Chapter VI

**Double-Diffusive Convection in a Rotating Fluid: Role of Soret Term**
double-diffusive convection in a rotating fluid

6.1 Introduction

The double-diffusive convection and its causes have been discussed elaborately in the previous chapters. We have studied the Stern type problems of double-diffusive instabilities due to double-diffusive convection in a horizontal nonporous layer (chapter II) and porous layer (chapter III) saturated with Newtonian fluid under the influences of cross-diffusion terms. The instabilities due to thermal and thermosolutal convection with non-Newtonian fluid saturated medium have been investigated at length in chapters IV and V. In geophysical applications, the positions and velocities are measured with respect to an inertial frame of reference fixed on the surface of the earth, which rotates with respect to an inertial frame stationary with respect to ‘distant stars’. In real life experiences, there are situations where the fluid motion is influenced by rotational axes. We are interested to study how rotation influences the motion of fluids in convective heat transfer. In this chapter we shall investigate the effects of rotation on the problem of double-diffusive instabilities in presence of Soret term.

The thermohaline convection in a horizontal layer of fluid heated from below with stable salinity gradient was first discussed using finite amplitude disturbances by Veronis (1965). Chandrasekhar (1981) pointed out that rotation brings a new phenomenon leading to the destoration of convection cells as well as generation of overstable oscillations under suitable conditions in the case of thermal convection. Veronis (1959) studied at length, using finite amplitude disturbances, the cell patterns in stationary thermal convection of a rotating fluid heated from below. Also, the fluid motion at the subcritical values of the Rayleigh number in a rotating fluid was investigated by Veronis (1966a). The Bénard convection and the Bénard convection with rotating fluid with large amplitude disturbances were studied by Veronis (1966b, 1968). Sengupta and Gupta (1971) extended the analysis of Veronis on thermohaline convection including the effect of uniform rotation and found that for infinitesimal disturbances in the form of rolls, the marginal state is oscillatory and rotation parameter tends to stabilize the double-diffusive convection. They also studied thermohaline convection in a rotating fluid using finite amplitude disturbances, according to Veronis (1959) and found the possibility of subcritical instability. Chakraborti and Gupta (1981) studied thermohaline convection in a rotating porous
medium using nonlinear stability analysis and showed that the system exhibit subcritical instability in the presence of rotation. Steady convective motions in a Boussinesq fluid with unstable thermal and stable salinity stratification with diffusivities $\left( \kappa_I / \kappa_T \right) \ll 1$ was investigated by Proctor (1981). Using perturbation theory with finite amplitude disturbances, it was established that steady subcritical convection occurred for stress-free and rigid boundaries. The linear and nonlinear stability analysis of thermohaline convection in a horizontal porous layer with Boussinesq-Darcy equation heated from below were made by Rudraiah et al. (1982).

With linear analysis they showed that vertical solute gradient sets up overstable motions while with finite amplitude study it was established that the effect of Prandtl number is very weak in contrast to existing viscous fluid flow. The linear and nonlinear stability of rotating double-diffusive convection in a sparsely packed porous medium considering a non-Darcy equation were investigated by Rudraiah et al. (1986). Linear stability and weak nonlinear theories were used to investigate analytically the Coriolis Effect on three-dimensional gravity-driven convection in a rotating porous layer heated from below, (Vadasz, 1998). Sharma and Kumar (1994, 1998) investigated the effect of rotation on thermal convection in micropolar fluid respectively in a nonporous and porous fluid layer. Chand et al. (2011) studied the effect of rotation in a double-diffusive convection in a magnetized ferrofluid with internal angular momentum.

In this study we are interested to investigate the role of Soret parameter on the double-diffusive convection in a horizontal layer of rotating fluid heated and salted from below. We use linear stability analysis with infinitesimal disturbances first to find the critical value of the Rayleigh number at the onset of instability and the type of the instability. The analysis reveals that in the marginal state oscillatory convection prevails at the onset. The rotation parameter tends to stabilize the flow while the Soret parameter shows destabilizing effect on the system. We also study finite amplitude stability analysis pivoted around a steady marginal state to explore the possibility of subcritical instability. For this we consider finite amplitude disturbances to the variables governing the fluid motion when the convection is steady. The analysis results the possibility of subcritical steady convection and thus the system becomes unstable with finite amplitude disturbances before it is unstable with infinitesimal
disturbances. The Rayleigh number in the subcritical state increases as the Soret parameter increase resulting a possibility of steady supercritical instability with higher values of the Soret number.

6.2 Basic equations of the problem

We consider an infinite horizontal layer of fluid of vertical depth \( h \) confined between two stress-free boundaries maintained at constant temperature and salinity. The fluid layer is heated and salted from below so that the bottom surface \( z=0 \) is kept at a temperature \( T_0 \) and concentration \( C_0 \) with the upper surface \( z=h \) at temperature \( (T_0-\Delta T) \) and concentration \( (C_0-\Delta C) \). The fluid layer is subjected to rotation with constant angular velocity \( \Omega \) about the vertical \( z \)-axis. For simplicity, we restrict ourselves to two-dimensional rolls only so that all the physical quantities are independent of \( y \).

The density decreases with increase in temperature but it slightly increases with increase in concentration of the salt so that the co-efficient of thermal expansion \( \alpha_t > 0 \) and the co-efficient of solutal expansion \( \beta_t > 0 \). The equation of state is then given by

\[
\rho = \rho_0 [1 - \alpha_t (T - T_0) + \beta_t (C - C_0)].
\]  

(6.1)

The layer of fluid is heated and salted from below in the field of gravity. The basic governing equations of motion with Oberbeck-Boussinesq approximations may be written as

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0
\]  

(6.2)

\[
\frac{\partial u_t}{\partial t} + u_j \frac{\partial u_t}{\partial x_j} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x_i} - g (1 - \alpha_t (T - T_0) + \beta_t (C - C_0)) k_T + \nu \nabla^2 u + 2\varepsilon_{ik} u_j \Omega_k
\]  

(6.3)

\[
\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} = \kappa_T \nabla^2 T
\]  

(6.4)

\[
\frac{\partial C}{\partial t} + u_j \frac{\partial C}{\partial x_j} = \kappa_T \nabla^2 C + \beta_s \nabla^2 T
\]  

(6.5)

Here \( u_i \equiv (u, v, w) \) are the velocity components in \( x, y, z \) directions and \( \rho_0 \) is the reference density at the reference temperature \( T_0 \) and concentration \( C_0 \); \( \beta_s \) is the
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Soret parameter, treated as constant and positive. The other symbols have their usual meanings.

The boundary conditions are

\[ T = T_0 \text{ and } C = C_0 \text{ at } z = 0 \text{ (at the bottom surface)} \]

\[ T = T_0 - \Delta T \text{ and } C = C_0 - \Delta C \text{ at } z = h \text{ (at the upper surface)} \] (6.6)

A physical sketch of the problem is shown below (Fig6.1)

![Physical sketch of the problem](image)

Fig6.1 Physical sketch of the Problem.

We investigate the stability of the above double-diffusive system under the infinitesimal disturbances in the form of rolls with the idealized stress-free boundary conditions. The variables in the perturbed motion is decomposed into a steady state of no motion and the perturbation quantities denoted by superscripts (') as

\[
\begin{align*}
    u_i &= 0 + u_i'(x,t), \\
    T &= T_h(z) + T'(x,t), \\
    p &= p_h(z) + p'(x,t), \\
    C &= C_h(z) + C'(x,t),
\end{align*}
\] (6.7)

where z-axis is taken vertically upwards. Using the basic state solutions and applying the infinitesimal disturbances (6.7), the equations (6.2) to (6.5) become

\[
\frac{\partial u_i'}{\partial t} + u_j' \frac{\partial u_i'}{\partial x_j} = - \frac{1}{\rho_0} \frac{\partial p'}{\partial x_i} - g(\alpha_i T' - \beta_i C') \hat{k} + \nu \nabla^2 u_i' + 2e_{ik} u_j' \Omega_k
\] (6.9)
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\[
\frac{\partial T'}{\partial t} + u'_j \frac{\partial T'}{\partial x_j} - \frac{w'}{h} \Delta T = \kappa_T \nabla^2 T' \tag{6.10}
\]

\[
\frac{\partial C'}{\partial t} + u'_j \frac{\partial C'}{\partial x_j} - \frac{w'}{h} \Delta C = \kappa_s \nabla^2 C' + \beta_s \nabla^2 T' \tag{6.11}
\]

We introduce the dimensionless variables with the following transformation

\[
(x, y, z) \rightarrow (x^* h, y^* h, z^* h), t \rightarrow \frac{h^2}{\kappa_T} t^*, u'_i \rightarrow \frac{\kappa_T}{h} u^*_i, T' \rightarrow \Delta T \theta^*,
\]

\[
C' \rightarrow \Delta CC^*, p' \rightarrow \frac{\rho \kappa_T}{h^2} p^*.
\tag{6.12}
\]

Using these transformations to make the variables of the governing equations dimensionless and dropping '*'; the equations become

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{6.13}
\]

\[
\frac{\partial u}{\partial t} + u_j \frac{\partial u}{\partial x_j} = -\frac{\nu}{\kappa_T} \frac{\partial p}{\partial x} + \left( \frac{g \alpha_i \Delta T h^3}{\kappa_T} - \frac{g \beta_i \Delta C h^3}{\kappa_T} \right) \frac{\theta}{\kappa_T} + \frac{\nu}{\kappa_T} \nabla^2 u + \frac{2h^2}{\kappa_T} e_{ij} u_j \Omega_k \tag{6.14}
\]

\[
\frac{\partial \theta}{\partial t} + u_j \frac{\partial \theta}{\partial x_j} - w = \nabla^2 \theta \tag{6.15}
\]

\[
\frac{\partial C}{\partial t} + u_j \frac{\partial C}{\partial x_j} - w = \frac{\kappa_s}{\kappa_T} \nabla^2 C + \frac{\beta_s \Delta T}{\kappa_T} \nabla^2 T. \tag{6.16}
\]

Let us introduce a stream function \( \psi \) defined as

\[
u = \frac{\partial \psi}{\partial z}, \quad w = -\frac{\partial \psi}{\partial x} \tag{17}
\]

which is consistent with the equation (6.13).

Substituting (6.17) and eliminating the pressure term by taking certain mathematical manipulations in order to eliminate the unknown pressure, the equations (6.14) to (6.16) become

\[
\left( \frac{1}{\text{Pr}} \frac{\partial}{\partial t} - \nabla^2 \right) \nabla^2 \psi = -Ra \frac{\partial \theta}{\partial x} + \frac{1}{\text{Le}} \frac{\partial C}{\partial x} + R_s + Y \frac{\partial \psi}{\partial z} + \frac{1}{\text{Pr}} J(\psi, \nabla^2 \psi), \tag{6.18}
\]

\[
\left( \frac{1}{\text{Pr}} \frac{\partial}{\partial t} - \nabla^2 \right) \nabla^2 \psi = -Y \frac{\partial \psi}{\partial z} + \frac{1}{\text{Pr}} J(\psi, \psi), \tag{6.19}
\]
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\[
\left( \frac{\partial}{\partial t} - \nabla^2 \right) \theta = -\frac{\partial \psi}{\partial x} + J(\psi, \theta), \quad (6.20)
\]

\[
\left( \frac{\partial}{\partial t} - \frac{1}{Le} \nabla^2 \right) C = -\frac{\partial \psi}{\partial x} + S_r \nabla^2 \theta + J(\psi, C), \quad (6.21)
\]

where

\[
\begin{align*}
\text{Pr} = & \nu / \kappa_T, \quad \text{Le} = \kappa_T / \kappa_s, \quad \text{Ra} = (g \alpha T \Delta h^3) / (\nu \kappa_T), \\
S_r = & (\beta_s \Delta T) / (\kappa_T \Delta C), \quad Y^2 = (4 \Omega^2 h^4) / v^2.
\end{align*}
\]

Here the dimension-less parameters Ra and Rs are respectively the Rayleigh and solutal Rayleigh number, Y^2 is the Taylor number and v the azimuthal velocity developed due to rotation.

Using the conditions of stress-free boundaries and the equation of continuity the boundary conditions are

\[
\psi = \frac{\partial^2 \psi}{\partial z^2} = \theta = C = \frac{\partial v}{\partial z} = 0 \text{ at } z = 0 \text{ and } z = 1. \quad (6.23)
\]

**6.3 Linear stability analysis**

The linear perturbation equations are obtained as follows

\[
\begin{align*}
\left( \frac{1}{\text{Pr}} \frac{\partial}{\partial t} - \nabla^2 \right) \nabla^2 \psi & = -Ra \frac{\partial \theta}{\partial x} + \frac{1}{\text{Le}} \frac{\partial C}{\partial x} Rs + Y \frac{\partial v}{\partial z}, \quad (6.24) \\
\left( \frac{1}{\text{Pr}} \frac{\partial}{\partial t} - \nabla^2 \right) v & = -Y \frac{\partial \psi}{\partial z}, \quad (6.25) \\
\left( \frac{\partial}{\partial t} - \nabla^2 \right) \theta & = -\frac{\partial \psi}{\partial x}, \quad (6.26) \\
\left( \frac{\partial}{\partial t} - \frac{1}{\text{Le}} \nabla^2 \right) C & = -\frac{\partial \psi}{\partial x} + S_r \nabla^2 \theta. \quad (6.27)
\end{align*}
\]

We consider the solutions of the equations (6.24) to (6.27) satisfying the boundary conditions (6.23) are given below

\[
\begin{align*}
\psi & = \psi_0 e^{\sigma t} \sin(\pi \alpha x) \sin(n \pi z), \quad v = v_0 e^{\sigma t} \sin(\pi \alpha x) \cos(n \pi z), \\
\theta & = \theta_0 e^{\sigma t} \cos(\pi \alpha x) \sin(n \pi z), \quad C = C_0 e^{\sigma t} \cos(\pi \alpha x) \sin(n \pi z),
\end{align*}
\]

where \( \alpha \) is the wave number, \( \sigma \) is a complex constant, \( n \) is an integer and \( \psi_0, v_0, \theta_0, C_0 \) are constants.
Substituting (6.28) in the linearized equations (6.24) to (6.27) and eliminating \( \psi_0, v_0, \theta_0, C_0 \) leads to a single equation characterizing the instability of the system, at the threshold of convection \( n=1 \), as

\[
\begin{bmatrix}
\left(\frac{\sigma}{Pr} + \pi^2 + \pi^2 \alpha^2\right)^2(\sigma + \pi^2 + \pi^2 \alpha^2)(\pi^2 + \pi^2 \alpha^2) + Y^2 \pi^2(\sigma + \pi^2 + \pi^2 \alpha^2) \\
-\pi^2 \alpha^2 Ra(\sigma + \pi^2 + \pi^2 \alpha^2) \\
-\pi^2 \alpha^2 Rs(\sigma + \pi^2 + \pi^2 \alpha^2)\left(\sigma + (1 - S_r)(\pi^2 + \pi^2 \alpha^2)\right)
\end{bmatrix}
(Le\sigma + \pi^2 + \pi^2 \alpha^2) =
\]

(6.29)

Equation (6.29) is a biquadratic equation in \( \sigma \), where the coefficients of the equation involve the governing parameters are all real. This biquadratic equation with real coefficients has four roots, so if complex roots exist there must be two pairs of complex roots or one pair of complex roots with two real roots; otherwise all four roots are real. We are interested in the marginal stability analysis and in that case \( \sigma_r = 0 \), where \( \sigma_r \) is the real part of the complex roots \( \sigma = \sigma_r \pm \omega \) and it is apparent that the marginal state \( (\sigma_r = 0) \) occurs with two cases \( \omega = 0 \) and \( \omega \neq 0 \), where \( \omega \) is the imaginary part of the complex roots.

When \( \omega = 0 \) then the marginal state is characterized by the steady (cellular) convection and when \( \omega \neq 0 \) then the instabilities are characterized by a marginally oscillatory mode and then the instability sets in as oscillatory convection of growing amplitude, known as overstability.

When the marginal state \( (\sigma_r = 0) \) is characterized by a stationary pattern of convection for which \( \omega = 0 \), we have \( \sigma = 0 \). Then one of the roots of the biquadratic equation (6.29) is zero and the condition for that gives the Rayleigh number for steady convection as

\[
Ra^{(steady)} = \frac{\pi^4(1 + \alpha^2)^3}{\alpha^2} + \frac{Y^2}{\alpha^2} + (1 - S_r)Rs.
\]

(6.30)

Thus the marginal state with cellular convection is governed by the relation (6.30) and it depends on cross-diffusive term viz. the Soret number along with the Taylor number and the solutal Rayleigh number. The critical value of the Rayleigh number for steady convection at the marginal state is obtained from this relation.
The relation (6.30) manifested that the Rayleigh number increases with increasing value of the Taylor number, the rotation parameter and the solutal Rayleigh number while decreases with the increasing value of the Soret number. Thus the rotation parameter Taylor number and the solutal Rayleigh number tends to stabilize whereas the cross-diffusion parameter the Soret number destabilizes this rotating double-diffusive system.

A marginally oscillatory mode \((\sigma_\nu = 0 \text{ and } \omega \neq 0)\) occurs and the instability sets in as oscillatory of growing amplitude, when there exists a pair of purely imaginary roots of the biquadratic equation (6.29), then from the equation (6.29), according to Chandrasekhar (1981) the frequency of the periodic convection and the Rayleigh number for oscillatory convection are obtained respectively as

\[
\omega^4 + \left[ \pi^4 \Pr(1+\alpha^2)^2 + \pi^4 \frac{1}{Le} (1+\alpha^2)^3 + \frac{\alpha^2 \Pr (1-Le(1-S_r))}{Le^2 (1+Pr)(1+a^2)} \right. \\
+ \left. \frac{Pr(Pr-1)\pi^2 + Y^2}{\pi^2 (Pr+1)(1+\alpha^2)} \right] \omega^2 + \frac{\pi^8 \Pr^2 (1+\alpha^2)^4}{Le^2} \\
+ \frac{\pi^4 \alpha^2 \Pr^3 (1-Le(1-S_r)) (1+\alpha^2)}{Le^2 (1+Pr)} R_s + \frac{\pi^4 \Pr^2 (Pr-1)(1+\alpha^2)Y^2}{Le^2 (1+Pr)} = 0
\]

(6.31)

and

\[
Ra_{osc} = \frac{\pi^4 (1-S_r)(1+\alpha^2)^2 Le \omega^2 \alpha^2}{\pi^4 (1+\alpha^2)^2 + Le^2 \omega^2 R_s} + \frac{\pi^4 \Pr (1+\alpha^2)^3 + \omega^2 \Pr Y^2}{\pi^4 \Pr^2 (1+\alpha^2)^2 + \omega^2 \alpha^2} \\
+ \frac{\{\pi^4 \Pr (1+\alpha^2)^3 - \omega^2\} (1+\alpha^2)}{\alpha^2 \Pr}.
\]

(6.32)

The relation (6.31) gives the square of the frequency of the oscillatory convection, which strongly depends on the Soret parameter along with Prandtl number and the rotation parameter Taylor number. Since \(\omega\), the frequency of the oscillatory convection is real then \(\omega^2\) must be always real positive. The condition, for which the relation (6.31) gives positive real \(\omega^2\), would be the condition for the occurrence of oscillatory convection in this thermohaline convection.

The relation (6.32) establishes that the Rayleigh number for oscillatory convection at the marginal state depends on the cross-diffusive term, the Soret number. It also depends on the Prandtl number, Taylor number and the solutal Rayleigh number. It is observed from the relation (6.32) that the value of the Rayleigh
number for oscillatory convection increases with the increasing value of the Taylor number and the solutal Rayleigh number, but its value decreases with the increasing value of the Soret number. Hence the Taylor number and the solutal Rayleigh number stabilize the system while the Soret number tends to destabilize the double-diffusive system.

It reveals that the steady convection as well as the oscillatory convection in thermohaline convection with a rotating fluid depends on the cross-diffusive term, the Soret parameter and the Soret parameter tends to destabilize the double-diffusive system in both cases.

In the absence of Soret effect, the results thus obtained for steady and oscillatory convection are in good agreement with the results which were obtained by Sengupta and Gupta (1971).

### 6.4 Results and discussion

The equation characterizing the stability analysis of the double-diffusive system at the threshold of convection, obtained from the relation (6.29), can be represented as

\[ A_4 \sigma^4 + A_2 \sigma^3 + A_3 \sigma^2 + A_4 \sigma + A_5 = 0 \]  

(6.33)

where

\[
A_1 = \left( \frac{Le J}{Pr} \right), \\
A_2 = \left( \frac{1 + Le}{Pr^2} + \frac{2Le}{Pr} \right) J^2, \\
A_3 = \left( \frac{1}{Pr^2} + 2 \frac{(1 + Le)}{Pr + Le} \right) J^3 + \frac{\pi^2 Y^2 Le + \pi^2 \alpha^2 (Rs - LeRa)}{Pr}, \\
A_4 = (1 + Le + 2Pr) J^4 + \left( \frac{1 + Le}{Pr} \pi^2 Y^2 + \pi^2 \alpha^2 (Rs - LeRa) + \pi^2 \alpha^2 \left( \frac{1 - S_r}{Rs - Ra} \right)/Pr \right) J, \\
A_5 = J^5 + \pi^2 Y^2 J^2 + \pi^2 \alpha^2 \left( \frac{1 - S_r}{Rs - Ra} \right) J^2
\]

and considering \( J = \pi^2 (1 + \alpha^2) \).

The stability of the double diffusive system depends on the co-efficients of the characteristic equation (6.33) containing a large number of dimensionless parameters characterizing the fluid flow. The task of discussing stability of the system or the instabilities set in can be simplified by fixing some of the parameters and accordingly some of the parameters are fixed by us. We are interested to investigate the influences of Soret effect on the type of instability at the onset and on the critical value of the Rayleigh number, manifested the onset of instability in the double-diffusive system.
Since the Rayleigh number for the steady as well as oscillatory convection and the frequency of the oscillatory convection contain a large number of governing parameters, thus it is not possible to find the critical value of the Rayleigh number at the onset of instability or the frequency of the oscillatory convection analytically. Thus we analyze these relations numerically by studying the graphs of the corresponding relations. The results thus obtained for steady and oscillatory convection in the relations (6.30), (6.31) and (6.32) in the absence of Soret effect that is $S_r = 0$ coincide with results of Sengupta and Gupta (1971).

To study the influences of rotation parameter, the Taylor number ($Y^2$) and the cross-diffusion parameter, the Soret number ($S_r$) numerically, we consider the fixed values of the dimensionless parameters Lewis number ($Le$), Prandtl number ($Pr$) and the solutal Rayleigh number ($Rs$) with varying values of the Taylor number and Soret number. The variations of Rayleigh number with square of the wave number for the marginally steady and oscillatory convection are evaluated from the relations (6.30), (6.31) and (6.32) with $Rs=10^4$, $Le=10^2$, $Pr=7.0$ and for different values of the rotation parameter $Y^2$ in the absence of Soret number and plotted in the Fig6.2. It is observed from the graphs of Fig6.2 that each of the curves shows a minimum which corresponds to the critical Rayleigh number against a certain wave number signifying the onset of instability; it can also be observed that for a given value of $Y^2$, the curve giving $Ra^{(steady)}$ corresponding to the marginally steady convection lies above the curve for $Ra^{(osc)}$ corresponding to the oscillatory convection and $Ra^{(steady)}_{cr} > Ra^{(osc)}_{cr}$ for all curves. It is also observed that the critical Rayleigh number increases with increasing Taylor number. This justifies that for infinitesimal disturbances the marginal state will be always oscillatory without Soret effect and the rotation parameter, Taylor number tends to stabilize the system. The results and the graphs in Fig6.2 coincide with those as obtained by Sengupta and Gupta (1971). We obtain the same results in the case of this thermohaline convection with the Soret effect, which is depicted in the Fig6.4. Thus in both the cases with or without Soret effect, the onset is manifested by the oscillatory convection and there is a tendency of stabilizing influence of rotation on the fluid flow.
Fig6.3 and Fig6.5 illustrate the variations of the square of frequency of the oscillatory convection with the square of wave number for increasing values of rotation parameter the Taylor number \((Y^2)\) without and with Soret effect respectively. In both cases with or without Soret effect, the frequency decreases as the Taylor number increases justifying the stabilizing influence of the rotation parameter.

Fig6.6 and Fig6.7 depict that the critical value of Rayleigh number decreases with the increasing value of Soret parameter in the steady convection as well as in the oscillatory convection. The influence of Soret number on the oscillatory convection is very small in comparison to the stationary convection as observed in the Fig6.6 and Fig6.7. The variation of the frequency of oscillatory convection with wave number is being illustrated in the Fig6.8 and it is observed that frequency decreases prominently with the increase of Soret number. Hence the influence of Soret parameter has a tendency to destabilize the fluid flow in both the cases steady as well as oscillatory convection.

The relations (6.30) and (6.32) reveal that the solutal Rayleigh number has stabilizing influences on the steady and oscillatory convection respectively in this double-diffusive system. The effects of solutal Rayleigh number, in presence of Soret effect, on the Rayleigh number justifying steady convection or oscillatory convection, are being illustrated respectively in the Fig6.9 and Fig6.10. The effects of solutal Rayleigh number, in presence of Soret number, on the frequency of periodic convection is illustrated in Fig6.11.

### 6.5 Finite amplitude steady convection

Now we study a finite amplitude stability analysis pivoted around steady marginal state to find the possibility of subcritical or supercritical instability. When the disturbances are of finite amplitude and the convection is steady, then the governing equations for stability are (6.18) to (6.21) with \(\partial/\partial t = 0\). Now we use the perturbation method due to Veronis (1959) to solve these equations. We express all the dependent variables in powers of \(\varepsilon\), a finite amplitude disturbance parameter, as
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\[
\psi = \varepsilon \psi_0 + \varepsilon^2 \psi_1 + \varepsilon^3 \psi_2 + \ldots \quad \left(6.34\right)
\]

\[
v = \varepsilon v_0 + \varepsilon^2 v_1 + \varepsilon^3 v_2 + \ldots \quad \left(6.18\right)
\]

\[
\theta = \varepsilon \theta_0 + \varepsilon^2 \theta_1 + \varepsilon^3 \theta_2 + \ldots \quad \left(6.21\right)
\]

\[
C = \varepsilon C_0 + \varepsilon^2 C_1 + \varepsilon^3 C_2 + \ldots \quad \left(6.30\right)
\]

\[
R = \varepsilon R_0 + \varepsilon^2 R_1 + \varepsilon^3 R_2 + \ldots \quad \left(6.30\right)
\]

where \( R_0 (= Ra^{(steady)}) \) is the critical Rayleigh number for steady convection according to the linear stability analysis and given by (6.30). Substituting (6.34) in the equations (6.18) to (6.21) with \( \partial / \partial t = 0 \), and then equating the co-efficients of \( \varepsilon, \varepsilon^2 \) and \( \varepsilon^3 \) respectively we get the following equations as

\[
- \nabla^4 \psi_0 = - R_0 \frac{\partial \theta_0}{\partial x} + \frac{1}{Le} \frac{\partial C_0}{\partial x} \left( Y \frac{\partial v_0}{\partial z} \right)
\]

\[
- \nabla^4 v_0 = - \frac{\partial \psi_0}{\partial z}, \quad \left(6.35\right)
\]

\[
- \nabla^4 \theta_0 = - \frac{\partial \psi_0}{\partial x},
\]

\[
- \frac{1}{Le} \nabla^4 C_0 = - \frac{\partial \psi_0}{\partial x} + S \nabla^2 \theta_0.
\]

\[
- \nabla^4 \psi_1 = - \left( R_0 \frac{\partial \theta_0}{\partial x} + R_i \frac{\partial \theta_0}{\partial x} + R_2 \frac{\partial \theta_0}{\partial x} \right) + \frac{1}{Le} \frac{\partial C_1}{\partial x} + Y \frac{\partial v_1}{\partial z} + \frac{1}{Pr} J(\psi_0, \nabla^2 \psi_0), \quad \left(6.36\right)
\]

\[
- \nabla^4 v_1 = - \frac{\partial \psi_1}{\partial z} + \frac{1}{Pr} J(\psi_0, v_0),
\]

\[
- \nabla^4 \theta_1 = - \frac{\partial \psi_1}{\partial x} + J(\psi_0, \theta_0),
\]

\[
- \frac{1}{Le} \nabla^4 C_1 = - \frac{\partial \psi_1}{\partial x} + S \nabla^2 \theta_1 + J(\psi_0, C_0).
\]

and

\[
- \nabla^4 \psi_2 = - \left( R_0 \frac{\partial \theta_0}{\partial x} + R_i \frac{\partial \theta_0}{\partial x} + R_2 \frac{\partial \theta_0}{\partial x} \right) + \frac{1}{Le} \frac{\partial C_2}{\partial x} + Y \frac{\partial v_2}{\partial z} + \frac{1}{Pr} J(\psi_0, \nabla^2 \psi_1) + \frac{1}{Pr} J(\psi_1, \nabla^2 \psi_0), \quad \left(6.37\right)
\]

\[
- \nabla^4 v_2 = - \frac{\partial \psi_2}{\partial z} + \frac{1}{Pr} J(\psi_0, v_1) + \frac{1}{Pr} J(\psi_1, v_0),
\]

\[
- \nabla^4 \theta_2 = - \frac{\partial \psi_2}{\partial x} + J(\psi_0, \theta_1) + J(\psi_1, \theta_0),
\]

\[
- \frac{1}{Le} \nabla^4 C_2 = - \frac{\partial \psi_2}{\partial x} + S \nabla^2 \theta_2 + J(\psi_0, C_1) + J(\psi_1, C_0).
\]

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Eliminating \( v_0, \theta_0, C_0 \) from the equations in (6.35) and expressing those equations as a single equation in \( \psi_0 \) as

\[
\left[ \nabla^2 + Y^2 \frac{\partial^2}{\partial z^2} - R_0 \frac{\partial^2}{\partial x^2} + (1 - S_r)R_s \frac{\partial^2}{\partial x^2} \right] \psi_0 = 0 \quad \text{or, } \mathcal{Z} \psi_0 = 0, \tag{6.38}
\]

where \( \mathcal{Z} \) is an operator such that

\[
\mathcal{Z} \equiv \nabla^2 + Y^2 \frac{\partial^2}{\partial z^2} - R_0 \frac{\partial^2}{\partial x^2} + (1 - S_r)R_s \frac{\partial^2}{\partial x^2}.
\]

The solutions of the equations in (6.38) satisfying the boundary conditions (6.23) at the threshold of convection \((n = 1)\) are

\[
\psi_0 = -\frac{2}{\pi \alpha} \sin(\pi \alpha x) \sin(\pi z), \quad v_0 = \frac{2Y}{\pi \alpha (1 + \alpha^2)} \sin(\pi \alpha x) \cos(\pi z), \quad \theta_0 = \frac{2}{\pi^2 (1 + \alpha^2)} \cos(\pi \alpha x) \sin(\pi z), \quad C_0 = \frac{2Le(1 - S_r)}{\pi^2 (1 + \alpha^2)} \cos(\pi \alpha x) \sin(\pi z). \tag{6.39}
\]

Eliminating \( v_1, \theta_1, C_1 \) from the equations in (6.36) and expressing those equations as a single equation in \( \psi_1 \) as

\[
\mathcal{Z} \psi_1 = R_1 \frac{\partial^2 \psi_0}{\partial x^2} + \frac{1}{Pr} \frac{Y}{\partial z} J(\psi_0, v_0) \left( \frac{\partial \psi_0}{\partial x} \right) \left( \frac{\partial \psi_0}{\partial x} \right) = 0 \tag{6.40}
\]

Using (6.39) the equation (6.40) becomes

\[
\mathcal{Z} \psi_1 = -\pi^2 \alpha^2 R_1 \psi_0. \tag{6.41}
\]

According to Veronis (1959), \( R_1 \) is evaluated in such a manner so as to cancel the terms of the equation (6.41) containing \( \psi_0 \), since a term of the form \( \psi_0 \) will be a secular term for this mode and its presence will affect the assumed periodicity of the solution and hence it requires \( R_1 = 0 \). Thus \( \mathcal{Z} \psi_1 = 0 \) and its solution subject to the boundary conditions (6.23) is \( \psi_1 = 0 \).

Using this we deduce the solutions of the system of equations (6.36) satisfying the conditions (6.23) are

\[
v_1 = \frac{Y}{2\pi^2 \alpha^3 Pr(1 + \alpha^2)} \sin(2\pi \alpha x), \quad \theta_1 = -\frac{1}{2\pi^2 (1 + \alpha^2)} \sin(2\pi z), \quad C_1 = \frac{Le\{Le(1 - S_r) - S_r\}}{2\pi^2 (1 + \alpha^2)} \sin(2\pi z). \tag{6.42}
\]
Eliminating $v, \theta, C$ from the equations in (6.37) and expressing those equations as a single equation in $\psi_2$ as

$$\mathcal{Z}\psi_2 = -\frac{\pi\alpha}{2} \left[ 2\pi R - \frac{\alpha R_0}{\pi(1+\alpha^2)} + \frac{\alpha Le^2 R_0}{\pi(1+\alpha^2)} + \frac{Y^2}{\pi^2 \alpha^3 Pr^2(1+\alpha^2)} - \frac{\alpha(1+Le+Le^2)S_r R_s}{\pi(1+\alpha^2)} \right] \psi_0 + \left[ \frac{\alpha R_0}{\pi(1+\alpha^2)} - \frac{\alpha Le^2 R_0}{\pi(1+\alpha^2)} \right] \sin(\pi\alpha x)\sin(3\pi z) - \frac{Y^2}{\pi^2 \alpha^3 Pr^2(1+\alpha^2)} \sin(3\pi\alpha x)\sin(\pi z)$$

The first term on the RHS of the equation (6.43) has the term $\psi_0$ and hence a secular term for this mode and must vanish. Thus $R_2$ can be evaluated in such a manner so that

$$2\pi R = -\frac{\alpha R_0}{\pi(1+\alpha^2)} + \frac{\alpha Le^2 R_0}{\pi(1+\alpha^2)} + \frac{Y^2}{\pi^2 \alpha^3 Pr^2(1+\alpha^2)} - \frac{\alpha(1+Le+Le^2)S_r R_s}{\pi(1+\alpha^2)}$$

i.e.

$$R_2 = \frac{R_0}{2\pi^2(1+\alpha^2)} - \frac{Le^2 R_0}{2\pi^2(1+\alpha^2)} - \frac{Y^2}{2\pi^2 \alpha^2 Pr^2(1+\alpha^2)} + \frac{(1+Le+Le^2)S_r R_s}{2\pi^2(1+\alpha^2)}.$$ (6.44)

where $R_0$ is the critical value of the Rayleigh number for steady convection with respect to infinitesimal disturbances. It reveals from the relation (6.44) that $R_2$, the subcritical Rayleigh number to this mode, may be negative for certain range of wave numbers with given solutal Rayleigh number $(Rs)$, Prandtl number $(Pr)$, Lewis number $(Le)$, Taylor number $(Y^2)$ and Soret number $(S_r)$. This establishes that the system becomes unstable to finite amplitude steady disturbances before it becomes unstable to disturbances of infinitesimal amplitude. Thus the relation (6.44) shows that the effects of stable salinity gradient and the rotation promote the possibility of subcritical instability. The relation (6.44) also establishes that the increase of Soret number increases the value of the subcritical Rayleigh number. Thus the Soret parameter delays the subcritical instability that is with finite amplitude disturbances the Soret number tends to stabilize the double-diffusive system, while with infinitesimal disturbances the Soret number destabilizes the system. Hence with finite amplitude disturbances for higher value of Soret parameter there is a possibility of supercritical instability. This is being illustrated through the Fig6.12, which depicts
that as the Soret number increases the Rayleigh number passes from negative to positive and the critical value of the Rayleigh number also increases gradually.
Fig 6.2 The variations of Rayleigh number with square of wave number for various values of $Y^2$ with $R_s=10^4$, $L_e=10^2$, $P_r=7.0$ and without Soret effect.

Fig 6.3 The variations of square of frequency with square of wave number for various values $Y^2$ for the oscillatory convection without Soret effect.
Fig6.4 The variations of Rayleigh number with square of wave number for various values of $Y^2$ with $Rs=10^4$, $Le=10^2$, $Pr=7.0$ and with $S_r=0.5$.

Fig6.5 The variations of square of frequency with square of wave number for various values $Y^2$ for the oscillatory convection with $S_r=0.5$. 
Fig6.6 The variations of Rayleigh number with square of wave number for various Soret number for the steady convection.

Fig6.7 The variations of Rayleigh number with square of wave number for various Soret number for the oscillatory convection.
**Fig 6.8** The variations of square of frequency with square of wave number for various values of Soret number for the oscillatory convection.

**Fig 6.9** The variations of Rayleigh number with square of wave number for various Solutal Rayleigh number for the steady convection.
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Fig6.10 The variations of Rayleigh number with square of wave number for various Solutal Rayleigh number for the oscillatory convection.

Fig6.11 The variations of square of frequency with square of wave number for various values of Solutal Rayleigh number for the oscillatory convection.
Fig 6.12 The variations of Rayleigh number for subcritical instability with square of wave number for various Soret number and fixed $Rs, Y^2$ for the steady convection.