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CHAPTER - I

INTRODUCTION

1.1. PRELIMINARY

Thermoelasticity deals with the study of thermodynamical system of bodies in equilibrium, whose interactions with the surroundings are limited to mechanical work, heat exchange, and external work.

In general the change of body temperature is not caused only due to the external and internal sources but also by the process of deformations itself. Under normal conditions of heat exchange, the flux produced by deformation gives rise to unsteady heating. In classical theory, the corresponding terms in the field equations; inertia terms in the elastic equation, are neglected due to the fact that the change of temperature is very small. Every non-stationary problem of thermoelasticity is dynamic. However, for small variations of temperature with time, the inertia terms in the elastic equations of motion can be neglected and the problem is treated as quasi-static. This is not true if the temperature undergoes a large and sudden change, such as, sudden heating and cooling of a body. In such cases the inertia terms must be considered in the equation of motion.
The theory of coupling of temperature and the strain fields was postulated by Duhamel [1,2] in 1837 shortly after the formulation of the theory of elasticity. The author derived the equations for the distribution of strains in an elastic medium subjected to temperature gradient and introduced the dilatational term in the thermal conductivity equation but the equation was not on a thermodynamical basis. The work of Boit [3] gave a satisfactory derivation of thermal conductivity equation which includes the dilatation term based on thermodynamics of irreversible processes.

The mutual interactions between the thermoelastic deformations of a solid body and externally applied magnetic field, give rise to the coupled field of magneto-thermoelasticity. The field of electro-thermoelasticity is similarly concerned with the flow of electric current in the conductors undergoing thermoelastic deformations and the interactions between them. Since electric currents also give rise to magnetic fields and vice versa, the combined field is some time known as electromagneto-thermoelasticity. Cagniard [4] discussed the propagation of seismic waves from the earth's mantle to its inner core and suggested that the earth's magnetic field may also be taken into account for explaining certain phenomena concerning these waves.
1.2 COUPLED THEORY OF THERMOELASTICITY

The coupling between thermal and strain fields gives rise to the coupled theory of thermoelasticity. For static problems this coupling vanishes and two fields become independent of each other. Weiner [5], Lesson [6], and Chadwick and Sneddon [7] published their works by taking into account the coupling effects. The latter discussed in detail the influence of volume and thermal changes, coupled with each other, in the form of plane harmonic waves.

Nowacki [8] discussed the propagation of longitudinal waves in an unbounded thermoelastic medium. A list of Nowacki's papers on the coupled theory of thermoelasticity can be found in his monumental books Nowacki [9,10].

Chadwick and Windle [11] studied the effects of heat conduction upon the propagation of Rayleigh waves in a semi infinite elastic solid for (i) when the surface of the solid is maintained at constant temperature and (ii) when the surface is thermally insulated. The same problem has again been considered by Chadwick and Atkin [12].

Roberts [13] obtained the stresses and displacements caused by a delta function temperature or stress line pulse moving with constant speed over the surface of a coupled thermoelastic half-space. Boley and Tolins [14]
have investigated the transient temperature and strain in
a thermoelastically coupled half-space by employing the
Fourier sine and cosine transforms with respect to time.

Chadwick and Seet [15] and Chadwick [16] investigated
the thermoelastic wave propagation in transversely isotropic
heat conducting as well as homogeneous an-isotropic heat
conducting elastic materials respectively. Chattopadhyay,
et.al [17] studied the transient solutions due to (i) a
step input of stress and zero temperature, and (ii) a step
input of temperature and zero stress, on the boundary of
the surface of the cylindrical hole in a transversely
isotropic thermoelastic medium in the context of coupled
thermoelastic theory. Kumar [18] studied the coupled
thermoelastic wave problem of infinitely extended elastic
plate of thickness 'd' subject to initially a state of
axially symmetric hydrostatic tension. The expression for
stresses and temperature for short and long time have been
derived by using the integral transform technique.

1.3 GENERALIZED THEORIES OF THERMOELASTICITY

The basic governing equations of thermoelasticity
in the usual frame work of linear coupled thermoelasticity
consists of the wave type (hyperbolic) equations of motion
and the diffusion type (parabolic) equation of heat
conduction.
It is observed that a part of the solution of the energy equation tends to infinity. This implies that if an isotropic homogeneous elastic medium is subjected to disturbances, the effects in the temperature and thermal or mechanical/displacement fields are felt at an infinite distance from the source of disturbance instantaneously. This implies that a part of the solution has an infinite velocity of propagation, which is physically impossible. To overcome this problem the following two theories have been developed.

1.3.1. **LORD-SHULMAN (L-S) THEORY**

Some researchers such as Kaliski [19], Lord and Shulman [20], Fox [21], Gurtin and Pipkin [22], and Meixner [23] have tried to modify the Fourier law of heat conduction so as to get a hyperbolic differential equation of heat conduction. These works include the time needed for acceleration of the heat flow in the heat conduction equation along with the coupling between the temperature and strain fields. The paradox in the existing coupled theory of thermoelasticity has also been discussed by Boley [24]. This new theory which is named as the 'Generalized Theory of Thermoelasticity,' eliminates the paradox of an infinite velocity of propagation and is based upon the more general linear functional relationship between the heat flow and the temperature gradients.

Lord and Shulman [20] have formulated a generalized dynamical theory of thermoelasticity (here in after called L-S theory) by using a form of the heat conduction
equation which includes the time needed for acceleration of the heat flow. Some researchers such as Ackerman, et.al [25], Guyer and Krumhansl [26], and Akerman and Overtone [27] proved experimentally for solid Helium that thermal waves (second sound) propagating with finite, though quite large, speed also exist although for most of the solids the corresponding 'frequency window' namely range of the frequency of thermal excitations in which thermal waves can be detected, is extremely limited.

Nayfeh and Nasser [28] have used the Maxwell's modified heat conduction equation to study plane harmonic waves in unbounded media as well as Rayleigh's surface waves propagating along a half-space consisting of linearly elastic materials that conduct heat. Singh and Singh [29] studied the generalized thermoelastic longitudinal waves and the temperature field set up due to coupling of the displacement and the temperature fields with heat wave travelling with certain finite velocity, in an unbounded medium.

Nayfeh and Nasser [30] have used the Cagniard De-Hoop [31] method to develop the displacement and temperature fields in a half-space subjected on its free surface an instantaneously applied heat source. Puri [32] studied the properties of two dilatational motions in the
context of generalized thermoelasticity. The exact solution to the frequency equation is obtained and the exact values of the real and imaginary parts of the wave number have been calculated. Sinha and Sinha [33] investigated the reflection of generalized thermoelastic waves from the free boundary of the half-space.

Basu [34] investigated the propagation of waves in an infinite homogeneous isotropic medium having a spherical cavity for the generalized dynamical theory of thermoelasticity after solving the basic field equations, Roy-choudhuri and Sain [35] applied the generalized dynamical theory of thermoelasticity to study the problem of determining the distribution of temperature, deformation, stress, and strain in an infinite isotropic elastic solid having distributed instantaneous heat sources with the help of integral transforms and obtained short time solutions. Mondal [36] obtained the frequency equations corresponding to a thermoelastic plane wave in an infinite thermoelastic plate immersed in an infinite liquid which is kept at uniform temperature, for symmetrical and antisymmetrical vibrations about the vertical axis, taking into account the thermal relaxations.

Banerjee and Pao [37] investigated the propagation of plane harmonic thermoelastic waves in an infinitely
extended anisotropic solids taking into account the thermal relaxations. Dhalivval and Sherief [38] derived the governing equations of generalized thermoelasticity for anisotropic media. They also proved that these equations are unique and also established a variational principle for these equations.

Singh and Sharma [39] studied the propagation of generalized thermoelastic waves in transversely isotropic media. The basic equations have been solved by a general method after decoupling the SH-wave, which is not affected by thermal variations and is independent of the rest of the motion. They found that, in general, there are three distinct waves apart from the SH-wave in transversely isotropic media. The results have been verified numerically and are represented graphically for a single crystal of zinc in the case of waves of assigned frequency.

Sharma and Singh [40] studied the propagation of surface waves in a transversely isotropic thermoelastic half-space taking into account the generalized form of heat conduction equation. They obtained the frequency equations by a general method. They discussed their results numerically and illustrated those graphically.
Sharma and Sidhu [41] discussed the propagation of plane harmonic waves in a homogeneous anisotropic generalized thermoelastic solid. They found that four dispersive wave modes namely, three quasi-elastic (E) and fourth quasi-thermal (T) which is diffusive in coupled thermoelasticity, now becomes wave like with finite velocity of propagation, are possible. They have also obtained the low and high frequency approximations for all the modes.

Sharma [42] studied the disturbance due to a line source in a homogeneous transversely isotropic thermoelastic half-space with one relaxation time. The exact closed algebraic expressions for the displacements and temperature as function of time and horizontal distances, valid for all epicentral distances have been obtained by using a combination of Fourier and Laplace transform technique and by inverting the integrals with the help of Cagniard's method [4].

Sharma and Singh [43] investigated the propagation of plane harmonic waves in a cubic crystals in the context of generalized thermoelastic theories. They found that in general, there are four waves namely, a quasi longitudinal, two quasi-transverse, and a thermal wave, which can propagate in these types of crystals. They verified the results numerically and represented graphically.
Sharma [44] studied the distribution of temperature, displacement, and stress in an infinite homogeneous transversely isotropic elastic solid having a cylindrical hole by taking (i) unit step in stress and zero temperature change and (ii) unit step in temperature and zero stress, at the boundary of the cylindrical hole. The short time solutions have been obtained by using the Laplace transform technique on time.

Sharma [45] discussed the distribution of temperature and displacement and the analysis based on the decoupled field equations of generalized thermoelasticity. The integral transform technique is used to solve the basic equations. The dynamic behaviour of an elastic half-space due to thermal shock on the boundary has been discussed. He considered small time approximations to obtain the solutions.

Massalas and Kalpakidis [46] investigated the propagation of thermoelastic waves in a wave guide in the specified cartesian space in the context of generalized theory of thermoelasticity. The solution of the problem has been expressed in terms of the Lame's scalar and vector potentials and frequency equations have been derived. They have also obtained numerical results for the wave motion characteristic.
Noda and Ashida [47] studied the transient thermoelastic problem for an axisymmetric transversely isotropic infinite solid containing a penny-shaped crack subject to heat absorption and heat exchange through the crack surface. A finite difference formulation based on time variable alone is proposed to solve a three dimensional transient heat conduction equation in orthotropic medium. The thermal stress field is analysed by means of transversely isotropic potential function method.

Anwar and Sherief [48] used the state space approach for the solution of one dimensional coupled thermoelastic problem in generalized thermoelasticity with one relaxation time. The technique is applied to a thermal shock in a half-space and a layered medium.

Noda et.al [49] studied the generalized thermoelastic problems for an infinite solid with a cylindrical hole and an infinite solid with a spherical hole by means of Laplace transform technique. They carried out the numerical calculations for temperature, displacement, and stress under the generalized formulation and compared with those of classical dynamic coupled theory.

1.3.2 GREEN-LINDSAY (G-L) THEORY OF THERMOELASTICITY

Green and Lindsay [50] and Green and Laws [51] have also obtained a generalization of the coupled theory of
thermoelasticity (hereinafter called G-L Theory) like Lord and Shulman theory, which proved that the 'second sound' effects are short lived. Their analysis is based on modified form of entropy production inequality. Green [52] proved the uniqueness of the equations derived by Green and Lindsay and studied the propagation of acceleration waves. The basic differences between the two theories are as under:

i) The L-S theory modifies only the energy equation of the coupled theory by taking into account the time needed for the acceleration of heat flow, whereas the G-L theory modifies both constitutive equations and the energy equation. Accordingly, the L-S theory involves only one relaxation time of the thermoelastic process and the G-L theory involves two relaxation times.

ii) The energy equation of the L-S theory depends both on the strain velocity and strain acceleration where as the corresponding equation of the G-L theory depends only on the strain velocity.

iii) In the linearized case, according to the G-L theory the heat can not propagate with a finite speed unless the stresses depend on the temperature velocity. According to the L-S theory the heat can propagate with a finite speed even though the stresses are independent of the temperature velocity.
Chandrasekhar [53] studied one dimensional disturbances in a half-space due to a thermal impulse on the boundary, based on L-S and G-L theories of generalized thermoelasticity. He deduced the short time solutions by using the Laplace transform technique and analysed the exact discontinuities in the thermal and mechanical fields. He [54] also studied the one dimensional dynamical disturbances in a thermoelastic half-space with plane boundary due to step in strain and step in temperature on the plane boundary in the context of the linearized G-L theory.

Chandrasekhar and Srikantaiah [55] have obtained the uniqueness of solutions, a generalized Hamilton's principle and a reciprocal theorem for dynamical mixed boundary value problems, in the context of linear anisotropic thermoelasticity theory, which predicts a finite speed of propagation of thermal wave.

Chandrasekhar and Srikantaiah [56] have discussed the plane waves in a homogeneous isotropic, unbounded thermoelastic solid rotating with uniform angular velocity in the context of G-L theory. They [57] investigated Rayleigh waves in a half-space with isothermal plane boundary taking into account the surface stresses exerted by the boundary, by employing the G-L theory. These authors [58] also studied the waves of general type propagating in a compressible non-viscous liquid bounded on both sides by two different generalized thermoelastic half-space, respectively.
Sharma [59] investigated the propagation of plane waves in a homogeneous transversely isotropic generalized thermoelastic medium taking into account the thermal relaxations with the help of the theory of algebraic functions. The low and high frequency approximations for the propagation speeds and attenuation coefficients have been obtained for quasi-longitudinal (QL), quasi-transverse (QT) and quasi-thermal (T-mode).

Sharma [60] studied the distribution of temperature and deformation in an homogeneous isotropic elastic solid having distributed instantaneous heat source taking into account the thermal relaxations. He obtained the short time solutions by employing the Laplace transform on time and Fourier transform on space.

Yadaiah and Shukla [61] studied the thermoelastic longitudinal vibrations of an infinite circular cylindrical solid, in terms of potential functions. They also compared the results obtained from conventional theory of thermoelasticity with the results discussed by Chadwick.

Dhaliwal and Roknè [62] investigated the distribution of displacement, stress, and temperature in an isotropic elastic half-space with its plane boundary either held rigidly fixed or stress free and subjected to a sudden temperature increase by considering the two
relaxation times. An approximate small time solutions are obtained by using the Laplace transform. The numerical values of the said distributions are obtained and are represented graphically. A good survey of literature on these theories can be found in Chandersekharaih [63].

1.4 MAGNETO-THERMOELASTICITY

In this field the contribution of the Polish applied mathematicians are of fundamental importance. Kaliski and Petykiewicz [64,65] and Kaliski [66] gave a systematic treatment of the magneto-electro-thermoelastic field equations and those of wave equations of thermoelctromagneto-elasticity. In their work the effects of anisotropy of the medium and also the viscoelastic relaxation phenomena were taken into consideration.

Nowacki [67] gave energy, uniqueness theorems and vibration, formation of magneto-thermoelasticity in linear coupled setting. Kaliski and Nowacki[68] have given the reciprocity theorems of magneto-thermoelasticity for an ideal and real conductors. Kaliski and Nowacki [69] discussed the propagation of discontinuities in one-dimensional problems for ideal as well as real conductors.

Sinha [70] discussed the problems of disturbances in a long thin piezo-electric rod placed in a magnetic field.
Mookerjee[71] studied the magneto-thermoelastic disturbances in a semi infinite bar with a variable cross-section.

Yuan [72] investigated the magneto-thermoelastic stresses in an infinitely long cylindrical conductor carrying a distributed axial current. Nayfeh and Namet [73] studied the electro-magneto-thermoelastic plane waves in solids with thermal relaxations.

Chandrasekharaih [74] investigated the transverse surface waves in a stratum of uniform thickness, bounded on both sides by deep layers of different materials, in the context of magneto-elasticity. He assumed that all the three materials are perfect conductors of electricity and found that the waves exist for all orientations of the initial magnetic field. He also found that the surface wave of the SH-type can be propagated without dispersion as in an isotropic case.

Bhattacharya [75] studied the wave propagation in an infinite, interacting random 'weakly conducting' magneto-viscoelastic medium, assumed to be slightly different from the homogeneous medium. He derived and analysed the dispersion equation to show that up to the first order of randomness is to alter the waves velocity and the attenuation coefficient.
Massalas and Dalmangas [76] investigated the magneto-thermoelastic waves produced by a thermal shock in a perfectly conducting half-space. They obtained the solution in the analytical form which is reduced to the previous results.

Roychoudhuri [77] discussed the propagation of electro-magneto-thermoelastic plane harmonic waves in an unbounded isotropic conducting field when the entire medium rotates with an uniform angular velocity. Chatterjee and Roychoudhuri [78] discussed the magneto-thermoelastic disturbances in a perfectly electrically conducting elastic half-space, in contact with vacuum, due to a thermal shock applied on the plane boundary of the half-space in the context of G-L theory of thermoelasticity.

Carbonaro and Russo [79] obtained the condition on the acoustic tensor of an unbounded electrically conducting body of infinite conductivity. A general domain of influence theorem has been used to achieve a strong uniqueness result.

Paul and Narasimhan [80] studied the electro-magnetic, thermomechanical vibration modes in homogeneous, initially unstressed and isotropic electrically conducting plate in a static magnetic field. They derived the frequency equations for both symmetric and antisymmetric waves and
computed the roots of equations, numerically.

Bhutani and Kumari [81] studied the wave propagation in magneto-thermo-micro-elasticity valid for isotropic, centro-symmetric media with no spin. The characteristics features regarding the interaction of various waves and their dependence on the dissipative processes have been brought out using Whitham's technique.

Verma, et.al [82] discussed the wave propagation in self-reinforced, anisotropic elastic half-space, when the direction of uniform magnetic field is different from the wave propagation. Paul and Muthiyalu [83] studied the free vibrations of an infinite isotropic elastic plate placed in a thermo-magnetic field.

Dass, et.al [84] discussed the solution of the problems of thermoelasticity and magneto-elasticity, by the eigenfunction expansion method. The theory for the solution of an inhomogeneous vector-matrix differential equation has been presented.

Yeh [85] investigated the distributions of the induced magnetic field generated by a line mechanical singularity in the magnetized elastic half plane. The Fourier transform technique has been used to obtain the exact solutions for the generated magnetic inductions due
to various mechanical singularities such as a single force, a dipole, and single couple. The said distributions on the surface were also explained with figures.

1.5 **BASIC EQUATIONS OF MAGNETO-THERMOELASTICITY**

If an electrically conducting anisotropic elastic solid is subjected to a mechanical load as well as a varying magnetic field, the resulting elastic and the electromagnetic fields, are determined by the usual laws of elasticity and the Maxwell's laws respectively. The interaction between these two fields modifies the body force term in the stress equations of motion and also Ohm's law of electro-magnetics. The additional body force term is called the Lorentz's pondermotive force.

The Maxwell's equations governing the electromagnetic field, in the rationalized M.K.S. Units, are \[86\]

\[
\begin{align*}
\text{Curl } \vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t}, \\
\text{Curl } \vec{E} &= -\frac{\partial \vec{B}}{\partial t}, \\
\text{div } \vec{B} &= 0, \\
\text{div } \vec{D} &= \rho, \\
\end{align*}
\]

along with the relations

\[
\begin{align*}
B_i &= \mu_0 H_j, \\
D_i &= \varepsilon_{ij} (E_j + \frac{1}{c} \frac{\partial (\vec{u} \times \vec{E})}{\partial x_j}) - \frac{1}{c} \epsilon_0 (\vec{u} \times \vec{H})_i, \\
\sigma_{ij} &= \varepsilon_{ijkl} e_{kl} - \kappa_{ij} \Theta, \\
\end{align*}
\]
\[ J_i = \eta_{ij} E_j + k_{jk}^{-1} (q_k - \Pi_{ks} J_s) + \eta_{ij} (\vec{u} \times B)_j + \rho_e u_i \]  
(1.5.4)

\[ \zeta \dot{q}_i + q_i = -\kappa_{ij} \Theta_{ij} + \Pi_{ik} J_k, \]  
(modified Fourier Law),

where

\[ \chi_{ij} = E_{ijkl} \chi_{okl}, \quad \alpha_{ij} = (\gamma^2)(u_i, j + u_j, i), \]  
(1.5.5)

\( \vec{H}, \vec{E}, \vec{J}, \vec{D}, \) and \( \vec{B} \) the sum of primary and induced magnetic field, electric field induced by primary magnetic field, induced electric current density, electric displacement vector and magnetic induction respectively; \( \rho_e, \mu_{ij}, E_{ij}, \)

\( K_{ij}, \chi_{ij}, \) and \( \zeta_{0ij} \) be the electric charge density, tensor of magnetic permeability, tensor of thermal conductivity, and tensors of thermal expansion, respectively and \( E_{ijkl}, \lambda_{ij}, \Pi_{ij}, I_{ij}, \) and \( H_0 \) be the tensor of the strain, the temperature field elastic moduli, tensors describing the action of the current intensity on heat flux, tensor connecting the temperature gradient and electric current, and primary magnetic field vector. The most general form of equations of motion and the energy equation in the context of L-S theory, are

\[ \sigma_{ij,j} + \rho_e E_i + (J \times B)_i + F_i = \rho \ddot{u}_i, \]  
(1.5.6)

\[ (K_{ij} \Theta_{ij}) - (\Pi_{ik} J_k)_i = \rho_0 (\dot{\Theta} + c_0 \dot{\Theta}) + \lambda_{ij} + (\Theta_{ij} + c_0 \Theta_{ij}) = Q, \]  
(1.5.7)
where $C_0$ is the thermal relaxation time, $\beta_0$ is a constant which is equal to $\gamma C_E$ in the isotropic case, $Q$ be the energy dissipation term, and $\rho_e E_i + (\tilde{J} \times \tilde{B})_i$ represents the Lorentz's pondermotive force and modify the body force term in the equations of motion.

For an isotropic elastic medium the above equations become

\begin{align}
\text{Curl} \cdot \tilde{H} &= \tilde{J} + \tilde{D}, \quad \text{Curl} \cdot \tilde{E} = -\tilde{B}, \\
\text{div} \tilde{B} &= 0, \quad \text{div} \tilde{D} = \rho_e, \\
\tilde{B} &= \mu_e \tilde{H}, \quad \tilde{D} = \tilde{E} + (\tilde{J} \times \tilde{B}) - \frac{1}{c^2} (\tilde{u} \times \tilde{H}) \quad (1.5.8) \\
\sigma_{ij} &= 2\mu \varepsilon_{ij} + \left(\lambda + \mu_{kk} - \gamma\theta\right) \delta_{ij} - \left(3\lambda + 2\mu\right) \varepsilon_{ij}, \quad (1.5.10) \\
\tilde{J} &= \eta \left(\tilde{E} + (\tilde{J} \times \tilde{B}) - I \cdot \text{grad} \theta - \left(I \cdot (\tilde{C}_0/K) \tilde{\theta} + \rho_e \tilde{u}\right) \quad (1.5.11) \\
\sigma_{ij} + \rho_e E_i + (\tilde{J} \times \tilde{B})_i + F_i &= \rho u_i, \quad (1.5.12) \\
\mathcal{K} \cdot \nabla^2 \theta + \Pi_0 \cdot \text{div} \tilde{J} &= \rho C_E (\tilde{\theta} + \tilde{C}_0 \tilde{\theta}) + \gamma T_0 (\varepsilon_{kk} + C_0 \varepsilon_{kk}) - Q, \quad (1.5.13)
\end{align}

The equations of motion and heat conduction equation in terms of displacements are written as

\begin{align}
\mu \nabla^2 \tilde{u} + (\lambda + \mu) \cdot \text{grad div} \tilde{u} - \gamma \text{grad} \theta + (\tilde{J} \times \tilde{B}) + \rho_e \tilde{E} + \tilde{F} = \rho \tilde{u}, \quad (1.5.14)
\end{align}
and
\[ k \nabla^2 \Theta - \Pi^0 \text{ div } \mathbf{j} = (\dot{\Theta} + C_0 \ddot{\Theta}) + \beta (\dot{e}_{kk} + C_0 \dddot{e}_{kk}) - \mathbf{w}, \quad (1.5.15) \]

where
\[ k = k/p^C e, \quad \beta = \gamma T_0 / \rho^C E, \quad \Pi = \Pi_0 / \rho^C E, \quad \mathbf{w} = \Omega / \rho^C E, \]

and superposed dot denotes time derivatives. In case of

G–L theory the equations of motion and heat conduction are
given as follow,

\[ \mu \nabla^2 \mathbf{u} + (\lambda + \mu) \text{ grad div } \mathbf{u} - \gamma \text{ grad } (\Theta + \kappa \dot{\Theta}) + (\mathbf{J} x \mathbf{B}) + \rho_e \dot{\mathbf{E}} + \mathbf{F} = \rho \dddot{\mathbf{u}}, \quad (1.5.16) \]

and

\[ k \nabla^2 \Theta - \Pi^0 \text{ div } \mathbf{j} = (\dot{\Theta} + \kappa \dddot{\Theta}) + \beta e_{kk} \mathbf{w}. \quad (1.5.17) \]

For homogeneous isotropic, perfectly electrically conducting
magneto-thermoelastic solid, the above equations of motion
and heat conduction, in the absence of body forces and
heat sources, are given by: (a) L–S theory,

\[ \mu \nabla^2 \mathbf{u} + (\lambda + \mu) \text{ grad div } \mathbf{u} - \gamma \text{ grad } (\Theta + \kappa \dot{\Theta}) + (\mathbf{J} x \mathbf{B}) + \rho_e \dot{\mathbf{E}} = \rho \dddot{\mathbf{u}}, \quad (1.5.18) \]

\[ k \nabla^2 \Theta - \Pi^0 \text{ div } \mathbf{j} = (\dot{\Theta} + C_0 \ddot{\Theta}) + \beta (\dot{e}_{kk} + C_0 \dddot{e}_{kk}), \quad (1.5.19) \]

and (b) G–L theory,

\[ \mu \nabla^2 \mathbf{u} + (\lambda + \mu) \text{ grad div } \mathbf{u} - \gamma \text{ grad } (\Theta + \kappa \dot{\Theta}) + (\mathbf{J} x \mathbf{B}) + \rho_e \dot{\mathbf{E}} = \rho \dddot{\mathbf{u}}, \quad (1.5.20) \]

\[ k \nabla^2 \Theta - \Pi^0 \text{ div } \mathbf{j} = (\dot{\Theta} + \kappa \dddot{\Theta}) + \beta e_{kk}. \quad (1.5.21) \]
If the medium is rotating with uniform angular velocity then the governing equations of motions in both respectively the theories are written as:

\[ \mu \nabla^2 \ddot{u} + (\lambda + \mu) \, \text{grad div } \ddot{u} - \gamma \, \text{grad } \dot{\Theta} + (\dot{J}\times B) + \rho_e \ddot{E} \]

\[ = \rho \left[ \dddot{u} + \dddot{x}(\dddot{x} \times \dot{u}) + 2 \dddot{x} \times \ddot{u} \right], \quad (1.5.22) \]

\[ \mu \nabla^2 \ddot{u} + (\lambda + \mu) \, \text{grad div } \ddot{u} - \gamma \, \text{grad } (\Theta + \dot{\Theta}) + (\dot{J}\times B) + \rho_e \ddot{E} \]

\[ = \rho \left[ \dddot{u} + \dddot{x}(\dddot{x} \times \dot{u}) + 2 \dddot{x} \times \ddot{u} \right], \quad (1.5.23) \]

where \( \dddot{x}(\dddot{x} \times \dot{u}) \) and \( 2 \dddot{x} \times \ddot{u} \) be the contributions from the centripetal acceleration and Coriolis acceleration.
1.6 PRESENT WORK

In the second chapter the transient magneto-thermoelastic waves due to (i) a step in stress and (ii) a thermal shock at the plane boundary, in a homogeneous isotropic, thermally and electrically conducting, elastic half-space, in context of generalized theory of thermoelasticity developed by Lord and Shulman [20]. The basic equations have been solved by using the Laplace transform with respect to time. As the 'second sound' effects are short lived, so the discussions have been confined to small time approximations. The results obtained for deformation, temperature, perturbed magnetic field, and stresses, analytically have been discussed at the various wavefronts. These results have also been verified numerically and are represented graphically for carbon steel [87].

In the third chapter the distribution of deformation, temperature, magnetic field, and stresses in a homogeneous isotropic, thermally and electrically conducting, uniformly rotating half-space, in contact with vacuum due to (i) a step in stress and zero temperature change and (ii) an impulse stress and zero temperature, acting on the plane boundary of the half-space, have been investigated in the context of generalized theory of thermoelasticity developed by Lord and Shulman [20].
The basic governing equations have been solved by using the Laplace transform with respect to time. Because the 'second sound' effects are short lived [50, 52], so we confined the discussions to small time approximations. It is found that there are, in general, three characteristic waves, namely, an elastic wave, a thermal wave, and a Alfvén-acoustic wave in the said medium. The qualitative results obtained analytically have been discussed at the various wave fronts. These results have also been verified numerically and are represented graphically for Carbon Steel [87].

In the fourth chapter the distribution of temperature, deformation, and magnetic field in a homogeneous isotropic, thermally and perfectly electrically conducting, elastic half-space, in contact with vacuum, has been investigated in the context of G-L theory [50] of thermoelasticity, by considering two types of boundary conditions: (i) a step in stress and zero temperature change and (ii) a thermal shock and zero stress, acting on the plane boundary of the half-space. The small time solutions have been obtained by employing the Laplace transform technique with respect to time to the basic governing equations.

The fifth chapter deals with the study of the propagation of electromagnetic thermoelastic plane waves in an initially unstressed, homogeneous isotropic, conducting plate under uniform static magnetic field, in the context of
generalized theory of thermoelasticity developed by Lord and Shulman [20]. The frequency equations for both symmetric and skew symmetric waves have been obtained by considering the plane wave solutions. For very large values of characteristic frequency, the symmetric and antisymmetric frequency equations coincide with each other and provide the surface solution for magneto-thermoelastic semi infinite solid at adiabatic conditions. If there is no thermo-mechanical coupling, then the thermal wave gets decoupled from rest of the motion, and in this case the symmetric and skew symmetric waves also vanish. Some particular cases have also been discussed.

In the sixth chapter the displacement, temperature, and stress due to a thermal shock in a homogeneous, transversely isotropic elastic medium with a cylindrical hole have been investigated in the context of generalized theories of thermoelasticity developed by Green and Lindsay [50] as well as Lord and Shulman [20]. The short time solutions have been obtained by using the Laplace transform technique. The temperature and stresses are found to be discontinuous in both the theories. The displacement is found to be continuous in L-S theory but found to be discontinuous in case of G-L theory. The stress experiences a strong discontinuity in case of G-L theory as compare to that in L-S theory. The jumps decay exponentially with respect to time.