PREFACE

A graph is a mathematical model which is applicable to any system involving binary relation. We can represent a road map of a city by a graph where the vertices are the prominent places of the city and two vertices are joined by an edge if they are along a road. But along a road, there can be other places of interest. To represent such systems which involve a relation that is not binary, some generalisation of graph is required.

There have been attempts to generalise this ‘graph model’ so that it is applicable to systems involving relations not necessarily binary. Hypergraph\[2\] is one such generalisation, in which an edge may contain more than two vertices. Semigraph, introduced by Sampathkumar[12] in 1996, is another generalisation in which an edge contains two or more vertices. While in hypergraphs, the order of vertices occurring in an edge is immaterial, in semigraphs the edges are ordered tuples.

Since the order in which the places of interest occur in a road is important, road maps are better represented by semigraphs. In electric circuits, the gadgets in a series combination have some order among them. Thus, we can represent the situation by semigraphs more effectively than graphs and hypergraphs.

The concept of a semigraph has been introduced recently and the subject is still in its nascent stage. To begin with, the problems in graph theory are reposed for semigraphs. Quite often the generalisation from graphs to semigraphs is not obvious. In this thesis, we investigate the following
problems for semigraphs-

1. Characterise the degree sequences of semigraphs.
2. Investigate the properties of complete semigraphs.
3. Define and characterise line semigraph of a semigraph.
4. Define vertex connectivity and edge connectivity of a semigraph and study their properties.

In Chapter 1, definitions of basic concepts in graphs, hypergraphs and semigraphs, which are needed in subsequent chapters, are listed. These definitions are illustrated with examples and figures wherever necessary.

In Chapter 2, a complete characterisation of ‘edge-degree’ sequences is given. We apply this result to linear hypergraphs, and get a characterisation of degree sequences of linear hypergraphs. A step by step procedure for constructing a semigraph for a given edge-degree sequence is given and illustrated by an example. We define a new type of degree, “me-degree”, of a vertex in a semigraph and give some necessary conditions for a sequence to be me-semigraphical. As in graphs, sufficient conditions are hard to come. Some results regarding the me-degree sequences of me-regular and uniform semigraphs are also given.

In Chapter 3, we classify the complete semigraphs in terms of the number of s-edges and give their edge-degree sequences and me-degree sequences. The number of nonisomorphic complete semigraphs for each of these classes is given. We identify the complete, e-regular, uniform semigraphs as BIBD’s and obtain results using this idea.
In Chapter 4, we define the line semigraph of a semigraph. According to our definition, the line semigraph of a semigraph is again a semigraph. As in the case of graphs, necessary and sufficient conditions for a semigraph to be line semigraph are given. Motivated by the work of Beineke[9, pg 75] on line graphs, we give a characterisation of line semigraph in terms of forbidden subsemigraphs. We also characterise isomorphic line semigraphs and discuss line semigraphs of some special types of semigraphs.

In Chapter 5, we define vertex connectivity and edge connectivity of a semigraph and study some of their properties. We also define \(n\)-connected and \(n\)-edge connected semigraphs and generalise the results about \(n\)-connected and \(n\)-edge connected graphs to semigraphs.

We earnestly hope that our investigation of semigraphs will initiate further useful studies in the topic, especially on their applications.

All the definitions, results, theorems and figures are numbered serially, chapterwise, sectionwise. For example, Definition 3.2.4 means the 4\(^{th}\) definition in Section 2 of Chapter 3. References are listed in an alphabetical order at the end of the thesis.