Chapter – 3

MODELING AND SIMULATION OF CLAW-POLE ALTERNATOR USING FINITE ELEMENT METHOD

3.1 Introduction

Boundary value problems occur in practically every engineering application like structural analysis, heat transfer, fluid flow, electromagnetic fields etc. As the geometry of devices tends to be more complicated and as the interaction between various fields, for example, electromagnetic and thermal fields in certain applications were established, an increase in accuracy of the performance prediction became necessary. The methods used from Maxwell onwards for the solution of the boundary value problem can be divided into four categories: analog, graphical, analytical and numerical.

Analogical procedures consist in obtaining the unknown field by experimental measurement on an analogue of the field region, i.e., on a field region governed by the same equations and with the same boundary and interface conditions. This method has a serious drawback, that it is practically impossible to model in-homogeneities, nonlinearities and so on using media different from that of the practical case.
Moreover, in its three dimensional version, this method is rather expensive and cumbersome.

Graphical procedures are restricted to Laplace’s equation for two dimensional geometries and also the accuracy is limited even when they are carefully applied.

Analytical methods have advanced a good deal but the major deficiency of these methods is the lack of generality. Besides, algorithms for inhomogeneous and non linear problems are practically non-existent except for some extremely simple and special problems. Another notable deficiency lies in the effort required to obtain the field solutions, such as developing special algorithms and discovering artifices.

The numerical methods have come to the fore with the advent of digital computers. The principal numerical methods that are in use are finite difference schemes, image methods, integral equation technique and variational formulation. Finite difference schemes in their traditional version monopolized the area of numerical methods in electrical engineering practically up to 1970. Alternative methods have also been proposed in the area of image methods and integral equation techniques which have intensive but limited application in special areas.
In the areas like civil engineering the drawbacks of traditional finite difference schemes have been recognised at an early stage and alternative methods such as variational procedures have been developed. This process has led to the finite element method.

Variational methods formulate the equations of boundary value problems in terms of variational expressions called energy functional which in electrical application often coincides with the energy stored in the field. The Euler equation of this functional will generally coincide with the original partial differential equation. It can be considered to be the growing technique in the area of electrical and electronics engineering.

In this Chapter, the author reports the use of the finite element method for modeling and simulation of the claw pole alternator.

3.2 Finite Element Method

The finite element method is concerned with the solution of mathematical or physical problem defined in a continuous domain either by a local differential equation or by equivalent global statements. Low frequency electromagnetic field
phenomena can be described by Poisson's equation (Laplace's equation for source free region) or the diffusion equation [55],[56]. The space of interest is subdivided into sub regions called elements which completely cover the space but do not overlap. The unknown quantities over this small region, such as for instance, a scalar or vector potential can be represented by suitable interpolation functions that contain the values of potential at the respective nodes of each element as unknowns. Polynomials are usually used and the order of the polynomial determines the order of the element. The method uses a minimization process to find values at the nodes, which minimizes a weighted error or energy functional. This process generates an algebraic system of equations and the potential values at the nodes can be determined by direct or iterative methods.

3.3 Theoretical Overview of the Analysis

The field theoretic formulation of the problem starts with Maxwell's equations

\[ \nabla \times E = -\frac{\partial B}{\partial t} \]  

(3.1)

\[ \nabla \times B = \mu_0 \left( J + \frac{\partial D}{\partial t} \right) \]  

(3.2)
\( \nabla \cdot B = 0 \) \hspace{1cm} (3.3)

It is convenient to pose the problem in terms of the magnetic vector potential \( \vec{A} \) which is defined by the relation:

\[ \nabla \times \vec{A} = \vec{B} \quad \text{and} \quad \nabla \cdot \vec{A} = 0 \] \hspace{1cm} (3.3a)

In the present case, \( \frac{\partial \Phi}{\partial t} = 0 \).

So (3.2) becomes \( \nabla \times \vec{B} = \mu_0 \vec{J} \).

\[ \nabla \times \nabla \times \vec{A} = \mu_0 \vec{J} \] \hspace{1cm} (3.3b)

This reduces to the vector Poisson Equation,

\[ \nabla^2 \vec{A} = -\mu_0 \vec{J} \] \hspace{1cm} (3.3c)

This gives the vector differential equation in terms of magnetic vector potential which governs the problem in the model. Now this differential equation is solved using finite element method. To get the solution for the equation, we minimize a functional corresponding to the above equation and boundary conditions.

The calculation of magnetostatic field in three dimensions is carried out in this thesis using the software EMAG. This can be done in two ways in the software [57]. First is the vector potential approach and second the reduced scalar potential approach.
### 3.3.1 The Vector Potential Approach

The energy related functional relating to this method is given by

\[
F(A) = \frac{1}{2} \int_{\Omega} \nabla \times (\nabla \times A) d\Omega - \int_{\partial \Omega} A \cdot j d\Omega
\]  
(3.4)

where \( \Omega \) is the region containing the source.

A current density \( j \) is regarded as given and the boundary condition is of the Dirichlet type where \( A \) is specified on some boundary surfaces.

### 3.3.2 The Reduced Scalar Potential Approach

In many cases, the magnetic field intensity \( H \) can be separated into two parts.

\[
H = H_s + H_m
\]  
(3.5)

where \( H_s \) is directly the result of the current sources and \( H_m \) is the magnetization enclosed in the material. \( H_s \) is entirely independent of the material and can be obtained from Biot-Savart law

\[
H_s(r) = \frac{1}{4\pi} \int_{\Omega} \frac{j \times (r - r')}{|r - r'|^3} d\Omega'
\]  
(3.6)
where $\vec{r}$ and $\vec{r}'$ are field and source point vector respectively and $\Omega_j$ is the region of space which contains all the current sources.

The Biot-Savart equation is in fact a solution of the equation

$$\nabla \times \vec{H}_s = \vec{J}$$

(3.7)

and since the Maxwell equations $\nabla \times \vec{H} = \vec{J}$ must hold it follows that $\vec{H}_m$ is irrotational.

i.e., $\nabla \times \vec{H}_m = 0$ (3.8)

Hence $\vec{H}_m$ can be written as $\vec{H}_m = -\nabla \phi_m$ where $\phi_m$ is the reduced magnetic scalar potential (A). A reduced scalar differential equation for the potential is obtained by substituting (3.4), (3.5) and the relation $B=\mu H$ into (3.3) and is given by

$$\nabla (\mu \nabla \phi_m) = \nabla \cdot (\mu \vec{H}_s)$$

(3.9)

where $\mu$ is the permeability of the media.

The corresponding energy functional is given by

$$F(\phi_m) = \frac{1}{2} \int_\Omega \mu (\nabla \phi_m)^2 \, d\Omega + \int_\Omega \phi_m (\nabla \mu H_s) \, d\Omega$$

(3.10)

if the space current density is given and if boundary condition is of Dirichlet type

The author has reported the use of the vector potential approach for solution.
3.3.3 Magnetostatic Analysis in Non linear Magnetic Material.

The application of material non linearity arises when the material used has a reluctivity that is dependent on the magnetic field, for example, while taking into account the saturation effects of magnetic materials.

\[
\xi_n = \int_{0}^{B} \overline{H}(B) \, dB
\]  

(3.11)

is recognized as the stored energy in the material. Hence the energy functional required to solve the non linear magnetostatic problem is given by

\[
F(A) = \int_{\Omega} \left[ \int_{0}^{B} \overline{H}(B) \, dB \right] d\Omega - \int_{\Omega} A \, J \, d\Omega
\]  

(3.12)

The non linear magnetostatic analysis is implemented using an iterative procedure based on the Newton-Raphson method. The iteration procedure is

\[
[S]_p \{\Delta A\}_p = \{J\}_p - \{R\}_p
\]  

(3.13)

\[
\{A\}_p+1 = \{A\}_p + \{\Delta A\}_p
\]  

(3.14)

where

- \([S]\) the Jacobian matrix
- \([J]\) the load vector
- \([R]\) residual load vector
\{A\} nodal potential

\{\Delta A\} incremental nodal potential

p iteration counter

The above iteration scheme is performed until convergence is achieved or maximum iterations are reached.

For every iteration, the Jacobian matrix is updated as shown in Fig. 3.1 (Full Newton-Raphson Scheme).

![Fig. 3.1 Full Newton-Raphson Scheme](image)

For non-linear magnetostatic problems the stored energy is

\[
W_h = \int_\Omega \left[ \int_0^B \bar{H}.dB \right] d\Omega \tag{3.15}
\]

And stored co-energy is given by,

\[
W_c = \int_\Omega \left[ \int_0^B \bar{B}.d\bar{H} \right] d\Omega \tag{3.16}
\]
Fig. 3.2 shows energies for a non-linear problem.

![Non linear B-H Curve and its energies](image)

Fig. 3.2 Non linear B-H Curve and its energies

### 3.4 Modeling

As opposed to the other types of electrical machines where analysis can be carried out in a plane perpendicular to the machine shaft, the topology of claw pole alternator emphasizes the need for allowing all three field components; radial, axial and tangential. So, though a variety of modeling methods have been attempted for analyzing the claw-pole configuration, the complex geometry of the machine in Fig. 2.2 compels a 3D finite element approach. In the present work, the author has used DISPLAY III/IV for 3D modeling and the solver EMAG for calculating the magnetic vector potential allowing for non linearity of the material.
3.4.1 General Features of the Software

EMAG is a finite element program developed and maintained by EMRC [57]. It can be used to perform 2D and 3D electric and magnetic field analysis in electromagnetic devices. EMAG is an integral part of the NISA (Numerically Integrated elements for System Analysis) family of programs. The two major analysis types included in the software are electric field analysis (EFIELD) and magnetic field analysis (MFIELD). These analysis types are subdivided into sub analysis types based on the excitation in the model.

3.4.2 Preprocessing

EMAG interfaces with the pre-processing module, the DISPLAY III program; a 3D graphics program with modeling capabilities for finite element model generation and problem definition. Highlights of the capabilities are:

- CAD interface, directly from geometry database or through IGES format.
- Both command and menu driven modes, with on-line help.
- 3D geometric modeling, including points, lines, arcs, curves, surfaces and solids as well as surface intersections.
Geometric transformations including translation, rotation, scaling, mirror imaging and dragging a curve along an arbitrary 3D point.

3D finite element meshes generation.

Mesh grading with uniform or non-uniform spacing.

Merging separate models into a larger one

Definition of element attributes including material and geometric properties

Specification of boundary conditions

Model editing capabilities

Plotting options including boundary line, hidden line removal and shrink element plots for selected elements or regions.

Model checking including calculation of element areas, volumes and distortion index.

EMAG data deck generation.

3.4.3 Input Data Structure

Modular input data structure.

The data deck consists of three data blocks. The executive commands specifying the overall control parameters in simple alphanumerical format, the model data block
describing the model characteristics and finally the analysis data block specifying the loads, boundary conditions and output options.

- Data groups (modules) may appear in any order within each data block, with very few exceptions.
- Annotated echo of the input data
- Data checking with diagnostic messages.

### 3.4.4 Element Library

A library of elements consisting of 2D, Axisymmetric and 3D elements is available in EMAG. Each element in the library is identified by two variables, NKTP and NORDR. The variable NKTP specifies the element type whereas NORDR specifies element shape and number of nodes. The elements with NKTP 104, models the three dimensional state of field distribution and are suitable for modeling solid structures. The element has the field potential (Φ or $\overline{A}$) as the only degree of freedom at each node.

The element can be shaped as 8 or 20 node hexahedron, 6 or 15 node wedge or tetrahedrons as in Fig. 3.3. An element reference guide is given in Table 3.1.
Fig. 3.3 Elements available for solid structures

Table 3.1 Element reference guide (NKTP 104)

<table>
<thead>
<tr>
<th>Element Type</th>
<th>NKTP=104, 3-D Solid Element for EMAG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysis Types</td>
<td>Electric and Magnetic field</td>
</tr>
<tr>
<td>NORDR (Element shape, nodes)</td>
<td>- Hexahedron (brick): 8 or 20 nodes (NORDR=1,2)</td>
</tr>
<tr>
<td></td>
<td>- Wedge: 6 or 15 nodes (NORDR=10,11)</td>
</tr>
<tr>
<td>Real constants</td>
<td>None</td>
</tr>
<tr>
<td>Material Properties</td>
<td>ESTAT</td>
</tr>
<tr>
<td>---------------------</td>
<td>--------</td>
</tr>
<tr>
<td>Isotropic</td>
<td>EXX</td>
</tr>
<tr>
<td>Orthotropic</td>
<td>EXX</td>
</tr>
<tr>
<td></td>
<td>EYY</td>
</tr>
<tr>
<td></td>
<td>EZZ</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Element Output</th>
<th>- Nodal field potential in the model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>- Element field distribution in the model</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Transient capabilities</th>
<th>- variable, user defined, or equal time step</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>increments with various time integration schemes</td>
</tr>
<tr>
<td></td>
<td>- Time dependent flux and current densities</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Non linear capabilities</th>
<th>- Magnetic field dependent material properties (B-H curve) for magnetic field analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>- Voltage dependent material property for electric field analysis</td>
</tr>
</tbody>
</table>

Referring to the table, we can see that the material properties to be given for different sub analysis are different. The sub analysis types denoted are ESTAT- Electrostatics, SCFL- Steady current flow, MGVP- 3D Magnetic analysis using Magnetic vector potential approach and MGSS- 3D Magnetic analysis using reduced scalar potential approach. The labels given for the material property are,
EXX, EYY, EZZ - dielectric permittivity in x, y and z directions.

SIXX, SIYY, SIZZ - electrical conductivity in x, y and z directions

MUXX, MUY, MUZZ - permeability in x, y and z directions

NUXX, NUYY, NUZZ - reluctivity in x, y and z directions

3.4.5 Boundary conditions

Available boundary conditions are

- Dirichlet boundary condition - where $\bar{A}$ is specified on some boundary surface.

- Neumann boundary condition – where $\frac{\partial \bar{A}}{\partial N} = 0$ on some boundary surface.

3.4.6 Analysis and Output Features

The following outputs are obtained during the magnetostatic analysis.

- Magnetic vector potential or scalar potential
- Magnetic flux density
- Stored magnetic energy and co-energy
- Magnetic force
- Inductance
Post processing is available for magnetic vector potential, magnetic field intensity and magnetic flux density in DISPLAY III.

### 3.4.7 Solution Techniques

- Built-in element re-sequecing algorithm for wave front minimization. All input specifications and output requests are in terms of the user’s element identification numbers.
- Conventional Newton-Raphson method and incremental procedure for non linear magnetostatic analysis.

### 3.4.8 Post Processing

Graphical representation of the results was obtained by the author interactively using the post-processing module of the 3D colour graphics DISPLAY III program following a successful EMAG run. A brief account of the post-processing features available is given below.

- Various geometric plotting options including hidden line removal, boundary and feature line plots and view manipulation including rotation, scaling and zooming.
- Contour plots for electric scalar potential (voltage), magnetic vector potential, current and flux densities, and
electric and magnetic fields can be obtained depending on the analysis types.

### 3.4.9 Input Data Setup

Fig 3.4 shows a global description of the input data setup used for a typical EMAG analysis run in the present work. The first data block (executive commands) should flag

![Diagram of input data setup]

- **Executive Commands**
- **Model Data Block**
  - Model data describing the model characteristics (starts with *TITLE data group)
- **Analysis Data Block**
  - Analysis Data (starts with *EMAGCNTL data group)
- **Data Terminator**
  - Input Terminator *ENDDATA

Fig 3.4 Input data setup used for EMAG analysis
the analysis and sub analysis types, and should also include
the appropriate or applicable commands for file settings,
element sensitivity, etc.

The second data block (the modal data) should start with
the TITLE data group in order to allow printing of the problem
as a heading on all pages of the output file. The model data
block then describes model characteristics, as for example,
element definition (ELEMENT), nodal coordinates (*NODES),
etc.

The third data block (the analysis data) must start with
the EMAGCNTL data group. The block also describes various
loading and boundary conditions. Depending on analysis and
sub analysis type the loading can be charge density (CHRDEN)
or current density (CURDEN) and boundary conditions are
given by specifying primary variable (voltage (*SPFPOT) or
vector potential (*SPFPOT) or its gradients (electric flux
density (*EFLUX) or magnetic flux density (*MFLUX)). The
block also defines various output and print control options
(*PRNTCNTL).

The last card of the input deck must always be data
terminator, the *ENDATA group.
3.5 No-load Analysis

Due to periodicity of geometry, only 1/6th of the claw pole machine was studied. A 12 pole 36 slot claw pole alternator was analyzed for an excitation current of 1A and 428 field turns for a typical rotor position where the centre of the rotor pole aligns with the centre of the stator tooth. The physical model of the machine was made using the pre-processor DISPLAY III. Meshing was manually done to form the finite element model. The finite element model of the machine for one pole pitch is given in the Fig 3.5 along with co-ordinate axis.

Fig. 3.5 Finite element model of stator and rotor for one pole pitch
Because of the highly complex shape of the machine and small air gap length when compared to the machine dimensions, 59528 elements of NKTP 104 had to be used to model two pole pitches, to keep skew, aspect ratio, twist, warp etc. of elements within allowable limits. The magnetic vector potential $A$ was set to zero at the outer boundaries of this model and source also was applied to form the input file (nis file) for the solver EMAG.

About 13 hours were taken to complete the analysis in a 3.0GHz processor. The flux density at the nodes in the air gap was taken during post processing. Using these values of flux density the generated voltage waveform was then calculated for a rotor speed of 2900 \( \text{rpm} \).

Fig. 3.6 to 3.8 shows the contour plots of radial flux density ($B_x$) in the stator, rotor and air gap of the machine obtained during post processing; along with numerical values for colour code and co-ordinates axis.

Corresponding plots of resultant flux density were also obtained and are given in Fig. 3.9 to 3.11. The generated voltage on no load obtained from the FEM analysis is shown in Fig. 3.12.
Fig. 3.6 Flux density distribution in stator (Bx)

Fig. 3.7 Flux density distribution in rotor (Bx)
Fig. 3.8 Flux density distribution in air gap (Bx)

Fig. 3.9 Resultant Flux density distribution in stator

Fig. 3.10 Resultant Flux density distribution in rotor
Fig. 3.10 Resultant Flux density distribution in rotor

Fig. 3.11 Resultant Flux density distribution in air gap
Fig.3.12 Induced voltage in stator winding

Jerzy and Arkadiusz [36] have conducted a similar study on a 12 pole 140 A, claw pole alternator using Maxwell 3D and reported comparable results as regards shape of the waveform in Fig.3.12 is concerned. The comparison is limited by the fact that the rating of the machines is different. Hence for comparison of the results, the investigator has normalized the waveforms over a cycle and overlapped the two. The results are shown in Fig.3.13 which are in generic agreement.

3.6 Conclusion

After recounting the different methods available for solving boundary value problems and the basic features of FEM, a detailed account is given how the FEA was implemented by the author in the present analysis. The procedure adopted by the
Fig. 3.13 Overlapped voltage waveforms of present study and Study of Jerzy and Arkadiusz

author for modeling and post processing is also discussed and finally the contour plots of the flux density obtained during post processing is given. From the contours it is seen that maximum flux density occurs at the centre of the poles and low flux density is seen at stator slot opening. The wave form of the generated voltage obtained from analysis is also given. The frequency spectrum of the voltage waveform and discussion of the results is given in chapter 8.