Chapter 2

RISE-UP IN NEUTRINO SURVIVAL PROBABILITY

2.1 Introduction

Neutrino physics is passing through a phase of spectacular development. Vast amount of solar and atmospheric neutrino data have been accumulated and the neutrino deficits have been established to be the consequence of non-standard neutrino physics. The neutral current measurements at SNO [1] have, conclusively, established the oscillations of solar neutrinos. After the evidence of terrestrial antineutrino disappearance in a beam of electron antineutrinos reported by KamLAND [2], all other [3] explanations of the solar neutrino deficit can, at best, be just subdominant effects. The analyses of all the available solar and reactor neutrino data including KamLAND and SNO salt phase data have been presented in [4]. The solar neutrino experiments have, already, entered a phase of precision measurements for oscillation parameters. On the other hand, the LMA solution is facing a deeper scrutiny. In fact, the completeness of the LMA solution is being questioned [5] and the scope for some possible subdominant transitions is being explored [6, 7] vigorously. There are, at least, two generic predictions of LMA [6] which point towards life beyond LMA. One of these
is the prediction of a high argon production rate, $Q_{Ar} \approx 3SNU$, for the Homestake experiment which is about 2σ above the observed rate. Another generic prediction of the LMA scenario is the 'spectral upturn' at low energies. Both these predictions of LMA can only be tested in the forthcoming phase of high precision measurements in the solar neutrino experiments and are crucial for confirmation of the LMA solution.

The distortions in the neutrino spectrum are an important factor in resolving the solar neutrino problem. These distortions arise due to the energy dependence of the survival probability as a result of which neutrinos with different energies survive in different proportions leading to distortions in the observed spectrum. Experimentally, the boron neutrinos are the most accessible source for the study of the distortions in the observed spectrum since the SK and SNO detect the boron neutrinos in the small energy bins over a wide energy range. Since, the LMA solution has emerged as a solution of the SNP, the spectrum distortions within the LMA scenario are of paramount importance for the final confirmation of the LMA as a solution of the SNP and, also, for possible physics beyond LMA.

### 2.2 The rise-up in LMA survival probability

The LMA survival probability [8], to a very good approximation, can be written as

$$P = \frac{1}{2} + \frac{1}{2} \cos 2\theta \cos 2\theta_m,$$

where the mixing angle in matter is given by

$$\cos 2\theta_m = \frac{\cos 2\theta - \beta}{\sqrt{(\cos 2\theta - \beta)^2 + \sin^2 2\theta}},$$

and the ratio of matter to vacuum effects 'β' is given by

$$\beta = \frac{2\sqrt{2}G_F N_e E}{\Delta m^2}.$$ 

$E$ is the energy of the neutrino and $N_e$ is the electron number density at the point of maximal boron neutrino production i.e. at $x = r/R_s = 0.05$ where $R_s$ is the solar
radius so that

\[ G_F N_e = 0.4714 \times 10^{-11} eV \]  \hspace{1cm} (2.4)

at this point [9]. The energy dependence of the LMA survival probability \( P \) given by Eq. (2.1) is shown in Fig. 2.1 (dashed line) along with its asymptotic value \( \sin^2 \theta \) (dotted line). The survival probability averaged over the production region of the boron neutrinos [9] has been plotted as a solid line. It can be easily seen that the analytical expression (2.1) is in fairly good agreement with the exact numerical result. The value of \( P \) is slightly increased by averaging over the production region. Moreover, the earth regeneration effects will, also, increase the survival probability only by a small amount. It can be seen that the percentage increase in the survival probability from the earth regeneration effects equals the day-night asymmetry. The expected day-night asymmetry at SNO is about 3% [10]. Thus, Eq. (2.1) is a fairly good approximation to survival probability.
Equation (2.1) can be written as

\[ P = \sin^2 \theta + R, \]  

where

\[ R = \cos 2\theta \cos^2 \theta_m \]

is the rise-up in the survival probability. Obviously, R is always positive and increases with decrease in energy. The survival probability P is an increasing function of both \( \Delta m^2 \) and \( \theta \) in the allowed LMA region in contrast to R which is an increasing function of \( \Delta m^2 \) and a decreasing function of \( \theta \), within this region. The 'rise-up' R becomes zero for maximal mixing. Since, maximal mixing is rejected at 5.4 standard deviations, the 'rise-up' cannot be zero. Hence, a non-zero 'rise-up' is an inescapable consequence of LMA scenario.

Global analysis of the SNO salt phase data along with other solar and reactor neutrino data yields [11]

\[ \Delta m^2 = 7.1^{+1.2}_{-0.6} \times 10^{-5} eV^2, \]  
\[ \theta = 32.5^{+2.4}_{-2.3} \text{ deg}. \]

For these LMA parameters, we have

\[ \sin^2 \theta = 0.289^{+0.038}_{-0.036}, \]  
\[ P = 0.362^{+0.035}_{-0.031}, \]  
\[ R = 0.074^{+0.044}_{-0.035}. \]

It is clear that R is about three standard deviations above zero and is large enough to be measured experimentally.

The value of the survival probability for the boron neutrinos can be calculated from the SNO CC and NC rates using the relation

\[ P = \frac{\phi_{SNO}^{CC}}{\phi_{SNO}^{NC}} \]
where we have assumed transitions into active flavors only. Transitions into sterile neutrinos [12] can be important and will be studied briefly at the end of this chapter and in details in the next chapter. Even though, neither SK nor SNO has reported any statistically significant 'rise-up', one can infer the 'rise-up' at 6.4 MeV from SNO CC/ NC ratio. Since, $\sin^2 \theta$ is constrained by Eq. (2.9), we can constrain $R$ using Eqs. (2.5) and (2.9). In this manner, we can obtain an indirect upper bound on $R$. However, the value of $\theta$ obtained from the global analyses is not model independent as a result of which the value of $R$ obtained in this manner will be model dependent and will be valid only within the LMA scenario. In the following, we use this value of $R$ to further constrain the LMA region allowed by SNO.

The pure $D_2O$ data from SNO [1] gives

$$\phi_{\text{CC}}^{SNO} = 1.76^{+0.108}_{-0.103} \times 10^6 \text{cm}^{-2}\text{s}^{-1},$$

$$\phi_{\text{NC}}^{SNO} = 6.42^{+1.66}_{-1.67} \times 10^6 \text{cm}^{-2}\text{s}^{-1},$$

where the statistical and the systematic errors have been combined in quadratures. From Eq. (2.12), we have

$$P = 0.274^{+0.073}_{-0.073}.$$  

Using the LMA value of $\theta$ and Eq. (2.5), one can obtain

$$R = -0.015^{+0.082}_{-0.081},$$

which is not, significantly, different from zero. However, one can obtain an upper bound on $R$ from Eq. (2.16) viz.

$$R \leq 0.120$$

at 90% C.L.. It may be worthwhile to mention that the NC rate given in Eq. (2.14) has been obtained without any assumptions regarding the energy dependence of the survival probability. If one assumes an undistorted boron neutrino spectrum and, hence, an energy independent survival probability, SNO pure $D_2O$ data gives

$$\phi_{\text{NC}}^{SNO} = 5.09^{+0.637}_{-0.603} \times 10^6 \text{cm}^{-2}\text{s}^{-1}.$$
Using this value instead of the value quoted in Eq. (2.14) would give

\[ P = 0.346^{+0.048}_{-0.046} \]  \hspace{1cm} (2.19)

and

\[ R = 0.057^{+0.061}_{-0.058} \]  \hspace{1cm} (2.20)

in agreement with the LMA values given in Eqs. (2.10) and (2.11). However, the LMA survival probability being energy dependent, the use of the value quoted in Eq. (2.18) for deriving constraints on neutrino parameters will not be internally consistent [13].

The most recent SNO salt phase data [11]

\[ \phi_{CC}^{SNO} - \phi_{NC}^{SNO} = 0.306^{+0.035}_{-0.035} \]  \hspace{1cm} (2.21)

can, also, be used to obtain the new bounds on P and R viz.

\[ P = 0.306^{+0.035}_{-0.035} \]  \hspace{1cm} (2.22)

\[ R = 0.017^{+0.052}_{-0.050} \]  \hspace{1cm} (2.23)

This value of ‘rise-up’ will be used henceforth. The value of P given in Eq. (2.22) is smaller than the mean LMA value by an amount

\[ 0.057^{+0.068}_{-0.056} \]  \hspace{1cm} (2.24)

which is one standard deviation above zero. We shall explore the allowed LMA region to reduce the difference between the LMA values of P, R and their experimental values given by Eqs. (2.22) and (2.23) respectively which imply the following upper bounds on P and R:

\[ P \leq 0.363 \]  \hspace{1cm} (2.25)

\[ R \leq 0.102 \]  \hspace{1cm} (2.26)

at 90\%C.L. As noted earlier, the ‘rise-up’ R becomes smaller for smaller values of \( \Delta m^2 \) and larger values of \( \theta \). However, a larger value of \( \theta \) leads to an increase in the value of P. In fact, the experimental value of P is already greater than the mean LMA
value and cannot be increased further. Hence, we consider the constraints (2.25) and (2.26) on P and R simultaneously. This can be achieved by plotting the constant P and constant R curves in the allowed parameter space. The curves corresponding to 90% C.L. upper bounds on P and R have been plotted in Fig. 2.2 within the LMA parameter space allowed by the SNO. The overlap region below P and R curves is the region of parameter space allowed by the bounds on 'rise-up' and survival probability obtained above. The resulting upper bounds on $\Delta m^2$ and $\theta$ are

$$\Delta m^2 \leq 7.9 \times 10^{-5} eV^2,$$

$$\theta \leq 33.7 \text{ deg},$$

at 90% C.L. Thus, the 'rise-up' in the boron neutrino spectrum can be used to further restrict the neutrino parameter space. In fact, the bound on the 'rise-up' derived from SNO salt-phase data selects lower values of $\Delta m^2$ consistent with the conclusions
reached by Aliani et al. [4] who incorporated the SNO spectrum data in the global analysis. Therefore, the 'pure LMA' scenario will get rejected at more than 90% C.L. if the future precision measurements favor $\Delta m^2 > 7.9 \times 10^{-5} eV^2$. The value of $\Delta m^2$ significantly larger than $7.9 \times 10^{-5} eV^2$ may be regarded as a signature of physics beyond LMA being manifest in the oscillations of solar boron neutrinos. The inclusion of the earth regeneration effects as well as the averaging over the production region will only decrease the value of $\Delta m^2$ and the upper bound mentioned above will, still, remain valid.

The curves $P=0.342$ and $R=0.070$ corresponding to $1.02\sigma$C.L. are also shown in Fig. 2.2 below which there is no overlap. These two curves intersect at

$$\Delta m^2 = 6.5 \times 10^{-5} eV^2,$$

$$\theta = 31.4 \text{ deg}.$$  \(2.29\)

$$2.30$$

For these values of $\Delta m^2$ and $\theta$, the difference between the LMA values of $P$, $R$ and their experimental values (2.22) and (2.23) is the least (about one standard deviation). This can be regarded as the best fit point in the SNO allowed parameter space. The values of $\Delta m^2$ and $\theta$ obtained from the global analyses of all the solar neutrino data [10] are very close to the values obtained here.

The KamLAND 766.3 Ty spectrum data [14] has been combined with the solar neutrino data by several authors [4, 10, 14]. The two main implications of the KamLAND data are the increase in the value of $\Delta m^2$ to $8.3^{+0.40}_{-0.37} \times 10^{-5} eV^2$ and a decrease in the best-fit value of $\theta$ to $31.3^{+1.9}_{-1.3} \text{ deg}$ [14]. The best fit value of $\Delta m^2$ obtained in [14] is larger than the upper bound derived here [Eq. (2.27)]. This can be regarded as an indication of physics beyond LMA.

The inclusion of the earth regeneration effects will increase the value of $P$ and $R$ by only about 3% which is too small as compared to the 'rise-up' (which is about 28% [see Fig. 2.3]). Moreover, the LMA values of $P$ and $R$ are, already, larger than their experimental values. The earth regeneration effect will, therefore, further increase their values enhancing the mismatch between the theory and experiment. This would
Figure 2.3: The same as Fig. 2.1 for the values of $\Delta m^2 = 8.3 \times 10^{-5} eV^2$ and $\theta = 31.3$ degrees.

make the upper bound on $\Delta m^2$ even more restrictive.

For these values of $\Delta m^2$ and $\theta$, $P$ and $R$ will, now, become

$$P = 0.376^{+0.017}_{-0.014},$$  \hspace{1cm} (2.31)

$$R = 0.106^{+0.023}_{-0.025},$$  \hspace{1cm} (2.32)

in place of Eqs. (2.10) and (2.11). The difference of $P$ from its experimental value [Eq. (2.22)] is given by

$$\Delta P = 0.076^{+0.039}_{-0.038}$$  \hspace{1cm} (2.33)

which is $2\sigma$ above zero. Hence, there is considerable difference between the experimental and theoretical values of $P$ in the LMA scenario. Since there are very stringent bounds on the solar antineutrino flux [15] the transitions into antineutrinos can not account for this difference. Hence, we attribute the whole of this difference to the
transitions into sterile neutrinos and obtain [16]

\[ P(\nu_e \to \nu_e) = \frac{x \sin^2 \alpha}{1 - x \cos^2 \alpha}, \] (2.34)

\[ P(\nu_e \to \nu_\mu) = (1 - P(\nu_e \to \nu_e)) \sin^2 \alpha, \] (2.35)

\[ P(\nu_e \to \nu_S) = (1 - P(\nu_e \to \nu_e)) \cos^2 \alpha, \] (2.36)

where

\[ x = \frac{\phi_{SNO}}{\phi_{NC}}, \] (2.37)

and

\[ P(\nu_e \to \nu_\mu) = 1 - P_{LMA}. \] (2.38)

Here, \( \alpha \) is the sterile mixing angle. From these equations, we obtain

\[ P(\nu_e \to \nu_e) = \frac{x (1 - P_{LMA})}{(1 - x)}, \] (2.39)

and

\[ \sin^2 \alpha = 1 - \frac{P_{LMA} - x}{1 - 2x + x P_{LMA}}. \] (2.40)

Using Eq. (2.21) for \( x \) and Eq. (2.31) for \( P_{LMA} \), we obtain

\[ \sin^2 \alpha = 0.861^{+0.091}_{-0.077}, \] (2.41)

and

\[ P(\nu_e \to \nu_e) = 0.275^{+0.055}_{-0.049}. \] (2.42)

The pure sterile solution (\( \sin^2 \alpha = 0 \)) is disfavored at 11.2 standard deviations. From Eq. (2.36), we obtain

\[ P(\nu_e \to \nu_s) = 0.101^{+0.066}_{-0.069}, \] (2.43)

which implies at 3 \( \sigma \) C.L.

\[ P(\nu_e \to \nu_s) \leq 0.299. \] (2.44)

The sterile flux is non-zero at about 1.5 standard deviations. A more elaborate analysis is needed to constrain the sterile component using the approach adopted here and will be performed in Chapter 3.
2.3 Conclusions

In conclusion, the ‘rise-up’ in the boron neutrino spectrum at low energies has been studied within the framework of the LMA scenario. Indirect bounds on the rise-up have been obtained from the available solar neutrino data. These bounds have been used to demonstrate as to how a precision measurement of the rise-up can be used to further constrain the neutrino parameter space allowed by the SNO salt phase data. It is found that the pure LMA solution is sufficient to explain the SNO salt phase data for $\Delta m^2 \leq 7.9 \times 10^{-5}\text{eV}^2$ and $\theta \leq 33.7\text{deg}$ since larger values of $\Delta m^2$ will violate the upper bound on the rise-up. However, the global analyses \cite{4, 10, 14} of the solar neutrino which include the KamLAND data favor a central value of $\Delta m^2$ which violates this upper bound. Consequently, pure LMA solution seems to be disfavored indicating the presence of other subdominant transitions. The theoretical and experimental values of the boron neutrino survival probability in the pure LMA scenario for the most recent LMA parameters differ by two standard deviations. This discrepancy is too large to be explained by the subdominant spin flavor precession (SFP) transitions into antineutrinos. This discrepancy has been attributed to the subdominant transitions into the sterile neutrinos. It is concluded that the sterile neutrino flux in this scenario could be as large as 0.3 times the boron neutrino flux at $3\sigma$ C.L. \cite{17}.

Future solar neutrino experiments \cite{18} like Borexino, GENIUS, Heron, X-mass, LENS, HELLAZ etc. which are aiming to measure low energy solar neutrinos ($E < 1\text{MeV}$) can test the ‘upturn’ with the necessary precision to pass a final judgment.
Bibliography


[18] The information on these experiments can be obtained from the website [http://www.NeutrinoOscillation.org](http://www.NeutrinoOscillation.org).