Synopsis of the Thesis Entitled

STUDIES ON DOUBLE DIFFUSIVE CONVECTION AND INSTABILITIES

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By

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The study of the onset of thermal instability in a horizontal layer of viscous fluid heated from below has its origin in the experimental observations of Bénard in 1900 and later the development of its mathematical theory by Lord Rayleigh in 1916. In the absence of surface tension, the instability occurs as a result of the buoyancy effect due to the heating. This phenomenon of buoyancy driven instability for Newtonian viscous fluids had been studied extensively as basic stability problem of heat transfer in fluid mechanics during the last century. On the other hand double-diffusive convection is a process when the density variations are caused by two different components, with different molecular diffusivities. Due to the sharp contrast between the diffusivities convection can be manifested from instability. Double-diffusive convections of a fluid layer in different contexts have been the subject of extensive theoretical and experimental interest because of their relevant applications in various physical phenomena which have attracted considerable attention of a number of researchers in many branches of science and engineering. In this thesis we are considering different problems on double-diffusive convective models in nonporous and porous medium saturated with Newtonian and non-Newtonian fluids and our study includes investigation of the criteria for the onset of instability due to double-diffusive convection using linear stability analysis and the normal mode method.

Firstly in chapter II, a details theoretical analysis is made to investigate the onset of double-diffusive convection at the marginal state in presence Soret and Dufour effects in a Newtonian horizontal fluid layer heated and salted from above with two stress-free boundaries. The principal aim of this work is to find the critical value of Rayleigh number at the onset of double-diffusive convection taking into account of Soret and Dufour effects and their roles on the instability. We are interested to study whether the onset of instability manifesting the well-known salt fingers at the threshold of stationary convection is possible or not under the influences of cross-diffusion terms and the possibility of oscillatory instability at the onset.

The analysis reveals that the onset of instability is manifested by the well-known salt fingers at the threshold of stationary convection and this thermo-solutal instability depends strongly on the cross diffusion terms. It is noted that the stationary convection is followed by the oscillatory convection provided the Dufour number is always less than the reciprocal of the Lewis number and with certain typical values of
Soret and Dufour parameters. It was found that in the case of stationary instability, the critical value of the Rayleigh number will decrease when the Soret number is increased, thus the Soret parameter has a destabilizing effect; while it will increase as the Dufour number increases, so the Dufour parameter has a stabilizing effect. The frequency of oscillatory pattern of convection and the critical values of Rayleigh numbers for the oscillatory convection against the wave numbers are obtained and effects for the presence of cross-diffusive terms on them are studied. It is also found that the presence of cross-diffusive terms in this double diffusive system, the oscillatory convection has been delayed at the marginal state.

Double-diffusive stationary and oscillatory instabilities at the marginal state in a Newtonian fluid saturated horizontal porous layer heated and salted from above are investigated theoretically under the frame work of Darcy for porous medium in chapter III. The contributions of Soret and Dufour co-efficients are taken into account in the analysis. We are interested to find the critical value of the Darcy-Rayleigh number, manifesting the onset of instability due to this double-diffusive convection, taking into account the Soret and Dufour effects and their roles on the onset of instabilities. This mechanism can be of interest in the transport of contaminants in ground water and since the ground water flow is relatively slow and so the Darcy model considered is quite appropriate.

It was found that in the case of stationary instability, the critical values of the Darcy-Rayleigh number will decrease when the Soret number is increased, thus the Soret parameter has a destabilizing effect; on the other hand, it will increase as the Dufour number increases, so the Dufour parameter has a stabilizing effect in this double-diffusive system. The critical value of Darcy-Rayleigh number at the onset of the oscillatory convection does not depend on the cross-diffusion co-efficients but depends on the porosity parameter and the porosity parameter has a destabilizing effect on the oscillatory convection. The effects of cross-diffusion parameters on the frequency of the oscillatory convection are also obtained. The analysis also reveals that the increasing Dufour number with higher value of the porosity parameter provokes oscillatory instability at the onset whereas the increasing Soret number instigates stationary instability at the onset. Thus the type of instability at the onset is influenced by the Soret and Dufour effects along with the porosity parameter.
In chapter IV, we study the instability at the onset of thermal convection in a horizontal non-Newtonian fluid layer heated underneath using Oldroyd model in the Navier-Stokes’ constitutive equations. The stress-free boundaries are considered with linear temperature distribution in the vertical direction. The critical condition for the onset of instability due to thermal convection under the principle of exchange of stabilities is investigated using linear stability analysis. Also we examine the effects of viscoelastic properties, the relaxation and retardation times, on the onset of convective instabilities. The onset of thermal convection of different viscoelastic fluid models, as limiting cases of Oldroyd model, has been considered.

In this study, the linear stability analysis reveals that the oscillatory convection is prevailed at the onset of thermal convection for a viscoelastic fluid with any Prandtl number provided the relaxation time is greater than the retardation time. It is found that the critical value of the Rayleigh number at the onset of steady convection is 657.5, similar to that of Newtonian fluid layer heated from below, which is independent of relaxational parameters as well as Prandtl number of the fluid. It was also found that in the case of oscillatory instability the critical value of the Rayleigh number will decrease with the increasing relaxation time, thus the relaxation time has a destabilizing effect. On the other hand it will increase as the retardation time increases, so the retardation time has a stabilizing effect. The study also reveals that, high Prandtl number, for common viscoelastic fluids, has almost no effect on the Rayleigh number while small Prandtl number possesses destabilizing influence. It is interesting to note that for low Prandtl number fluids with certain typical values (nearly equal) of relaxation and retardation times the stationary pattern of convection is observed at the onset. The analysis reveals another interesting result that the oscillatory convection at the onset is not possible for Rivlin-Ericksen viscoelastic fluid model.

A theoretical study is made to investigate the effects of cross-diffusive terms and the relaxation and retardation times of viscoelastic fluid on the onset of thermohaline convection in a saturated horizontal porous fluid layer heated and salted from above using modified Darcy-Oldroyd model (chapter V). We use linear stability analysis to find the critical value of Darcy-Rayleigh number for the stationary convection as well as for the oscillatory convection and analyze the influences of
relaxational parameters (the time dependent viscoelastic properties) and cross-diffusive parameters on the critical value of the Darcy-Rayleigh number. We are also interested to find the type of convection at the onset with effects of relaxational and cross-diffusive parameters on it.

Employing the linear stability analysis, the study reveals that the cross-diffusion parameters have influenced the pattern of instability at the onset. With increasing Soret number, the onset is manifested by the steady convection, while the oscillatory convection prevails at the onset for the Soret number less than 1 and with increasing Dufour number. The Soret number greater than and equals to 1, always steady convection prevails. The steady convection for this double-diffusive system is independent of the relaxational parameters and only depends on the cross-diffusive terms like the Newtonian fluids. So the influence of Soret and Dufour parameters on the steady convection is similar to that of the Newtonian fluid which we have already discussed in chapter III. The oscillatory pattern of convection depends on the relaxational characteristics; the relaxation time destabilizes the oscillatory convection while the retardation time stabilizes it. The analysis reveals an interesting feature, for Rivlin-Ericksen fluid model the pattern of instability at the onset is always oscillatory.

We have an interest to study the double-diffusive instabilities with ‘coriolis forces’ developed due to the effect of rotation on the convective motion of a horizontal fluid layer. So we consider a problem of double-diffusive convection in a rotating horizontal Newtonian fluid layer heated and salted from below in a nonporous medium with Soret effect in chapter VI. Both linear and non-linear stability analysis are carried out. We study the marginally steady and oscillatory convections.

Employing linear stability analysis, the study reveals that the onset of instability is always manifested by oscillatory instability. The rotation parameter, Taylor number, plays stabilizing influence on the instability whenever the Soret tends to destabilize the double-diffusive system. The solutal Rayleigh number, representing the solute gradient, has also a stabilizing influence on this double-diffusive convection.

We also study finite amplitude stability analysis of this double-diffusive system pivoted around a steady marginal state to explore the possibility of subcritical
instability. For this we consider finite amplitude disturbances to the variables governing the fluid motion when the convection is steady. The analysis results the possibility of subcritical steady convection and thus the system becomes unstable with finite amplitude disturbances before it is unstable with infinitesimal disturbances. The Rayleigh number in the subcritical state increases as the Soret parameter increases, so that there may be the possibility of steady supercritical instability with higher values of the Soret number.

In the earlier chapters (III and V) we have consider the modified Darcy law for the investigation of the onset of instabilities due to double-diffusive convection with cross-diffusion in a horizontal porous medium saturated with Newtonian and non-Newtonian fluids, where the system of governing equations describing the flow behavior includes the heat diffusion equation derived from the classical Fourier law of heat flux. But presently in so many biomedical applications the propagations of thermal waves due to heat conduction is being utilized e.g. laser treatment in cornea, destruction of tumours, radiofrequency heating for the elimination of cardiac arrhythmias etc. The classical Fourier law of heat conduction can not account for the apparent increase in thermal conductivity of the nanofluid suspensions. A key way of introducing finite temperature wave propagation it is emergent to incorporate the thermal relaxation theory. Now to incorporate the thermal relaxation theory, instead of Fourier law of heat flux the Cattaneo-Christov law of heat flux will be considered for the heat diffusion equation in the system of governing equations of the fluid flow through porous medium. In chapter VII, we consider the double-diffusive convection in a Newtonian fluid saturated porous medium with Cattaneo-Christov model. In this double-diffusive convection we incorporate the effect of cross-diffusion term, the Soret number.

We are interested to study the effects of thermal relaxation time parameter (Cattaneo number) and the cross-diffusive parameter (Soret number) on this double-diffusive convection employing Cattaneo-Christov model of heat flux in a horizontal layer of viscous incompressible fluid saturated porous layer under the frame work of modified Darcy, heated and salted uniformly from below. Linear stability analysis is employed to find the critical value of Rayleigh number for the onset of convection.
The stationary and oscillatory mode of instabilities at the threshold of convection is also investigated.

In this investigation we have found an interesting effect of thermal relaxation parameter (Cattaneo parameter) on the double-diffusive convection by employing the Cattaneo-Christov model of heat flux, for very small values of Cattaneo number instability is manifested by stationary convection only and convection cells have fixed aspect ratio. As the Cattaneo number increases there is a bifurcation and instability then switches to oscillatory convection (Hopf bifurcation) with narrower cells. Further increase in the Cattaneo number leads to decrease the critical value of the Rayleigh number and further narrowing of the convection cells. So the instability due to convection occurs more easily and hence the Cattaneo number tends to destabilize the double-diffusive system. The analysis reveals that the Soret number destabilizes the steady double-diffusive convection but stabilizes the oscillatory convection. It is also observed that with increasing Soret number the steady convection prevails at the onset of instability. Thus the increasing Cattaneo number destabilizes the double-diffusive convection through oscillatory convection while increasing Soret number destabilizes through steady convection.

For understanding instabilities better, it is necessary to go beyond the linear theory and consider the nonlinear terms in the lowest order. The linear theory does not tell us anything about the amplitudes of waves and instabilities. The linearized equations would be satisfied by any value of the amplitude. It often happens that instability saturates at certain amplitude due to the effect of the nonlinear terms, instead of growing indefinitely as suggested by the linear theory. The possibility was first discussed by Landau (1949), Malakus and Veronis (1958) carried out the nonlinear analysis of the Rayleigh-Bénard problem for Rayleigh numbers larger than the critical value $R_c$. It was found that the amplitude of the convective motions increases monotonically with $|R-R_c|$. So, when $|R-R_c|$ is increased further, more heat is transported by convection.

Keeping this in view we have an interest to precede our study of double-diffusive convection and instabilities there at using nonlinear stability analysis to get better insight into the physical phenomena of the fluid flow during transition towards turbulence due to the convective motion as our future work.