manufacturing System (FMS) in industries.	To find the best location for building a new plant	To find the best personnel who is suitable in a Telecom company.
4	To evaluate banking performance of commercial banks.	To find the best supplier to provide forging parts for Propeller shaft for the light and heavy commercial vehicles	To find the best suitable location for building a new warehouse.	To Find the best candidate for teaching in an University.
5	To select the best configuration alternative of a lathe machine.	Find the best supplier in the garment industry.	To find the best Sustainable city logistics initiative.	Student selection for achievement scholarship and underprivileged scholarship.
6	To evaluating an Intelligent Business System of IT department.	To select the best project manager in a certain company.		To select the best customer in a transport company.
7		To Select the suitable material supplier for purchasing martial of a new product.		
8		To find proper project delivery system for a large-scale water supply project.		
9		To find the most appropriate maintenance approach for air caster.		
10		To select the appropriate Software Life Cycle Model (SLCM) of software development process.		

Table2.7 describes the application fields of the FMCDM techniques and also gives us the clear essence about generalized the domains in which these techniques can be applied. The generalized domains divided into 4 parts. Under these domains, respective domain works are also enlisted. From the survey it has notified that 6, 10, 5 and 6 applications under commercial, industrial, environmental estimation and performance rating respectively. Though it will be

wrong to say that FMCDM methods are mostly used in industrial sectors but from this short survey it found a number of applications in an industrial area than any other areas. From this above Table 2.7. It is easily concluded that the application areas of these methods are numerous. In most of these decision-making problems, a fuzzy approach to MCDM is applied according to the complexity and the difficulty of the problem and due to its capability of handling uncertain situations and as it proves to be the best determination for the decision makers.

Table 2.8 :FMCDM methods and their respective number of occurrence

Sl. no	MCDM methods	Allowance
1	FAHP	4
2	FTOPSIS	8
3	FSAW	3
4	Combinational	7
5	FMCGDM	5

Table 2.8 gives the most widely used fuzzy MCDM techniques in MCDM problems and they are also ranked according to their usage and applicability in various domains. The allowance number shows their number of occurrences in different problems in this survey.

For better understanding, a graph has been plotted for indicating respective usages of different Fuzzy MCDM techniques as follows

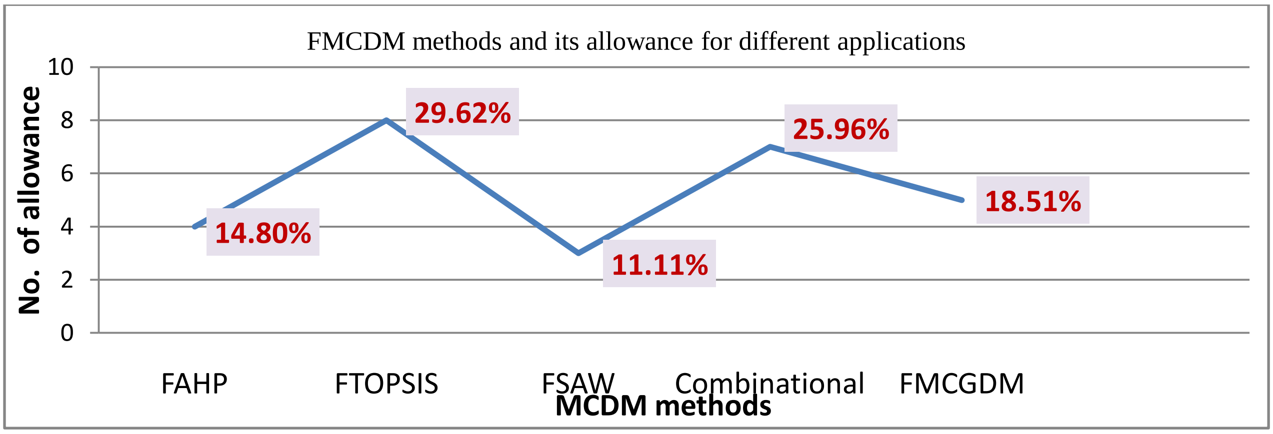

Figure2.4 FMCDM methods and its allowance for different applications

Most widely used Fuzzy MCDM techniques only are taken into consideration. This is basically the graphical representation of the table no 8. The Fuzzy MCDM techniques include Fuzzy AHP(FAHP), Fuzzy TOPSIS (FTOPSIS), Fuzzy SAW (FSAW), Fuzzy Combinational techniques and Fuzzy MCGDM (FMCGDM) techniques.

From the above figure 2.4, it seems that FTOPSIS and Fuzzy combinational techniques come among the most widely used FMCDM techniques in order to be used in some domain though it will be unfair to judge the usefulness of these techniques only on a small scale that based on our survey. In between these techniques there also many techniques which include some alterations in classical techniques and those alterations are new variations of those techniques which are actually altered for gaining better result and according to the problem analysis.

S. Saghafian and S.R Hejazi [28] proposed a modified Fuzzy TOPSIS procedure in which they have implemented a new approach for measuring distance using the fuzzy comparison function instead of simple vertex method. W. Zhuo, etal.[33] have used a new weight evaluation technique "entropy weight method" which modifies the expert's subjective weight and give the comprehensive weight, instead of using the attribute weight setting method.

A. Awasthi and S.S. Chauhan[10] previously used the simple Fuzzy TOPSIS method for location planning but for better evaluation later they have proposed a combinational approach towards city logistic planning.

Many other MCDM methods are also there such as Fuzzy BCC, FSROWA, Fuzzy SBM, COPRAS-G, VIKOR, Fuzzy DEMATEL, Grey theory, Data envelopment analysis (DAE), Aggregated Indices Randomization method (AIRM), Goal Programming(GP) etc. But as the world of MCDM is too vast to be restricted to a survey and only consider some of the methods under MADM which are vastly used methods.