CHAPTER - III

MAGNETOTHERMOELASTIC WAVES DUE TO LOADS IN A ROTATING HALF-SPACE

3.1 INTRODUCTION

This chapter deals with the investigations of distribution of deformation, temperature, magnetic field, and stresses in a homogeneous isotropic, thermally and electrically conducting, uniformly rotating elastic half-space, in contact with vacuum. This investigation has been done in the context of generalised theory of thermoelasticity developed by Lord and Shulman [20], by considering the two types of boundary conditions:

(i) A step in stress, and zero temperature change and (ii) an impulsive stress and zero temperature change, acting on the plane boundary of the half-space.

The basic governing equations have been solved by using the Laplace transform with respect to time. Because the "second sound" effects are short lived [50,52], so we have confined our discussions to small time approximations. The qualitative results obtained analytically have been discussed at the various wave-fronts. These results have also been verified numerically and are represented graphically for Carbon Steel [87].
3.2 FORMULATION OF THE PROBLEM

We consider a homogeneous isotropic, thermally and electrically conducting elastic medium initially at uniform temperature $T_0$, in contact with vacuum. We assume that the medium is rotating with uniform angular speed given by $\vec{\omega} = (0, 0, \omega)$ and an initial magnetic field $\vec{H}_0$ acting along $x_3$ - axis. When the medium undergoes dynamical deformation, the two additional terms which do not appear in the non-rotating medium: (i) the time dependent part of the centripetal acceleration $\vec{\omega} \times (\vec{\omega} \times \vec{u})$ and (ii) the Coriolis acceleration $2(\vec{\omega} \times \vec{u})$, where $\vec{u}$ is the displacement vector, will also appear in the governing equations. The simplified linear equations of electrodynamics of slowly moving bodies for a perfectly homogeneous conducting elastic medium are [76]

\[
\begin{align*}
\vec{\nabla} \times \vec{h} &= 4\pi J/c, \\
\vec{\nabla} \times \vec{E} &= \mu_0 \frac{\partial \vec{h}}{\partial t}/c,
\end{align*}
\]

\[\vec{\nabla} \cdot \vec{h} = 0, \quad \vec{E} = -\mu_0 (\vec{u} \times \vec{H}_0)/c, \] \hspace{1cm} (3.2.1)

The basic equations of motion and heat conduction in the linear theory of generalised thermoelasticity are [20]

\[
\begin{align*}
\mu \nabla^2 \vec{u} + (\lambda + \mu) \nabla (\nabla \cdot \vec{u}) + \frac{\mu_0}{4\pi} \left[ (\nabla \times \vec{h}) \times \vec{H}_0 \right] - \gamma \vec{\nabla} \Theta \\
= \rho \left[ \ddot{\vec{u}} + \vec{\omega} \times (\dot{\vec{\omega}} \times \vec{u}) + 2 \vec{\omega} \times \dot{\vec{u}} \right]. \hspace{1cm} (3.2.2)
\end{align*}
\]
and

$$\rho C_v(\dot{\theta}+c_o \dot{\theta})+\gamma \Gamma_o \nabla \cdot (\dot{\theta}+c_o \ddot{\theta})=K \nabla^2 \theta,$$  \hspace{1cm} (3.2.3)

where the symbols and notations used are same as defined in chapter - II. We take $H_o=(0,0,H_2),u_1=u$ and $x_1=x$, then equations (3.2.1) - (3.2.3) become

$$\vec{E} = \frac{\mu_o}{c}(0,\dot{u},0), \quad \vec{h} = -\frac{c_o}{\mu_o} \left(0,0,\frac{\partial E_2}{\partial x}\right), \quad \vec{J} = \frac{c_o}{4\pi} \left(0,\frac{-\partial H_3}{\partial x},0\right),$$ \hspace{1cm} (3.2.4)

$$(\lambda +2\mu+a_o^2) \frac{\partial u}{\partial x^2} = \gamma \frac{\partial \theta}{\partial x} = \rho(\ddot{u}+\Omega^2 u-2 \Omega \dot{u}),$$ \hspace{1cm} (3.2.5)

$$\rho C_v(\dot{\theta}+c_o \dot{\theta})+\gamma \Gamma_o \left(\frac{\partial^2 u}{\partial x \partial t}+c_o \frac{\partial^3 u}{\partial x \partial t^2}\right) = K \frac{\partial \theta}{\partial x^2},$$ \hspace{1cm} (3.2.6)

where $a_o = \sqrt{\mu_o \Omega^2/4\pi \rho}$ is the Alfvén wave velocity.

In vacuum, the system of equations of electrodynamics is expressed as

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right)E_2^0 = 0, \quad \left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right)H_3^0 = 0,$$ \hspace{1cm} (3.2.7)

where $x'=-x$.

The initial conditions are

$$u(x,0)=0, \quad \theta(x,0)=0, \quad \frac{\partial u(x,0)}{\partial x} = 0.$$ \hspace{1cm} (3.2.8)
The components $T_{11}$ and $T_0^{11}$ of Maxwell's stress tensor in elastic medium, in vacuum, and also the stress in the elastic medium, are respectively given by

$$T_{11} = -\mu_0 h_3 h_3/4\Pi, \quad T_0^{11} = -h_3^0 h_3/4\Pi,$$  \hspace{1cm} (3.2.9)

$$c_1 = (\lambda + 2\mu) \frac{du}{dx} - \gamma(\Theta + \Theta_0 \dot{\Theta}).$$  \hspace{1cm} (3.2.10)

The boundary conditions are given by

$$\sigma_{11} + T_{11} - T_0^{11} = f(t),$$
$$E_2 = E_2^0,$$  \hspace{1cm} (3.2.11)
$$\Theta(0, t) = 0,$$

where $f(t)$ is a sufficiently well behaved function of time. We define the quantities

$$\eta = \frac{c_o x}{k}, \quad \xi = \frac{c_o^2 t}{k}, \quad U = \frac{\rho c_o^3 u}{\gamma T_0 k},$$

$$\zeta = \Theta/T_0, \quad \ell = \frac{i^2 T_0}{\rho^2 c_o^2 c_v}, \quad C* = C_o w*,$$

where

$$c_1^2 = (\lambda + 2\mu)/\rho, \quad a_0^2 + c_1^2 = c_o^2, \quad K = K/\rho c_v, \quad w* = \rho c_v c_o^2/K.$$  

Using these quantities in equations (3.2.5) - (3.2.8), we obtain...
\[ \frac{d^2 U}{d \eta^2} - \frac{dZ}{d \tau} - \Omega_0^2 U + 2 \Omega_0 \frac{dU}{d \tau} = 0, \text{ for } \eta > 0, \]  \tag{3.2.12}

\[ \frac{dZ}{d \tau} - \frac{dZ}{d \tau} - \frac{dZ}{d \tau} - \left( \frac{dU}{d \tau \eta} + \frac{dU}{d \eta \tau} \right) = 0, \text{ for } \eta > 0, \]  \tag{3.2.13}

\[ \frac{\partial h_0^0}{\partial \eta^2} - \beta^2 \frac{\partial h_0^0}{\partial \eta^2} = 0, \text{ for } \eta' > 0, \]  \tag{3.2.14}

\[ U(\eta, 0) = 0, Z(\eta, 0) = 0, \quad \frac{\partial U(\eta, 0)}{\partial \eta} = 0, \]  \tag{3.2.15}

where \( \Omega_0 = k \Omega / c_0^2, \eta' = -\eta, \beta = c_0 / c. \)

The boundary conditions (3.2.11) become

\[ \frac{\partial U}{\partial \eta} - Z - \frac{\partial Z}{\partial \tau} + \beta_1 h_0^0 - f(\tau)/\Gamma_0 = 0, \]  \tag{3.2.16}

\[ \beta_2 \frac{\partial U}{\partial \tau} - \frac{\partial h_0^0}{\partial \eta} = 0, \quad \text{at } \eta = \eta' = 0, \]

\[ Z(0, \tau) = 0, \]

where \( \beta_1 = H_3 / 4 \pi \gamma \Gamma_0, \beta_2 = \mu_0 H_3 / \rho c^2, \gamma = -H_3 \frac{\partial U}{\partial x}. \)

We introduce the potential function \( \phi \) defined by

\[ U = \frac{\partial \phi}{\partial \eta}. \]  \tag{3.2.17}
in equations (3.2.12), (3.2.13) and (3.2.15), we obtain

\[ Z(\eta, \xi) = \left( \frac{\partial^2}{\partial \eta^2} - \frac{\partial^2}{\partial \xi^2} + \omega \frac{\partial}{\partial \xi} - \omega^2 \right) \phi, \text{ for } \eta > 0, \]  

(3.2.18)

\[ \frac{2}{\partial \eta^2} - \frac{2}{\partial \xi^2} - \xi \frac{\partial}{\partial \xi} - \xi \frac{\partial}{\partial \xi} - \xi \left( \frac{\partial \phi}{\partial \eta^2} + \xi \frac{\partial}{\partial \eta^2} \right) = 0, \text{ for } \eta > 0 \]  

(3.2.19)

\[ \phi(\eta, 0) = 0, \quad \frac{\partial \phi}{\partial \eta}(\eta, 0) = 0, \quad \frac{\partial^2 \phi}{\partial \eta^2}(\eta, 0) = 0. \]  

(3.2.20)

The boundary conditions (3.2.16), after using (3.2.17), become

\[ \frac{2}{\partial \eta^2} - \frac{2}{\partial \eta^2} - \xi \frac{\partial}{\partial \eta} + \frac{1}{\beta_2} \frac{\partial^2 h_3^0}{\partial \eta^2} - f(\xi) / Y_0 = 0, \]  

\[ \beta_2 \frac{\partial^2 \phi}{\partial \eta^2} - \frac{\partial h_3^0}{\partial \eta} = 0, \]  

\[ Z(0, \xi) = 0. \]  

(3.2.21)

### 3.3 SOLUTION OF THE PROBLEM

We apply Laplace transform defined by

\[ \tilde{f}(s) = \int_0^\infty f(t) e^{-st} \, dt, \]  

(3.3.1)
to equations (3.2.18), (3.2.19), and (3.2.14), we obtain

\[ Z(n, s) = \left( \frac{2}{\eta^2} - s^2 + 2 \eta s - \Omega^2 \right) \phi, \text{ for } \eta > 0, \]  
\[ (\frac{2}{\eta^2} s - \beta^2 s^2) Z = \Omega s(1 + \epsilon s) \phi, \text{ for } \eta > 0, \]

\[ h_3 = A_3 e^{-\beta s}, \text{ for } \eta' > 0. \]  

Applying the Laplace transform to equations (3.2.21), we get

\[ \frac{\partial^2 \phi}{\partial \eta^2} - \Omega s f(s)/\eta = 0, \]
\[ \beta^2 s^2 \phi - \phi = 0, \]
\[ Z(0, s) = 0. \]  

Eliminating \( Z \) from equations (3.3.2) and (3.3.3), we get

\[ \frac{\partial^4 \phi}{\partial \eta^4} = \left[ s(1 + \epsilon + 2 \Omega) + s^2 (1 + \epsilon + \Omega) + \Omega^2 \right] \frac{2}{\eta^2} \phi + \]

\[ \left[ s^2 (1 + \epsilon + 2 \Omega) - 2 \Omega \epsilon \right] \phi + s^2 (2 \Omega^2 - 2 \Omega) \phi = 0, \]  

The general solution of (3.3.6), which vanishes at \( \eta \rightarrow \infty \), is given by
\[ \bar{\Phi}(\eta, s) = A_1 e^{-\lambda_1 \eta} + A_2 e^{-\lambda_2 \eta}, \text{ for } \eta > 0, \quad (3.3.7) \]

where \( \lambda_1 \) and \( \lambda_2 \) are the roots of equation

\[ \lambda^4 - [s(1+\epsilon+2\Omega_o) + s^2(1+\epsilon \Omega_o + \Omega_o^2)] \lambda^2 + [s^3(1+\epsilon \Omega_o - 2\epsilon \Omega_o^2) + s^2(\epsilon \Omega_o^2 - 2 \Omega_o + s \Omega_o^2)] = 0. \quad (3.3.8) \]

From equations (3.3.2) and (3.3.7), we get

\[ \bar{Z}(\eta, s) = A_1 [\lambda_1^2 s^2 + \Omega_o (2s - \Omega_o)] e^{-\lambda_1 \eta} + A_2 [\lambda_2^2 s^2 + \Omega_o (2s - \Omega_o)] e^{-\lambda_2 \eta}, \quad (3.3.9) \]

Using equation (3.3.7) in (3.3.5), we get

\[ A_1 [s^2 (1+\Omega_o^2) - \Omega_o \lambda_1^2 s - (1+\Omega_o^2) \Omega_o (2s - \Omega_o)] + A_2 [s^2 (1+\Omega_o^2) - \Omega_o \lambda_2^2 s - (1+\Omega_o^2) \Omega_o (2s - \Omega_o)] + \beta A_3 = \delta(s) / \gamma T_0, \quad (3.3.9a) \]

\[ \beta_2 s^2 [A_1 \lambda_1 + A_2 \lambda_2] + \beta A_3 = 0, \quad (3.3.10) \]

\[ A_1 [\lambda_1^2 s^2 + \Omega_o (2s - \Omega_o)] + A_2 [\lambda_2^2 s^2 + \Omega_o (2s - \Omega_o)] = 0. \quad (3.3.11) \]
From equations (3.3.9a)-(3.3.11), we obtain

\[
\begin{align*}
A_1 &= \beta \bar{f}(s) \left[ \lambda_2^2 - s^2 + \Omega_o (2s - \Omega_o) / \gamma T_0 \right] s \Delta , \\
A_2 &= \beta \bar{f}(s) \left[ \lambda_1^2 - s^2 + \Omega_c (2s - \Omega_c) \right] / \gamma T_0 s \Delta , \\
A_3 &= \beta \bar{f}(s) \left[ \lambda_1 - \lambda_2 \right] \left[ \lambda_1 \lambda_2 - s^2 + \Omega_o (2s - \Omega_c) \right] / \gamma T_0 \Delta ,
\end{align*}
\]  

(3.3.12)

where

\[
\Delta = (\lambda_1 - \lambda_2) \left[ \beta \bar{f}(s) \left[ \lambda_1^2 - s^2 + \Omega_o (2s - \Omega_o) \right] e^{-\lambda_1^2 s} - \Omega_o (2s - \Omega_o) / \gamma T_0 \right] / \gamma T_0 s \Delta ,
\]  

(3.3.13)

Equations (3.3.12), (3.3.7), (3.3.9), (3.3.4), (3.3.13) and (3.2.17) provide us.

\[
\tilde{Q}(\eta, s) = \beta \bar{f}(s) \left[ \left( \lambda_1^2 - s^2 + \Omega_o (2s - \Omega_o) \right) e^{-\lambda_1^2 s} - \Omega_o (2s - \Omega_o) / \gamma T_0 \right] / \gamma T_0 s \Delta ,
\]  

(3.3.14)

\[
\tilde{Z}(\eta, s) = \beta \bar{f}(s) \left[ \left( \lambda_1^2 - s^2 + \Omega_o (2s - \Omega_o) \right) \left( \lambda_2^2 - s^2 + \Omega_o (2s - \Omega_o) \right) \right] \\
\left[ e^{-\lambda_2 \eta} - e^{-\lambda_1 \eta} \right] / \gamma T_0 \Delta ,
\]  

(3.3.15)

\[
\tilde{h}_3^0(\eta', s) = \beta \bar{f}(s) (\lambda_1 - \lambda_2) \left[ \lambda_1 \lambda_2 - s^2 - \Omega_o (2s - \Omega_o) \right] e^{-\beta \eta' s} / \gamma T_0 \Delta ,
\]  

(3.3.16)
The transformed stresses in vacuum and in the elastic medium are given by

$$\bar{T}(\eta, s) = \beta f(s) \left[ \lambda_1 (\lambda_2 - s^2 + \omega_0 (2s - \omega_o)) \right] e^{-\lambda_1 \eta} / \gamma T_0 s \Delta.$$  

(3.3.17)

$$\bar{r}_{11} = f(s) \left[ \lambda_1 (\lambda_2 - s^2 - \omega_0 (2s - \omega_0)) e^{-\lambda_1 \eta} \right] / \gamma T_0 s \Delta,$$  

(3.3.18)

$$\bar{\sigma}_{11} = \left[ e^{-\lambda_1 \eta} - (1 + \omega_0 s) \right] / s \Delta.$$  

(3.3.19)

3.4 SMALL TIME APPROXIMATIONS:

The dependence of $\lambda_1, \lambda_2$ on $s$ is very complicated due to the damping term in heat conduction equation and hence the Laplace transform is difficult. These difficulties, however, reduce if we use some approximate methods. As the thermal relaxation effects are short lived so we confine our discussions to small time approximations, i.e. we take $s$ large.
Now the roots \( \lambda_1 \) and \( \lambda_2 \) of equation (3.3.8) are given by

\[
\lambda_{1,2} = s v^{-1}_{1,2} + \beta_{1,2} + \Omega_{1,2} (s^{-1}) + 0 (s^{-2}),
\]

where

\[
v_{1,2}^{-1} = (K_2^* \pm \Gamma^{1/2}) \gamma^{1/2}/\gamma_2,
\]

\[
B_{1,2} = \frac{[K_1^* \pm (K_1^* K_2^*-2)/\Gamma^{1/2}] 2 / \sqrt{2} (K_2^* \pm \Gamma^{1/2}) \gamma_{1/2}}{2(K_2^* \pm \Gamma^{1/2}) \gamma_{1/2} / 4 / \sqrt{2} (K_2^* \pm \Gamma^{1/2}) \gamma_{1/2}},
\]

and

\[
K_1^* = (1 + \ell + 2 \Omega o), \quad K_2^* = (1 + \ell o + \ell o),
\]

\[
\Gamma = K_2^* - 4 \ell o = (1 + \ell o + \ell o)^2 - 4 \ell o = (1 + \ell o - \ell o)^2 + 4 \ell o^2,
\]

is a positive quantity. The wave propagating with speed \( v_1 \) will be elastic and that with \( v_2 \) will be thermal wave and also the third one travelling with velocity \( c_0 \) is the Alfvén-acoustic wave (Ch.II) we shall now consider two cases:
CASE-I: NORMAL LOAD AT THE BOUNDARY

The step in stress at the boundary of the half space produces disturbance. In this case we take $f(t) = \sigma_0 \cdot H(t)$ so that $\bar{f}(s) = \sigma_0 / s$, \hspace{1cm} (3.4.7)

where $H(t)$ being Heaviside function. Using equations (3.4.1) and (3.4.7) in equations (3.3.14) to (3.3.19), we obtain.

\[ \phi(\eta, s) = \frac{\sigma_0 \beta}{\gamma_0} \left[ \frac{\sigma'(1-v_1^2)}{v_1^2} \frac{1}{s} + \frac{1}{s^4} \left( \frac{2B_1+2\alpha_0 v_1}{v_1^2} \frac{p'}{v_1} + \frac{1-v_1^2}{v_1^2} Q' \right) \right] \]

\[ + 0(s^{-5}) \cdot e^{-\lambda_2 \eta} \cdot \left\{ \frac{(1-v_2^2)}{v_2^2} \frac{p'}{s^3} + \frac{1}{s^4} \left( \frac{2B_2+\alpha_0 v_2}{v_2^2} + \frac{1-v_2^2}{v_2^2} Q' \right) \right\} \]

\[ + 0(s^{-5}) \cdot e^{-\lambda_2 \eta} \right] , \hspace{1cm} (3.4.8) \]

\[ \tilde{Z}(\eta, s) = \frac{\sigma_0 \beta}{\gamma_0} \left[ \frac{Lp'}{s} + \frac{1}{s^2} \left( MP_1+LQ_1 \right) + 0(s^{-3}) \left[ e^{-\lambda_2 \eta} - e^{-\lambda_1 \eta} \right] \right] \hspace{1cm} (3.4.9) \]

\[ h_3(\eta', s) = \frac{\sigma_0 \beta_2}{\gamma_0} \left[ \frac{P(1-v_1^2)}{v_1 v_2} \right] \frac{1}{s^2} \left\{ \frac{B_1 v_1+B_2 v_2-2 \alpha_0 v_1 v_2}{v_1 v_2} \right\} \]

\[ + 0(s^{-3}) \cdot e^{-\beta \eta'} \right] , \hspace{1cm} (3.4.10) \]

\[ \tilde{U}(\eta, s) = \frac{\sigma_0 \beta}{\gamma_0} \left[ \frac{1-v_2^2}{v_1 v_2^2} \frac{p'}{s^2} + \frac{1}{s^3} \left( \frac{B_1(1-v_2^2)}{v_2^2} + 2 \frac{B_2+\alpha_0 v_2}{v_1 v_2} \right) p' \right] \]

\[ + 0(s^{-4}) \cdot e^{-\lambda_1 \eta} - \left\{ \frac{1-v_2^2}{v_1 v_2^2} \frac{p'}{s^2} + \frac{1}{s^3} \right\} . \]
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\[ T_{11} = \frac{\sigma_0 \beta_2}{4 \pi r T_0} \left[ \left( \frac{1+v_1 v_2}{v_1 v_2} \right) \frac{P}{s} + \frac{1}{s^2} \left( \frac{\beta_1 V_1 + \beta_2 V_2 - 2 \alpha \nu_1 v_2}{v_1 v_2} \right) \right] + \frac{Q(1+v_1 v_2)}{v_1 v_2} + O(s^{-3}) \right] e^{-\beta \eta} s, \quad (3.4.12) \]

\[ \bar{\sigma}_{11} = \sigma_0 \left[ c^p L + (c_o^* LQ' + (L+c_o^* M)P' - \beta_3^2 P'A_1) \right] \frac{1}{s^2} + \frac{1}{s^2} \left( \frac{\beta_1 V_1 + \beta_2 V_2 - 2 \alpha \nu_1 v_2}{v_1 v_2} \right) \right] + \frac{Q(1+v_1 v_2)}{v_1 v_2} + O(s^{-3}) \right] e^{-\beta \eta} s, \quad (3.4.13) \]

where
\[ p = \frac{1}{\beta_1 \beta_2} \left[ 1 + \frac{\beta^2}{\beta_1 \beta_2} \left( \frac{v_1 + v_2}{v_1 v_2} \right)^2 + \frac{2\beta}{\beta_1 \beta_2} \frac{v_1 + v_2}{v_1 v_2} - \frac{\beta^3}{\beta_1 \beta_2} \left( \frac{v_1 + v_2}{v_1 v_2} \right)^3 \right] - \frac{1}{v_1 v_2} + \frac{1}{v_1 v_2} + \frac{\beta^4}{\beta_1 \beta_2} \left( \frac{v_1 + v_2}{v_1 v_2} \right)^4 - \frac{3\beta^2}{\beta_1 \beta_2} \left( \frac{v_1 + v_2}{v_1 v_2} \right)^2 - \frac{\beta}{\beta_1 \beta_2} \left( \frac{v_1 + v_2}{v_1 v_2} \right) \right], \quad (3.4.14) \]
\[ Q = \frac{1}{B_1 B_2} \left[ \frac{2\beta (v_1 + v_2)(B_1 + B_2)}{B_1 B_2^2 v_1^2 v_2^2} v_1 v_2 \right] - \frac{(B_1 v_1 + B_2 v_2)}{v_1 v_2} + 2 \, \omega_o - \frac{\beta}{B_1 B_2} \left( B_1 + B_2 \right) \\
+ \frac{4\beta \, \omega_o (v_1 + v_2)}{B_1 B_2 v_1 v_2} + \frac{2\beta}{B_1 B_2} \left( \frac{(v_1 + v_2)(B_1 v_1 + B_2 v_2) + (B_1 + B_2)v_1 v_2}{v_1^2 v_2} \right) \\
+ \frac{4 \, \omega_o}{v_1 v_2} - \frac{3\beta}{B_1 B_2} \left( \frac{(B_1 v_1 + B_2 v_2)}{v_1 v_2} \right) + \frac{6\beta^2 \omega_o}{B_1 B_2} \left( \frac{v_1 + v_2}{v_1 v_2} \right) \\
+ \frac{2(B_1 v_1 + B_2 v_2)}{v_1 v_2} - \frac{12(v_1 + v_2)}{B_1 B_2} \, \omega_o - \frac{3\beta^2}{B_1 B_2} \left( \frac{(v_1 + v_2)(B_1 v_1 + B_2 v_2)}{v_1^3 v_2^3} \right) \\
+ \frac{2(B_1 + B_2)(v_1 + v_2)}{v_1 v_2} + \frac{4\beta^4}{B_1 B_2} \left( \frac{v_1 + v_2}{v_1 v_2} \right)(B_1 + B_2) - \frac{8\beta^3 \omega_o}{B_1 B_2} \left( \frac{v_1 + v_2}{v_1 v_2} \right)^3 ] \]

\( P' = v_1 v_2 p' (v_2 - v_1), \quad Q' = (v_1 v_2 Q/(v_2 - v_1)) - v_1^2 v_2^2 (B_1 B_2) p'/(v_2 - v_1) \]

\( L = (1 - v_1^2)(1 - v_2^2)/v_1^2 v_2^2 \cdot M = \frac{2(B_2 + \omega_o v_2)(1 - v_1^2)}{v_1 v_2} \)

\[ + \frac{(2B_2 + \omega_o v_2^2)(1 - v_2^2)}{v_1 v_2^2} \quad (3.4.16) \]
\( \beta_3 = \frac{c_1}{c_0}, A_1 = (1-v_2^2)/v_1^2 - v_2^2, A_1' = (1-v_1^2)/v_1^2 v_2^2 \),
\( A_2 = 2 \left\{ \left( B_2 + \frac{\alpha_o v_2}{v_2} \right) v_2 + B_1 (1-v_2^2) v_1 \right\}/v_1^2 v_2^2 \),
\( A_2' = 2 \left\{ \left( B_1 + \frac{\alpha_o v_1}{v_2} \right) v_1 + B_2 (1-v_1^2) v_2 \right\}/v_1^2 v_2^2 \),
\( M' = \{ 8v_1 v_2 (B_1 + \frac{\alpha_o v_1}{v_2}) (B_2 + \frac{\alpha_o v_2}{v_2}) + 4v_1 (1-v_2^2) (B_1 + \frac{\alpha_o v_1}{v_2}) \}
+ 4v_2 (1-v_1^2) (B_2 + \frac{\alpha_o v_2}{v_2}) }/v_1^2 v_2^2 \) \quad (3.4.17)

Inverting the Laplace transforms, equations (3.4.8) to (3.4.13) provide us

\[
\Phi(\eta, c) = \frac{\sigma_o \beta}{T_o} \left[ \left\{ \frac{p' (1-v_1^2)}{v_1^2} \right\} (c-\eta/v_2) H(c-\eta/v_2) \left( \frac{2B_1 + 2\alpha_o v_1}{v_1^2} p' + \frac{1-v_1^2}{v_1^2} Q' \right) \right.

\left. + \frac{2B_2 + 2\alpha_o v_2}{v_2} p' + \frac{1-v_2^2}{v_2^2} \right] (c-\eta/v_2)^2 H(c-\eta/v_2) \left( c-\eta/v_1 \right) H(c-\eta/v_1)
\]

\[
Z(\eta, c) = \frac{\sigma_o \beta}{T_o} \left[ \left\{ p' L \delta(c-\eta/v_2) + (MP' + LQ') H(c-\eta/v_2) \right\} e^{-B_1 \eta} \right.

\left. + p' L \delta(c-\eta/v_1) + (MP' + LQ') H(c-\eta/v_1) \right\} e^{-B_2 \eta} \right] \right\}
\]

(3.4.18)

(3.4.19)
\[ h_3^0(\eta', v_1) = \frac{\sigma_0^{\beta}}{f_T^0} \left[ \frac{P(1+v_1 v_2)}{v_1 v_2} H(\varepsilon-\eta' \beta) + \frac{B_1 v_1 + B_2 v_2 - 2 \alpha \varepsilon v_1 v_2}{v_1 v_2} P + \eta(1+v_1 v_2) \right] \]

\[ (\varepsilon-\eta' \beta) H(\varepsilon-\eta' \beta) \] \hspace{1cm} (3.4.20)

\[ u(\eta, v_1) = \frac{\sigma_0^{\beta}}{f_T^0} \left[ \frac{(1-v_1^2) P'}{v_1 v_2} H(\varepsilon-\eta \beta) + \left( \frac{B_1 (1-v_1^2)}{v_2^2} + \frac{2B_2 + 2 \alpha \varepsilon v_2}{v_1 v_2} \right) P' \right] \]

\[ + \frac{1-v_1^2}{v_2^2} Q(\varepsilon-\eta \beta) H(\varepsilon-\eta \beta) \varepsilon^{-B_1} \eta^{-\frac{P'(1-v_1^2)}{v_1 v_2} H(\varepsilon-\eta \beta)} \]

\[ + \left[ \frac{B_2 (1-v_1^2)}{v_1^2} + \frac{2B_1 + 2 \alpha \varepsilon v_2}{v_1 v_2} \right] P' \varepsilon^{-B_1} \eta^{-\frac{1-v_1^2}{v_1 v_2} Q(\varepsilon-\eta \beta) H(\varepsilon-\eta \beta)} \]

\[ x e^{-B_2 \eta} \] \hspace{1cm} (3.4.21)

\[ T_{11}^0 = -\sigma_0^{\beta h} [P(1+v_1 v_2) H(\varepsilon-\eta' \beta) + \{P(B_1 v_1 + B_2 v_2 - 2 \alpha \varepsilon v_1 v_2) + Q(1+v_1 v_2)\}(\varepsilon-\eta' \beta) H(\varepsilon-\eta' \beta)] / 4 \Pi \gamma \left( T_0 v_1 v_2 \right) \] \hspace{1cm} (3.4.22)

\[ \sigma_{11} = \sigma_0^{\beta} \left[ \{C^0 p^L + (\varepsilon-\eta v_1) + (C^0 L Q^I + (L+C^0 M) P') p^2 - p^2 A_1) H(\varepsilon-\eta \beta) \right] \]

\[ + \left( (L+C^0 M) + (M+C^0 M') P' - p^2 (P'A_2 + Q'A_1) \right)(\varepsilon-\eta \beta) H(\varepsilon-\eta \beta) \]

\[ e^{-B_1 \eta} \{C^0 p^L + (\varepsilon-\eta v_2) + (C^0 L Q^I + (L+C^0 M) P' - p^2 P'A_1') \} \]

\[ H(\varepsilon-\eta \beta) + (M+C^0 M') P' + (L+C^0 M) Q^I - p^2 (P'A_2 + Q'A_1') \} \] \hspace{1cm} (3.4.23)
CASE II IMPULSIVE LOAD AT THE BOUNDARY

The impulsive load at the boundary of the half-space also produces disturbances. Here we take
\[ f(t) = \sigma_0 \delta(t), \quad \delta(t) \text{ being Dirac delta function,} \]
then
\[ f(s) = \sigma_0. \]  
(3.4.24)

Using equations (3.4.1) and (3.4.24) in equations (3.3.14) to (3.3.19), we obtain

\[
\begin{align*}
\bar{\Phi}(\eta,s) &= \frac{\sigma_0 \beta}{\gamma I_0} \left[ \frac{1-\nu_1^2}{\nu_1^2} \frac{p'}{s^2} + \frac{1}{s^3} \left( \frac{2B_1 + 2\alpha_0 v_1}{v_1} p' + \frac{1-\nu_2^2}{v_2^2} q' \right) ight. \\
&\quad + 0(s^{-4}) e^{-\lambda_2 \eta} - \left\{ \frac{1-\nu_2^2}{v_2^2} \frac{p'}{s^2} + \frac{1}{s^3} \left( \frac{2B_2 + 2\alpha_0 v_2}{v_2} p' \\
&\quad + \frac{1-\nu_2^2}{v_2^2} q' \right) \right\} e^{-\lambda_1 \eta} \biggr] \biggr], \\
\bar{Z}(\eta,s) &= \frac{\sigma_0 \beta}{\gamma I_0} \left[ \left( \frac{1}{s} + 0(s^{-2}) \right) \left[ e^{-\lambda_2 \eta} - e^{-\lambda_1 \eta} \right] \right], \\
\bar{\Sigma}_0(\eta',s) &= \frac{\sigma_0 \beta_2}{\gamma I_0} \left[ \frac{p(1+\nu_1 v_2)}{v_1 v_2} + \frac{1}{s^3} \left( \frac{B_1 (1+\nu_1 v_2) + B_2 (1+\nu_2 v_2)}{v_1 v_2} \right) \\
&\quad + 0(s^{-2}) \right] \times e^{-\beta \eta' s}, \\
\bar{U}(\eta,s) &= \frac{\sigma_0 \beta}{\gamma I_0} \left[ \left( \frac{1-\nu_1^2}{v_1^2} \frac{p'}{s} + \frac{1}{s^2} \left( \frac{B_1 (1-\nu_2^2)}{v_2^2} + \frac{2B_2 + 2\alpha_0 v_2}{v_1 v_2} \right) p' \right) \right].
\end{align*}
\]
The values of $P, P', Q, Q', \beta_3, \beta, A_1, A_1', A_2, A_2'$, and $M'$ are given by equations (3.4.14) to (3.4.17). Applying the inverse Laplace transform to equations (3.4.26) to (3.4.30), we get

$$\phi(\eta, \tau) = \frac{\sigma_0 B}{T_0} \left[ \frac{p'(1-v_2^2)}{v_2^2} (\varepsilon-\eta/v_2) H(\varepsilon-\eta/v_2) + \frac{2B_1+2\varepsilon_0 v_1}{v_1} P' \right]$$
\[
\frac{1-v_1^2}{v_1^2} Q' (c-\eta/v_2)^2 H(c-\eta/v_2) \quad e^{-B_2/2} \{ \frac{p'(1-v_2^2)}{v_2} (c-\eta/v_1) \\
H(c-\eta/v_1) + \left( \frac{2B_2 + 2 \omega_0 v_2}{v_2^2} \right) p' + \frac{1-v_2^2}{v_2^2} Q' (c-\eta/v_1^2) \}
\]
\[
H(c-\eta/v_1) \quad e^{-B_1 \eta} 
\] (3.4.31)

\[
Z(\eta,c) = \frac{\sigma_0^b}{\Gamma_0} \left[ \{ P'L \; (c-\eta/v_2) + (MP' + LQ')H(c-\eta/v_2) \} \; e^{-B_2 \eta} \\
- \{ P'L \; (c-\eta/v_2) + (MP' + LQ')H(c-\eta/v_1) \} \; e^{-B_1 \eta} \right], \quad (3.4.32)
\]

\[
h_3^0(\eta',c) = \frac{\sigma_0^b \omega_2}{\Gamma_0} \left[ \frac{P(1+v_1 v_2)}{v_1 v_2^2} \; G(c-\eta',\beta) \quad \left( \frac{B_1 v_1 + B_2 v_2 - 2 \omega_0 v_1 v_2}{v_1 v_2^2} \right) p + (1+v_1 v_2) Q \right] \\
H(c-\eta'/\beta) \quad \right], \quad (3.4.33)
\]

\[
U(\eta,c) = \frac{\sigma_0^b \beta}{\Gamma_0} \left[ \frac{P'(1-v_2^2)}{v_1 v_2^2} \; H(c-\eta/v_1) + \left[ \frac{B_1 (1-v_2^2)}{v_2^2} + \frac{2B_2 + 2 \omega_0 v_2}{v_1 v_2^2} p' \right] \\
+ \frac{1-v_2^2}{v_2^2} Q' \right] (c-\eta/v_1) H(c-\eta/v_1) \; e^{-B_1 \eta} - \left\{ \frac{p'(1-v_1^2)}{v_1 v_2} \right\} \\
H(c-\eta/v_2) \quad + \left[ \frac{B_2 (1-v_1^2)}{v_1^2} + \frac{2B_1 + 2 \omega_0 v_1}{v_1 v_2^2} p' \right] + \frac{1-v_2^2}{v_2} Q' \right] \\
(c-\eta/v_2) H(c-\eta/v_2) \; e^{-B_2 \eta} \quad \right], \quad (3.4.34)
\]

\[
T_{11}^0 = -\sigma_0^b \omega_3 \left[ P(1+v_1 v_2) \; \delta(c-\eta'/\beta) + \{ P(B_1 v_1 + B_2 v_2 - 2 \omega_0 v_1 v_2) + \\
(1+v_1 v_2) Q \} H(c-\eta'/\beta) \right] /4\Pi \, \gamma \Gamma_0 \, v_1 v_2, \quad (3.4.35)
\]
\[ \sigma_{11} = \sigma_0 \beta \left[ \varepsilon \lambda P' \delta' (c-\eta/v_1) + (\varepsilon \lambda LQ') + (L+c \delta M) P' - \beta^2 P' A_1 \right] \]

\[ \delta (c-\eta/v_1) + \left( (M+c \delta M') P' + (L+c \delta M) Q' - \beta^2 (P' A_2 + Q' A_1) \right) \]

\[ H(c-\eta/v_1) e^{-B^1 \eta} + \left[ -2 \varepsilon \lambda P' \delta' (c-\eta/v_2) + (\beta^2 P' A_1 - c \lambda LQ') - (L+c \delta M) P' \right] \delta (c-\eta/v_2) + (\beta^2 P' A_2 + Q' A_1) - (M+c \delta M') P' - (L+c \delta M) Q' \] \(H(c-\eta/v_2) \} e^{-B^2 \eta} \).

(3.4.36)

3.5 DISCUSSION OF THE RESULTS AT THE WAVE-FRONTs

It is clear from the short time solutions that they consist of three types of waves, i.e. the elastic wave, thermal wave, and the Alfvén–acoustic wave travelling with velocities \( v_1, v_2, \) and \( c_0 \) respectively. The terms containing \( H(c-\eta/v_1) \) represent the contribution of the elastic wave, containing \( H(c-\eta/v_2) \) represent the contribution of the thermal wave, and those containing \( H(c-\eta' \beta) \) represent the contribution of the Alfvén-acoustic wave in the vicinity of their wavefronts \( \eta = v_1 \beta, \eta = v_2 \beta, \) and \( \eta' = c/\beta \) respectively.

In case of normal load, the deformation is found to be continuous but temperature, perturbed magnetic field,
stresses in vacuum as well as in elastic medium are found to be discontinuous. The discontinuities are given by

\[ [Z^+ - Z^-] = -\sigma_0 B \frac{L' P'}{r T_0}, \quad \eta = \nu_2 c \]  
(3.5.1)

\[ [Z^- Z] = \sigma_0 B \frac{L' P'}{r T_0}, \quad \eta = \nu_2 c \]  
(3.5.2)

\[ [h_3^+ - h_3^-] = \sigma_0 B (1 + \nu_1 \nu_2) \frac{P'}{r T_0} \nu_1 \nu_2, \quad \eta' = \epsilon / \beta \]  
(3.5.3)

\[ [T_{11}^+ - T_{11}^-] = -\sigma_0 B H_3 (1 + \nu_1 \nu_2) \frac{P'}{r T_0} \nu_1 \nu_2 4 T_1, \quad \eta' = \epsilon / \beta \]  
(3.5.4)

\[ [\sigma_{11}^+ - \sigma_{11}^-] = \sigma_0 B [\epsilon L Q' + (L + \epsilon M) P' - \beta_3^2 P' A_1] \exp(-B_1 \nu_1 c), \quad \eta = \nu_1 c \]  
(3.5.5)

\[ [\sigma_{11}^+ - \sigma_{11}^-] = \sigma_0 B [\beta_3^2 P' A_1 - \epsilon L Q' - (L + \epsilon M) P'] \exp(-B_2 \nu_2 c), \quad \eta = \nu_2 c \]  
(3.5.6)

In case of impulsive load the deformation, temperature, perturbed magnetic field, stresses in vacuum and in the elastic medium all are found to be discontinuous.
The discontinuities are given by

\[
[U^+-U^-]_{\eta=v_1c} = \sigma_0 \beta (1-v_2^2) P' \exp(-B_1v_1c)/\tau T_0 v_1^2 v_2^2, \quad (3.5.7)
\]

\[
[U^+-U^-]_{\eta=v_2c} = -\sigma_0 \beta (1-v_1^2) P' \exp(-B_2v_2c)/\tau T_0 v_1^2 v_2^2, \quad (3.5.8)
\]

\[
[Z^+-Z^-]_{\eta=v_1c} = -\sigma_0 \beta (MP'+LQ') \exp(-B_1v_1c)/\tau T_0, \quad (3.5.9)
\]

\[
[Z^+-Z^-]_{\eta=v_2c} = \sigma_0 \beta (MP'+LQ') \exp(-B_2v_2c)/\tau T_0, \quad (3.5.10)
\]

\[
[h^+_3-h^-_3]_{\eta'} = c/\beta \sigma_0^B_2 \left[ (B_1v_1+B_2v_2-2v_1v_2) P + (1+v_1v_2) Q \right]/\tau T_0 v_1^2 v_2^2, \quad (3.5.11)
\]

\[
[\tau^+_H+\tau^-_H]_{\eta'} = -\sigma_0 \beta^H_3 \left[ (B_1v_1+B_2v_2-2v_1v_2) P + (1+v_1v_2) Q \right]/4\tau^2 T_0 v_1^2 v_2^2, \quad (3.5.12)
\]

\[
[\sigma^+_{11}-\sigma^-_{11}]_{\eta=v_1c} = \sigma_0 \beta \left[ (M+c_0^* M') P + (L+c_0^* M) + -\beta^2_3 (P'A_2+Q'A_1) \right] \exp(-B_1v_2c), \quad (3.5.13)
\]

\[
[\sigma^+_{11}-\sigma^-_{11}]_{\eta=v_2c} = \sigma_0 \beta \left[ \beta^2_3 (P'A_2+Q'A_1) - (M+c_0^* M') P - (L+c_0^* M) Q' \right] \exp(-B_2v_2c), \quad (3.5.14)
\]
The above discontinuities decay exponentially with time except for perturbed magnetic field and stress in vacuum.

3.6 PARTICULAR CASES

(a) If the medium is non-rotating:

In this case $\Omega = 0$, the results in case of normal load will reduce to those obtained in Chapter-II. The corresponding results for non-rotating medium in case of impulsive load can be deduced by setting $\Omega = 0$.

(b) If $c_0 = 0$, then we have

$$K_1^* = 1 + \epsilon + 2\Omega, \quad K_2^* = 1, \quad \Gamma = 1, \quad v_1 = 1, \quad v_2 \to \infty$$

$$B_1 = (\epsilon + 2\Omega)/2, \quad B_2 \to \infty, \quad D_1 = [2\Omega(\epsilon - 2\epsilon - \Omega) + \epsilon(4 - \epsilon)]/8, \quad D_2 \to \infty$$

It is observed that in case of normal load, the temperature at both the wave fronts and stress in the elastic medium at the thermal wavefront are found to be continuous. The perturbed magnetic field, stress in vacuum at the Alfvén-acoustic wavefront, and stress in the elastic medium at the elastic wavefront experience finite jumps, given by
In case of impulsive load the deformation, temperature, and stress in the elastic medium are found to be continuous at the thermal wave front but the perturbed magnetic field, stress in vacuum at the Alfvén-acoustic wavefront and stress in the elastic medium at the elastic wavefront experience an infinite jumps. The deformation and temperature experience finite jumps at the elastic wavefront, given by

\[ U^+ - U^- = \sigma_0 \beta P \exp \left[ -(\epsilon + 2 \Omega_0) \frac{c}{2} \right] / \gamma T_0, \]

\[ U^+ - U^- = \sigma_0 \beta \left( \epsilon + 4 \Omega_0 \right) P \exp \left[ -(\epsilon + 2 \Omega_0) \frac{c}{2} \right] \gamma T_0, \]

(c) If \( \epsilon_0 = 0, \Omega = 0 \), then we have the case of conventional coupled thermoelasticity in the absence of rotation and thus

\[ K_1^* = 1 + \epsilon_1, K_2^* = 1, \Gamma = 1, v_1 = 1, v_2 \to \infty, B_1 = \epsilon/2, B_2 \to \infty, \]

\[ D_1 = (4 - \epsilon)/9, D_2 \to \infty. \]
In this case the results obtained for normal load agree with those obtained in Chapter-II, except for the stress in vacuum in which the result agrees with case (b), and the stress in the elastic medium has a discontinuity at the elastic wavefront, given by

\[
[s_{11}^+ - s_{11}^-] = \sigma_0 \beta_3 \exp(-\xi \tau / 2) \eta = \nu_1 \tau
\]

In case of impulsive load the results agree with case (b), except the deformation and temperature have discontinuities at the elastic wavefront, given by

\[
[u^+ - u^-] = -\sigma_0 \beta \exp(-\xi \tau / 2) / \gamma T_0 \eta = \nu_1 \tau
\]

\[
[z^+ - z^-] = \sigma_0 \beta \exp(-\xi \tau / 2) / \gamma T_0 \eta = \nu_1 \tau
\]

(d) If \( \xi = 0, \tau = 0, \) and \( \Omega \neq 0, \) then we have

\[
K_1^* = 1 + 2 \Omega_0, \quad K_2^* = 1, \quad \Gamma = 1, \quad \nu_1 = 1, \quad \nu_2 \to \infty, \quad B_1 = \Omega_0,
\]

\[
B_2 \to \infty, \quad D_1 = \Omega_0 (3 - \Omega_0) / 4, \quad D_2 \to \infty.
\]

The results obtained in case of normal load are same as in case (b), except the stress in the elastic medium has a discontinuity, given by

\[
[s_{11}^+ - s_{11}^-] = \sigma_0 \beta_3 \eta = \nu_1 \tau
\]
In case of impulsive load the results obtained are same as in case (b), except the deformation and temperature have discontinuities at the elastic wavefront, given by

\[
[U^+ - U^-] = -\sigma_0^p \frac{\exp (-\tau_o c)}{\gamma T_o}, \quad \eta = v_1 c
\]

\[
[Z^+ - Z^-] = 4\sigma_0^p \frac{\tau_o^2 \rho \exp (-\tau_o c)}{\gamma T_o}, \quad \eta = v_1 c
\]

(e) If \(\xi = 0\), \(\zeta = 0\), and \(\omega = 0\), then we have

\[K_1^* = 1, \ K_2^* = 1, \ \Gamma = 1, \ v_1 = 1, \ v_2 \to \infty, \ B_1 = 0, \ B_2 \to \infty,\]

\[D_1 = 0, \ D_2 \to \infty.\]

In case of normal load the results obtained are same as in case (b), except stress in the elastic medium has a discontinuity, given by

\[
[{\sigma^+_{\eta} - \sigma^-_{\eta}}] = \sigma_0^p \frac{\beta_3^2}{\rho} \eta = v_1 c
\]

In case of impulsive load the results obtained agree with case (b) but temperature in this case also become continuous at the elastic wavefront, and the deformation has a discontinuity at the elastic wavefront, given by

\[
[U^+ - U^-] = -\sigma_0^p \frac{\beta_3^2}{\gamma T_o}, \quad \eta = v_1 c
\]
(f) If the magnetic field is ignored, i.e. $H_3 = 0$, then

$$\beta_1 = 0, \beta_2 = 0.$$ 

It is observed that the perturbed magnetic field $h_3^0 = 0$, which agrees with the result obtained in chapter - II.
## Table II

Physical data of Carbon - Steel [87].

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>( 9.3 \times 10^{10} )</td>
<td>N m(^{-2})</td>
</tr>
<tr>
<td>( \mu )</td>
<td>( 8.4 \times 10^{10} )</td>
<td>N m(^{-2})</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>( 13.2 \times 10^{6} )</td>
<td>deg(^{-1})</td>
</tr>
<tr>
<td>( \mu_0 )</td>
<td>( 1.3 \times 10^{-6} )</td>
<td>Hr m(^{-1})</td>
</tr>
<tr>
<td>( \rho )</td>
<td>( 7.9 \times 10^{3} )</td>
<td>Kg m(^{-3})</td>
</tr>
<tr>
<td>( c )</td>
<td>( 6.4 \times 10^{2} )</td>
<td>J Kg(^{-1}) deg(^{-1})</td>
</tr>
<tr>
<td>( T_0 )</td>
<td>293.1</td>
<td>°K</td>
</tr>
<tr>
<td>( \xi )</td>
<td>0.34</td>
<td>-</td>
</tr>
<tr>
<td>( H_3 )</td>
<td>1.0</td>
<td>A m(^{-1})</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( 747.1 \times 10^{10} )</td>
<td>-</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>( 4.767 \times 10^{10} )</td>
<td>-</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>( 311.479 \times 10^{-25} )</td>
<td>-</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>1.0</td>
<td>-</td>
</tr>
<tr>
<td>( c_0 )</td>
<td>2214.4</td>
<td>J Kg(^{-1}) deg(^{-1})</td>
</tr>
<tr>
<td>( c_1 )</td>
<td>2214.4</td>
<td>J Kg(^{-1}) deg(^{-1})</td>
</tr>
<tr>
<td>( a_0 )</td>
<td>( 3.62 \times 10^{-6} )</td>
<td>J Kg(^{-1}) deg(^{-1})</td>
</tr>
<tr>
<td>( \omega )</td>
<td>8.0</td>
<td>r.p.s.</td>
</tr>
</tbody>
</table>
In this section, the results obtained theoretically in the previous sections, for normal load, are verified numerically for carbon-steel [See Table -II]. The variations of temperature, deformation, and stress in the elastic medium with respect to different values of relaxation times are plotted for various values of epicentral distances and are compared with those obtained in Chapter -II. They in general, first grow and then decay exponentially with distance for various values of time whereas the variations of perturbed magnetic field and stress in vacuum vary linearly.

From figures 3.1 to 3.5 it is observed that all these quantities are less affected in the rotating case than the non-rotating one, with respect to thermal relaxation times. The variations of these quantities with respect to distance follow the same trend as discussed in Chapter-II.
For various values of relaxation time, the graph shows the variation of particle magnetic field at a given time with distance.

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>Field (μT)</th>
<th>Particle Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2</td>
<td>For coating medium</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>For non-coating medium</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>For coating medium</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>For non-coating medium</td>
</tr>
<tr>
<td>25</td>
<td>2</td>
<td>For coating medium</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>For non-coating medium</td>
</tr>
</tbody>
</table>

**Figure 3.1**
Fig. 3.2

Variations of temperature at a given time with distance for various values of relaxation time (Normal Load).

For non-contacting medium.

$0.0 = \theta_0$, $0.1 = \theta_1$

$0.2 = \theta_0$, $0.5 = \theta_1$
FIG. 3.4

Variation of stress in vacuum at a given time with distance

For various values of relaxation time (Normal load).

For rotating medium

For non-rotating medium
Figure 3.5

Variation of stress in elastic medium at a given time with distance for various values of relaxation time (Normal load).

For rotation medium

For non-rotation medium

\[ \sigma = 0, t = 0 \]

\[ \sigma = 0.5, t = 2 \]

\[ \sigma = 0.1, t = 2 \]
3.8 CONCLUSIONS

The short time solutions to the basic equations of the short time solutions to the basic equations consist of three types of waves namely, the elastic wave, thermal wave, and the Alfven - acoustic wave travelling with velocities \( v_1, v_2, \) and \( c_0 \) respectively. In case of normal load the deformation is found to be continuous but temperature, perturbed magnetic field, and stresses in vacuum as well as in the elastic medium are found to be discontinuous. In case of impulsive load all these quantities are found to be discontinuous. And the discontinuities decay exponentially with respect to time except perturbed magnetic field and stress in vacuum.

In case of conventional coupled thermoelectricity the results obtained agree with those in Chapter-II, for normal load. In case of impulsive load the deformation and stress in the elastic medium at the thermal wavefront and temperature at both the wavefronts are found to be continuous; stress in vacuum at the Alfven - acoustic wavefront and stress in the elastic medium at the elastic wavefront experience an infinite jumps. The deformation experience a finite jump at the elastic wavefront. If there is no coupling between the electromagnetic and strain fields, then the perturbed magnetic and stress in vacuum vanish, which agree with the results obtained in Chapter-II
From the comparison of various figures, it is observed that the perturbed magnetic field and stress in vacuum vary linearly with distance at given times for various values of relaxation times whereas the other quantities vary exponentially. These variations are more dominant at $\tau^* = 10$ and the peak values shift towards the higher values of distance for temperature, deformation, and stress in the elastic medium. It is also observed that all these quantities are less affected in the case when the medium is rotating with constant speed.