5.1 INTRODUCTION

In this particular chapter the propagation of electromagnetic thermoelastic plane waves in an initially unstressed, homogeneous isotropic, conducting plate under uniform static magnetic field, has been investigated in the context of generalised theory of thermoelasticity developed by Lord - Shulman [20]. The investigation has been done by assuming electrical behaviour as quasi-static and the mechanical behaviour as dynamical. The basic equations have been solved by considering the plane wave solutions.

5.2 FORMULATION OF THE PROBLEM

We consider an infinite homogeneous isotropic, elastic and conducting plate of thickness $2L$, initially at uniform temperature $T_0$ and a large static magnetic field $\overrightarrow{H}_0$ acting along $x_1$ - axis. We choose the origin of the co-ordinate system $(x_1, x_2, x_3)$ in the middle of the surface and the $x_2$-axis along the thickness of the plate. The surfaces $x_2 = \pm L$ are assumed to be stress free, insulated, and the perturbed magnetic field also vanishes on these
boundaries. It is also assumed that the electromagnetic field is quasi-static.

The basic governing equations of generalised thermoelasticity and electromagnetic interactions, in the absence of heat sources and body forces, are

\[
(\lambda + 2\mu) \nabla (\nabla \cdot \mathbf{u}) + \mu \nabla^2 \mathbf{u} + \mu_0 \sigma \frac{\partial y}{\partial t} - \mu_0^2 \mathbf{e} \cdot \mathbf{H}_0 = 0,
\]

\[
\mathbf{p} \left( \frac{\partial \theta}{\partial t} + C_0 \ddot{\theta} \right) + \Gamma \nabla \cdot \left( \dot{\mathbf{u}} + \xi \mathbf{u} \right) = \kappa \nabla^2 \theta,
\]

\[
\nabla \times \mathbf{e} = 0,
\]

\[
\nabla \times \mathbf{h} = \sigma \left( \mathbf{e} + \mu_0 \dot{x} \mathbf{H}_0 \right),
\]

\[
\nabla \cdot \mathbf{h} = 0,
\]

where \( \mathbf{H}_0 = (H_1, 0, 0) \), \( \mathbf{e} \) and \( \mathbf{h} \) are the perturbations in the electric and magnetic fields, \( \sigma \) and \( \mu_0 \) be the constant electrical conductivity and the magnetic permeability, \( \mu \) and \( \lambda \) are Lame' constants; \( \mathbf{u} \), \( \mathbf{e} \), \( \rho \), \( K \) and \( C_V \) be the displacement, the dilatation, density of the material, coefficient of heat conduction and specific heat at constant volume; \( \gamma = (3\lambda + 2\mu) \alpha_T \), \( \kappa \) be the coefficient of thermal linear expansion.
The equations (5.1.1) can be rewritten as

\[
\left(\lambda + 2\mu\right) \frac{\partial^2 u_1}{\partial x_1^2} + (\lambda + \mu) \frac{\partial^2 u_2}{\partial x_1 \partial x_2} + \frac{\partial^2 u_1}{\partial x_2^2} - \gamma \frac{\partial \Theta}{\partial x_1} - \rho \dot{u}_1 = 0,
\]

\[
(\lambda + 2\mu) \frac{\partial^2 u_2}{\partial x_2^2} + (\lambda + \mu) \frac{\partial^2 u_1}{\partial x_1 \partial x_2} + \frac{\partial^2 u_2}{\partial x_1^2} - \mu_0^2 H_1^2 \sigma \ddot{u}_2 = 0,
\]

\[
\mu \left( \frac{\partial^2 u_3}{\partial x_1^2} + \frac{\partial^2 u_3}{\partial x_2^2} \right) - \mu_0 \sigma H_1 e_2 - \mu_0^2 \sigma H_1^2 u_3 - \rho \ddot{u}_3 = 0,
\]

\[
\kappa \left( \frac{\partial^2 \Theta}{\partial x_1^2} + \frac{\partial^2 \Theta}{\partial x_2^2} \right) = \rho c_v \left( \dot{\Theta} + c_o \ddot{\Theta} \right) + \Gamma_0 \gamma \left( \frac{\partial}{\partial t} + c_o \frac{\partial^2}{\partial t^2} \right) \left( \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} \right),
\]

\[
\frac{\partial e_2}{\partial x_1} - \frac{\partial e_1}{\partial x_2} = 0,
\]

\[
\sigma e_1 - \frac{\partial h_3}{\partial x_2} = 0,
\]

\[
\sigma e_2 + \mu_0 \sigma H_1 \dot{u}_3 + \frac{\partial h_3}{\partial x_1} = 0,
\]

\[
\mu_0 \sigma \ddot{u}_2 + \frac{\partial h_2}{\partial x_1} - \frac{\partial h_1}{\partial x_2} = 0,
\]

\[
\frac{\partial h_1}{\partial x_1} + \frac{\partial h_2}{\partial x_2} = 0.
\]
The boundary conditions are

\[ \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_2} = 0, \]

\[ \frac{\partial u_3}{\partial x_2} = 0, \]

\[ -\frac{\beta_1^2}{2} \frac{\partial u_2}{\partial x_1} + (\beta_1^2 - 2) \frac{\partial u_1}{\partial x_1} - (\bar{R}_H - \bar{R}_H) \beta_1^2 \frac{\partial h_1}{\partial x_1} - \frac{\partial \beta_1^2 \Theta}{\partial x_2} = 0, \quad \text{at } x_2 = \pm L, \]

\[ \frac{\partial \Theta}{\partial x_2} = 0, \]

\[ h_3 = 0, \]

\[ h_1 \mp i \left( \frac{R_H}{\bar{R}_H} \right) h_2 = 0. \]

We introduce the following dimensionless quantities

\[ x_j = \frac{w_s x_j}{c_1}, t' = \frac{w_s t}{\bar{t}_0}, u_j' = \frac{w^* c_1 u_j}{\bar{t}_0}, h_j' = \frac{h_j}{H_1}, \]

\[ e_j' = e_j / \mu_0 c_1 H_1, \quad z' = \frac{\bar{z}}{\bar{t}_0}, \quad \xi = \frac{\bar{t}_0 \gamma^2}{\gamma c_1^2}, \quad \xi_v = \frac{\rho c_1^2}{\gamma}, \]

\[ \xi_H = \frac{w^*}{\mu_0 \gamma c_1^2}, \quad R_H = \frac{\mu_0 H_1^2}{\rho c_1^2}, \quad w^* = \frac{\rho c_1^2}{\gamma}. \]

where \( c_1^2 = (\lambda + 2\mu) / \rho \), \( c_2^2 = \mu / \rho \).

Introducing the above said quantities in equations (5.2.2), we get (on suppressing the dashes)
\[ \beta_1^2 \frac{\partial u_1}{\partial x_1} + (\beta_1^2 - 1) \left( \frac{\partial u_2}{\partial x_1} \frac{\partial u_2}{\partial x_2} + \frac{\partial u_1}{\partial x_2} \frac{\partial u_1}{\partial x_2} \right) - \beta_1^2 u_1 - \beta_1^2 u_2 - \frac{\partial z}{\partial x_1} = 0 , \]

\[ \beta_1^2 \frac{\partial u_2}{\partial x_2} + (\beta_1^2 - 1) \left( \frac{\partial u_1}{\partial x_1} \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \frac{\partial u_2}{\partial x_2} \right) - \beta_1^2 u_1 - \beta_1^2 u_2 - \frac{\partial z}{\partial x_2} = 0 , \]

\[ - \beta_1^2 \left( \frac{\partial z}{\partial t} - \xi_{\|} \right) u_2 = 0 , \]

\[ \xi \xi_v \left( \frac{\partial u_3}{\partial x_1} + \frac{\partial u_3}{\partial x_2} \right) - \xi_{\|} \beta_1^2 e_2 / \xi_{\|} - \xi_{\|} \xi_v \beta_1^2 u_3 / \xi_{\|} - \xi_v \xi_{\|} \beta_1^2 u_3 = 0 , \]

\[ (\dot{z} + c_0 \omega \ddot{z}) + \xi \left[ \left( \frac{\partial^2}{\partial t^2} + c_0 \omega \frac{\partial}{\partial t} \right) \left( \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} \right) \right] = \frac{\partial z}{\partial x_1} + \frac{\partial z}{\partial x_2} , \]

\[ \frac{\partial e_2}{\partial x_1} - \frac{\partial e_1}{\partial x_2} = 0 , \]

\[ e_1 - \xi_{\|} \frac{\partial h_3}{\partial x_2} = 0 , \]

\[ e_2 + \xi_v \dot{u}_3 + \xi_{\|} \frac{\partial h_3}{\partial x_2} = 0 , \]

\[ \xi \xi_v \dot{u}_3 + \xi_{\|} \frac{\partial h_2}{\partial x_1} - \xi_{\|} \frac{\partial h_1}{\partial x_2} = 0 , \]

\[ \frac{\partial h_1}{\partial x_1} + \frac{\partial h_2}{\partial x_2} = 0 , \]

where \( \beta_1 = c_1 / c_2 \).
5.3 SOLUTION OF THE PROBLEM

We consider the plane wave solutions as

\[ u_j = u_j \exp \left[ \eta x_2 + i \left( \xi x_1 - \xi t \right) \right], \]
\[ e_j = e_j \exp \left[ \eta x_2 + i \left( \xi x_1 - \xi t \right) \right], \]
\[ h_j = h_j \exp \left[ \eta x_2 + i \left( \xi x_1 - \xi t \right) \right], \]
\[ z = z \exp \left[ \eta x_2 + i \left( \xi x_1 - \xi t \right) \right]. \]

Using (5.3.1) in equations (5.2.4), we get

\[
\begin{bmatrix}
  (m^2 - \eta^2) & -m_2 \eta & 0 & 0 & i \beta_1^2 \\
  m_2 \eta & (\beta_1^2 \eta^2 - m_3) & 0 & 0 & \beta_1^2 \eta \\
  0 & -i \xi \epsilon_{\chi} - i \epsilon_{\eta} & i \xi \epsilon_{\xi} & 0 & 0 \\
  0 & 0 & i \xi \eta & 0 & 0 \\
  i \xi \epsilon_{\chi} & -i \xi \eta \epsilon_{\chi} & 0 & 0 & (m_3 - \eta^2)
\end{bmatrix}
\begin{bmatrix}
  \tilde{u}_1 \\
  \tilde{u}_2 \\
  \tilde{h}_1 \\
  \tilde{h}_2 \\
  \tilde{z}
\end{bmatrix} = 0, (5.3.2)
\]
and

\[
\begin{bmatrix}
\varepsilon \varepsilon_v (\eta^2 - m_3) & -R_i \beta_1^2 / \varepsilon_H & 0 & 0 \\
-i \varepsilon_v \chi & 1 & 0 & i \varepsilon_H \xi \\
0 & 0 & 1 & -\varepsilon_H \eta \\
0 & i \xi & -\eta & 0
\end{bmatrix}
\begin{bmatrix}
\tilde{u}_3 \\
\tilde{e}_2 \\
\tilde{e}_1 \\
\tilde{h}_3
\end{bmatrix}
= 0, \quad (5.3.3)
\]

where

\[
C = 1 - i \omega \xi, \quad m_1 = \beta_1^2 (\xi^2 - \chi^2), \quad m_2 = (\beta_1^2 - 1), \quad m_3 = -i R_i \chi \beta_1^2 / \varepsilon_H \varepsilon_H \beta_1^2 \chi^2, \quad m_4 = \xi^2 - i \chi C, \quad \chi = \omega / \omega^* 
\]

Equations (5.3.2) and (5.3.3) have non-trivial solution iff

\[
(\xi^2 - \eta^2) [\beta_1^2 \eta^2 - \eta^4 (m_3 + m_1^2 + \beta_1^2 (m_1 + m_4 - i \xi C \chi))] + \eta^2 \{ m_4 m_2 + m_3 (m_1 + m_4) + \beta_1^2 m_1 (m_4 - i \xi C \chi) - i \xi C \beta_1^2 \chi \} - m_3 (m_1 m_4 - i \xi C \beta_1^2 \chi \xi^2) = 0 
\]

and

\[
\eta^4 - \eta^2 (\xi^2 + m_3 + i \beta_1^2 R_i \chi / \varepsilon_H) + m_3 \xi^2 = 0. \quad (5.3.6)
\]
Let the roots of (5.3.5) and (5.3.6) be $\pm \eta_k, k=1$ to 3, and $\eta_4 = \pm \xi$ and $k=5,6$ respectively. The characteristic functions are
\[
\exp \left[ \pm \eta_k x_2 + i \left( \xi x_1 - \chi t \right) \right], \ k = 1 \text{ to } 6. \tag{5.3.7}
\]

We conclude that to each $\pm \eta_k$, there corresponds a set of constants. The superposition of the particular solutions with each such constant leads to the general solution. We assume the general solution of $\tilde{u}_1$ and $\tilde{u}_3$ as
\[
\tilde{u}_1 = \sum_{k=1}^{6} \left[ a_{1k} \exp(\eta_k x_2) + b_{1k} \exp(\eta_k x_2) \exp i \left( \xi x_1 - \chi t \right) \right], \tag{5.3.8}
\]
\[
\tilde{u}_3 = \sum_{k=5}^{6} \left[ a_{6k} \exp(\eta_k x_2) + b_{6k} \exp(\eta_k x_2) \exp i \left( \xi x_1 - \chi t \right) \right].
\]

The solutions of equations (5.3.5) and (5.3.6) can be expressed in terms of fifty-six coefficients out of which forty-four coefficients can be expressed in terms of twelve coefficients, associated with $\tilde{u}_1$ and $\tilde{u}_3$. Thus, we get

\[
\begin{align*}
\tilde{u}_1 &= \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{bmatrix} \begin{bmatrix}
a_{11} \exp(\eta_1 x_2) \\
\cdot \\
\cdot \\
a_{15} \exp(\eta_5 x_2) \\
\cdot \\
\cdot \\
an_{16} \exp(\eta_6 x_2)
\end{bmatrix} \exp i \left( \xi x_1 - \chi t \right), \\
\tilde{u}_2 &= \begin{bmatrix}
a_{21} & a_{22} & a_{23} & a_{24} & -a_{21} & -a_{22} & -a_{23} & -a_{24}
\end{bmatrix} \begin{bmatrix}
a_{11} \exp(\eta_1 x_2) \\
\cdot \\
\cdot \\
an_{15} \exp(\eta_5 x_2) \\
\cdot \\
\cdot \\
an_{16} \exp(\eta_6 x_2)
\end{bmatrix} \exp i \left( \xi x_1 - \chi t \right), \\
\tilde{h}_1 &= \begin{bmatrix}
a_{31} & a_{32} & a_{33} & 0 & a_{31} & a_{32} & a_{33} & 0
\end{bmatrix} \begin{bmatrix}
a_{11} \exp(\eta_1 x_2) \\
\cdot \\
\cdot \\
an_{15} \exp(\eta_5 x_2) \\
\cdot \\
\cdot \\
an_{16} \exp(\eta_6 x_2)
\end{bmatrix} \exp i \left( \xi x_1 - \chi t \right), \\
\tilde{h}_2 &= \begin{bmatrix}
a_{41} & a_{42} & a_{43} & 0 & -a_{41} & -a_{42} & -a_{43} & 0
\end{bmatrix} \begin{bmatrix}
a_{11} \exp(\eta_1 x_2) \\
\cdot \\
\cdot \\
an_{15} \exp(\eta_5 x_2) \\
\cdot \\
\cdot \\
an_{16} \exp(\eta_6 x_2)
\end{bmatrix} \exp i \left( \xi x_1 - \chi t \right), \\
\tilde{z} &= \begin{bmatrix}
 a_{51} & a_{52} & a_{53} & a_{54} & a_{51} & a_{52} & a_{53} & a_{54}
\end{bmatrix} \begin{bmatrix}
a_{11} \exp(\eta_1 x_2) \\
\cdot \\
\cdot \\
an_{15} \exp(\eta_5 x_2) \\
\cdot \\
\cdot \\
an_{16} \exp(\eta_6 x_2)
\end{bmatrix} \exp i \left( \xi x_1 - \chi t \right). \tag{5.3.9}
\end{align*}
\]
and

\[
\begin{bmatrix}
\tilde{u}_3 \\
\tilde{e}_2 \\
\tilde{e}_1 \\
\tilde{h}_3
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 & 1 \\
a_{75} & a_{76} & a_{75} & a_{76} \\
a_{85} & a_{86} & -a_{85} & -a_{86} \\
a_{95} & a_{96} & +a_{95} & a_{96}
\end{bmatrix}
\begin{bmatrix}
a_{65} \exp(\eta_5 x_2) \\
a_{66} \exp(\eta_6 x_2) \\
b_{65} \exp(\eta_5 x_2) \exp(i x_1 - \lambda t), \\
b_{66} \exp(\eta_6 x_2)
\end{bmatrix}
\tag{5.3.10}
\]

where

\[
a_{2k} = \eta_k (m_1 - n_k^2 + i \xi m_2) / [m_2 \eta_k^2 - i (\beta_1^2 n_k^2 - m_3)] ,
\]

\[
a_{3k} = i \xi \epsilon_v \eta_k a_{2k} / \ell_H (\xi^2 - n_k^2) ; \ a_{34} = 0 ,
\]

\[
a_{4k} = \xi \epsilon_v \eta_k a_{2k} / \ell_H (\xi^2 - n_k^2) ; \ a_{44} = 0 ,
\]

\[
a_{5k} = i \xi \epsilon_v \eta_k (m_1 + i m_2 \xi - n_k^2) / m_2 (m_4 - n_k^2 + \xi \epsilon v \beta_1^2 \xi) ,
\]

and

\[
a_{7k} = \xi \epsilon_v \ell_H (n_k^2 m_3) / R_H \beta_1^2 ,
\]

\[
a_{8k} = i \xi \epsilon_v \ell_H (n_k^2 - m_3) / R_H n_k \beta_1^2 ,
\]

\[
a_{9k} = i \xi \epsilon_v \ell_H (n_k^2 m_3) / R_H n_k^2 \beta_1^2 .
\]

For convenience, we define
\[
\begin{align*}
\bar{a}_{1k} &= \frac{a_k + b_k}{2}; \quad \bar{b}_{1k} = \frac{a_k - b_k}{2}; \quad k = 1 \text{ to } 4, \\
\bar{a}_{6k} &= \frac{a_k + b_k}{2}; \quad \bar{b}_{6k} = \frac{a_k - b_k}{2}; \quad k = 5, 6.
\end{align*}
\] (5.3.13)

Using (5.3.13) into (5.3.8) and dropping the factor \(\exp i (\xi x_1 - \chi t)\), we obtain

\[
\bar{u}_1 = \sum_{k=1}^{4} \left[ a_k \cosh \eta_k x_2 + b_k \sinh \eta_k x_2 \right],
\] (5.3.14)

\[
\bar{u}_2 = \sum_{k=1}^{4} \left[ a_{2k}(a_k \sinh \eta_k x_2 + b_k \cosh \eta_k x_2) \right],
\]

\[
\bar{h}_1 = \sum_{k=1}^{4} \left[ a_{3k}(a_k \cosh \eta_k x_2 + b_k \sinh \eta_k x_2) \right],
\]

\[
\bar{h}_2 = \sum_{k=1}^{4} \left[ a_{4k}(a_k \sinh \eta_k x_2 + b_k \cosh \eta_k x_2) \right],
\]

\[
\bar{z} = \sum_{k=1}^{4} \left[ a_{5k}(a_k \cosh \eta_k x_2 + b_k \sinh \eta_k x_2) \right],
\]

and

\[
\bar{u}_3 = \sum_{k=5}^{6} \left[ a_k \cosh \eta_k x_2 + b_k \sinh \eta_k x_2 \right],
\]

\[
\bar{e}_2 = \sum_{k=5}^{6} \left[ a_{7k}(a_k \cosh \eta_k x_2 + b_k \sinh \eta_k x_2) \right],
\]

\[
\bar{e}_1 = \sum_{k=5}^{6} \left[ a_{8k}(a_k \sinh \eta_k x_2 + b_k \cosh \eta_k x_2) \right],
\]

\[
\bar{h}_3 = \sum_{k=5}^{6} \left[ a_{9k}(a_k \cosh \eta_k x_2 + b_k \sinh \eta_k x_2) \right].
\] (5.3.15)
The non-dimensional boundary conditions required to be satisfied on both the surfaces \( x_2 = \pm L^* \) of the plate, are (after suppressing the dashes)

\[
\frac{\partial \tilde{u}_1}{\partial x_1} - \frac{\partial \tilde{u}_2}{\partial x_2} = 0 , \\
\frac{\partial \tilde{u}_3}{\partial x_2} = 0 , \\
\beta_1^2 \frac{\partial \tilde{u}_2}{\partial x_2} + (\beta_1^2 - 2) \frac{\partial \tilde{u}_1}{\partial x_1} - (\bar{R}_H - \bar{R}_H) \beta_1^2 \frac{\partial \tilde{h}_1}{\partial x_1} - \frac{\partial \tilde{v}_1}{\partial x_1} \beta_1^2 \frac{\partial \tilde{z}_1}{\partial x_1} = 0 ; \\
\frac{\partial \tilde{z}_1}{\partial x_2} = 0 , \\
\tilde{h}_3 = 0 , \\
\tilde{h}_1 + i (\bar{R}_H / \bar{R}_H) \tilde{h}_2 = 0 , \quad (5.3.16) \\
\tilde{h}_1 - i (\bar{R}_H / \bar{R}_H) \tilde{h}_2 = 0 , \quad (5.3.17) \\
\text{where } \bar{R}_H = \tilde{\mu} \frac{H^2}{\rho c_1^2} , \tilde{\mu} \text{ is the permeability of the free space and } L^* = \omega^* L/c_1 .
\]

Using (5.3.14) and (5.3.15) in equations (5.3.16) - (5.3.18) and simplifying, we get

\[
| \alpha_{jk} | = 0 , \text{ for symmetric wave,} \quad (5.3.19)
\]

and

\[
| \beta_{jk} | = 0 , \text{ for antisymmetric wave,} \quad (5.3.20)
\]

\( j, k = 1 \text{ to } 4 . \)
where
\[ \alpha_{1k} = a_{3k} + i \left( R_{H} / \bar{R}_{H} \right) a_{4k} p_k , \]
\[ \alpha_{2k} = (\eta_k + i \xi a_{2k}) p_k , \]
\[ \alpha_{3k} = \delta_k , \]
\[ \alpha_{4k} = \eta_k a_{5k} p_k , \]

and
\[ \beta_{1k} = a_{3k} p_k + i \left( R_{H} / \bar{R}_{H} \right) a_{4k} , \]
\[ \beta_{2k} = \eta_k + i \xi a_{2k} , \]
\[ \beta_{3k} = \delta_k p_k , \]
\[ \beta_{4k} = \eta_k a_{5k} , \]

where
\[ \delta_k = \beta_1^2 a_{2k} \eta_k + i \xi (\beta_1^2 - 2) - (R_{H} - \bar{R}_{H}) \xi \beta_1 a_{3k} - \xi \nu \beta_1^2 \xi a_{5k} , \]
\[ p_k = \tanh \eta_k L . \]
5.4 PARTICULAR CASES

(a) If $\mathcal{C}_0 = 0$, then we have $m_4 = \frac{\sqrt{2}}{2} - i\chi$, equation (5.3.5) provide us

\[ (\xi^2 - \eta^2) \left( \beta_1^2 \eta^2 - \eta^4 \right) \{ m_3 + m_2 + \beta_1^2 (m_1 + m_4 - 1 \{ \xi \chi \}) \} + \eta^2 \{ \\
\]

\[ m_2^2 m_4 + m_3 (m_1 + m_4) + \beta_1^2 m_1 (m_4 - 1 \{ \xi \chi \} - i\chi \beta_1^2 \xi^2 \} \]

\[ - m_3 (m_1 + m_4 - i\chi \beta_1^2 \xi^2 ) = 0, \tag{5.4.1} \]

which agrees with the results of [80].

(b) If there is no thermo-mechanical coupling, i.e. $\xi = 0$, then equation (5.3.5) provide us

\[ (\xi^2 - \eta^2) \left( \beta_1^2 \eta^2 - \eta^4 \right) \{ m_1 \beta_1^2 + m_3 + m_2^2 \} + m_4 m_3 \} = 0, \tag{5.4.2} \]

and $\eta^2 - m_4 = 0$, i.e. $\eta^2 - \frac{\xi^2}{2} + i\chi \mathcal{C} = 0$. \tag{5.4.3}

whereas equation (5.3.6) remains unchanged. Equation (5.4.1) and (5.4.2) represent the magneto-elastic and thermal (T-mode) waves respectively. Therefore, the thermal wave gets decoupled from rest of the motion in case of uncoupled thermoelasticity. In this case the symmetric and skew symmetric waves disappear.

(c) When the strain and thermal fields are uncoupled and the thermal relaxation is absent, i.e. $\xi = 0$ and $\mathcal{C}_0 = 0$, then in this case, from equations (5.4.2) and (5.3.5), we have

\[ m_4 = \frac{\sqrt{2}}{2} - i\chi, \tag{5.4.4} \]
\[(s^2-\eta^2) \left[ \mu_1^2 \eta^4 - \eta^2 \left( m_1 \mu_1^2 + m_2^2 \right) + m_1 m_2 \right] = 0 \tag{5.4.5}\]

and
\[\eta^2 - s^2 + i\chi = 0 \tag{5.4.6}\]

which agrees with the results obtained by [80].

(d) If the magnetic field is absent, i.e. when there is no magnetic coupling, then $H_1 = 0$ and $E_4 = 0$. The equation (5.4.5) reduces to equation of generalised thermoelasticity and the results can easily be worked out for that case.

(e) For small wave length or large frequency, the value of $L^*$ becomes very large and so terms containing hyperbolic tangent become very large and hyperbolic tangent term can be approximated to unity, then symmetric and skew symmetric wave frequency coincide with each other and which provide the surface wave solution for magneto-thermoelastic semi-infinite solid at adiabatic conditions, which agrees with the result obtained by [80].

5.5 CONCLUSIONS

The frequency equations for both symmetric and skew symmetric waves have been obtained by considering the plane wave solutions. For very large values of characteristic frequency, the symmetric and antisymmetric frequency equations coincide with each other and provide the surface wave solution for magneto-thermoelastic semi-infinite solid at adiabatic conditions. When the strain and thermal fields are uncoupled, i.e. in case of uncoupled theory of thermoelasticity
and the thermal relaxation effects are also absent, then the results obtained here agree with the results discussed in section 5.4 [ c.f. particular case(c)]. If there is no thermomechanical coupling, then the thermal wave gets decoupled from rest of the motion in this case the symmetric and antisymmetric waves also disappear. In the absence of the magnetic field, the results obtained are similar to those of generalised theory of thermoelasticity.