Chapter 6

PHENOMENOLOGICAL IMPLICATIONS OF A CLASS OF NEUTRINO MASS MATRICES

6.1 Introduction

Mass matrices provide important tools for the investigation of the underlying symmetries and the resulting dynamics. The first step in this direction is the reconstruction of the neutrino mass matrix in the flavor basis. However, the reconstruction results in a large variety of possible structures of mass matrices depending strongly on the mass scale, mass hierarchy and the Majorana phases. However, the relatively weak dependence on some oscillation parameters ($\theta_{23}$ and $\delta$) results in the degeneracy of possible neutrino mass matrices since all the parameters are not known at present.

The mass matrix for Majorana neutrinos contains nine physical parameters including the three mass eigenvalues, three mixing angles and the three CP-violating phases. The two squared-mass differences ($\Delta m_{21}^2$ and $\Delta m_{32}^2$) and the two mixing angles ($\theta_{12}$ and $\theta_{23}$) have been measured in solar, atmospheric, reactor and accelerator experiments. The third mixing angle $\theta_{13}$ and the Dirac-type CP-violating phase $\delta$ are
expected to be measured in the forthcoming neutrino oscillation experiments. The possible measurement of the effective Majorana mass in neutrinoless double $\beta$ decay searches will provide an additional constraint on the remaining three neutrino parameters viz. the neutrino mass scale and two Majorana CP violating phases. While the neutrino mass scale will be determined by the direct beta decay searches and cosmological observations, the two Majorana phases will not be uniquely determined even if the absolute neutrino mass scale is known. Under the circumstances, it is natural to employ other theoretical inputs for the reconstruction of the neutrino mass matrix. Several proposals have been made in the literature to restrict the form of the neutrino mass matrix and to reduce the number of free parameters which include presence of texture zeros [1, 2, 3, 4], requirement of zero determinant [5, 6] and the zero trace condition [7] to name a few. However, the current neutrino oscillation data are consistent only with a limited number of texture schemes [1, 2, 3, 4]. In particular, the current neutrino oscillation data disallow all neutrino mass matrices with three or more texture zeros in the flavor basis. Out of the fifteen possible neutrino mass matrices with two texture zeros, only seven are compatible with the current neutrino oscillation data. The seven allowed two texture zero mass matrices have been classified into three categories. The two class A matrices of the types $A_1$ and $A_2$ give hierarchical neutrino masses. The class B matrices of types $B_1$, $B_2$, $B_3$ and $B_4$ yield a quasi-degenerate spectrum of neutrino masses. The single class C matrix corresponds to inverted hierarchy of neutrino masses. Furthermore, only the mass matrices of class A and C can accommodate maximal 2-3 mixing and are, hence, favored by the data. Thus, only matrices belonging to class A can accommodate maximal 2-3 mixing and normal hierarchy of neutrino masses.

Even though the grand unification on its own does not shed any light on the flavor problem, the Grand Unified Theories (GUTs) provide the optimal framework in which possible solutions to the flavor problem could be embedded. This is because the GUTs predict definite group theoretical relations between the fermion mass matrices. For this purpose, it is useful to find out possible leading order forms of the neutrino mass matrix in a basis in which the charged lepton mass matrix is diagonal.
Such forms of neutrino mass matrix provide useful hints for model building which will eventually shed important light on the dynamics of lepton mass generation and flavor mixing. For example, a phenomenologically favored texture of quark mass matrix has been presented by Gupta and Rajput [8]. In the spirit of quark-lepton similarity, the same texture has been prescribed for the charged lepton and Dirac neutrino mass matrices. The same texture for the right handed neutrino mass matrix in the see-saw mechanism might follow from universal flavor symmetry hidden in a more fundamental theory of mass generation. Thus, the texture zeros in different positions of the neutrino mass matrix, in particular and fermion mass matrices, in general could be consequence of an underlying symmetry. Such universal textures of fermion mass matrices can, theoretically, be obtained in the context of GUTs based on SO(10). The Gupta-Rajput mass matrices are basically of type $A_2$ in the terminology followed in the present work. Thus, the selection of $A_2$ type neutrino mass matrices will have important implications for GUTs. On the other hand, there exist models based on minimal supersymmetric SO(10) [9], for example, which predict neutrino mass matrices of type $A_1$. An interesting way of realizing the textures zeros is the triplet Higgs model based on an abelian discrete symmetry [10] under which the transformation properties of $\nu_2$ and $\nu_3$ get interchanged in neutrino mass matrices of type $A_1$ and $A_2$. In another extension of the Standard Model (SM) with three SU(2) scalar triplets with horizontal symmetry $Z_6$ [11], mass matrices of types $A_1$ and $A_2$ have different transformation properties for SU(2) triplets. Thus, the identification of phenomenologically admissible forms of neutrino mass matrices will have important implications for the possible unification schemes at the GUT scale. Furthermore, neutrino mass matrices of types $A_1$ and $A_2$ will have, significantly, different implications for leptonic CP violation which is a crucial ingredient of leptogenesis to explain the cosmological baryon asymmetry. The sign of correlation between the Jarlskog parameter $J$ and the baryon asymmetry is positive for $A_1$ type mass matrices while the sign of this correlation could be both positive or negative for $A_2$ type mass matrices leading to vastly different leptogenesis scenarios [12] for the two categories of mass matrices.
6.2 Neutrino mass matrices of class A

There are two types of mass matrices in class A which are consistent with the neutrino oscillation experiments. For neutrino mass matrices of type $A_1$

$$m_{ee} = m_{e\mu} = 0$$ (6.1)

whereas

$$m_{ee} = m_{e\tau} = 0$$ (6.2)

for neutrino mass matrices of type $A_2$.

The neutrino mass matrix, in the flavor basis, is given by

$$(m_\nu)_{ij} = (U m^d_\nu U^T)_{ij}; \quad i, j = e, \mu, \tau$$ (6.3)

for Majorana neutrinos where $m^d_\nu = \text{Diag}\{m_1, m_2, m_3\}$ is the diagonal neutrino mass matrix and $U$ is the neutrino mixing matrix which is given by

$$U = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} \begin{pmatrix}
1 & 0 & 0 \\
0 & e^{i\alpha} & 0 \\
0 & 0 & e^{i(\beta+\delta)}
\end{pmatrix}$$ (6.4)

in PDG representation [13]. Here, $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$. For the above parameterization, the $ee, e\mu$ and $e\tau$ elements of the neutrino mass matrix are given by

$$m_{ee} = c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{2i\alpha} + s_{13}^2 m_3 e^{2i\beta},$$ (6.5)

$$m_{e\mu} = c_{13} s_{13} s_{23} e^{i\delta} (e^{2i\beta} m_3 - s_{12}^2 e^{2i\alpha} m_2) - c_{12} c_{23} s_{12} (m_1 - e^{2i\alpha} m_2) - c_{12}^2 s_{13} s_{23} e^{i\delta} m_1$$ (6.6)

and

$$m_{e\tau} = c_{13} s_{13} c_{23} e^{i\delta} (e^{2i\beta} m_3 - s_{12}^2 e^{2i\alpha} m_2) + c_{12} s_{23} s_{12} (m_1 - e^{2i\alpha} m_2) - c_{12}^2 s_{13} c_{23} e^{i\delta} m_1.$$ (6.7)

It will be helpful to note that $m_{e\tau}$ can be obtained from $m_{e\mu}$ by exchanging $s_{23}$ with $c_{23}$ and $c_{23}$ with $-s_{23}$. In other words, the transformation $\theta_{23} \to \theta_{23} + \frac{\pi}{2}$ transforms $m_{e\mu}$.
to $m_{\nu_\tau}$. However, the transformation $\theta_{23} \rightarrow \frac{\pi}{2} - \theta_{23}$ followed by the transformation $\delta \rightarrow \delta + \pi$, also, transforms $m_{\nu_e\mu}$ to $-m_{\nu_\tau}$. Therefore, if $m_{\nu_{e\mu}}$ vanishes for $\theta_{23}$ and $\delta$, then $m_{\nu_\tau}$ vanishes for $\frac{\pi}{2} - \theta_{23}$ and $\delta + \pi$. This symmetry argument clearly shows that the predictions of neutrino mass matrices for types $A_1$ and $A_2$ will be identical for all neutrino parameters except $\theta_{23}$ and $\delta$. The predictions for $\theta_{23}$ and $\delta$ can be parameterized as

\begin{align*}
A_1 & : \theta_{23} = \frac{\pi}{4} + x, \quad \delta = y, \\
A_2 & : \theta_{23} = \frac{\pi}{4} - x, \quad \delta = \pi + y.
\end{align*}

(6.8)

Since, the two squared-mass differences $\Delta m_{12}^2$ and $\Delta m_{23}^2$ are known experimentally from the solar, atmospheric and reactor neutrino experiments, we treat $m_1$ as the free parameter and obtain $m_2$ and $m_3$ from $m_1$ using the following relations:

\begin{equation}
\begin{aligned}
m_2 &= \sqrt{m_1^2 + \Delta m_{12}^2} \\
\end{aligned}
\end{equation}

(6.9)

and

\begin{equation}
\begin{aligned}
m_3 &= \sqrt{m_2^2 + \Delta m_{23}^2}.
\end{aligned}
\end{equation}

(6.10)

Our present knowledge [14] of the oscillation parameters has been summarized below:

\begin{align*}
\Delta m_{12}^2 &= 7.9^{+0.3}_{-0.3} \times 10^{-5} eV^2, \\
\sin^2 \theta_{12} &= 0.31^{+0.03}_{-0.03} \times 10^{-3}, \\
\Delta m_{23}^2 &= 2.2^{+0.6}_{-0.6} \times 10^{-3} eV^2, \\
\sin^2 \theta_{23} &= 0.50^{+0.06}_{-0.05} \times 10^{-3}, \\
\sin^2 \theta_{13} &< 0.012(0.046)
\end{align*}

(6.11)

where the errors are given at 1 and 3 standard deviations. Since, there is only an upper bound on $\theta_{13}$, we treat it as an unknown parameter in the beginning and use the CHOOZ bound given above to constrain the allowed parameter space only at the end. Thus, the element $m_{ee}$ is a function of four unknown parameters viz. $m_1, \alpha, \beta$ and $\theta_{13}$ while the elements $m_{e\mu}$ and $m_{e\tau}$ are the functions of five unknown parameters viz. $m_1, \alpha, \beta, \delta$ and $\theta_{13}$.
The condition $m_{ee} = 0$ [15] has, already, been examined in detail in the last chapter. For $\theta_{13} = 0$, $m_{ee}$ vanishes for $\alpha = 90^\circ$ and $m_1 = s_{12}^2\sqrt{\Delta m_{12}^2 \cos 2\theta_{12}}$. For non-zero $\theta_{13}$, the element $m_{ee}$ can vanish only in the normal hierarchy. The real and imaginary parts of $m_{ee}$ are separately zero if

$$s_{13}^2 = \frac{c_{12}^2 m_1 + s_{12}^2 m_2 \cos 2\alpha}{\mu_1}$$  \hspace{1cm} (6.12)$$

and

$$s_{13}^2 = \frac{s_{12}^2 m_2 \sin 2\alpha}{\mu_2}$$ \hspace{1cm} (6.13)

where, $\mu_1$ and $\mu_2$ are given by

$$\mu_1 = m_1 c_{12}^2 + m_2 s_{12}^2 \cos 2\alpha - m_3 \cos 2\beta,$$ \hspace{1cm} (6.14)

$$\mu_2 = m_2 s_{12}^2 \sin 2\alpha - m_3 \sin 2\beta.$$ \hspace{1cm} (6.15)

The two values of $s_{13}^2$ given in Eq. (6.12) and Eq. (6.13) are equal if

$$\sin 2\beta = -\frac{s_{12}^2 m_2 \sin 2\alpha}{M}$$ \hspace{1cm} (6.16)

and

$$\cos 2\beta = -\frac{c_{12}^2 m_1 + s_{12}^2 m_2 \cos 2\alpha}{M}$$ \hspace{1cm} (6.17)

and the value of $s_{13}^2$ is given by

$$s_{13}^2 = \frac{M}{M + m_3}$$ \hspace{1cm} (6.18)

where

$$M = \sqrt{m_1^2 c_{12}^4 + m_2^2 s_{12}^4 + 2m_1 m_2 c_{12}^2 s_{12}^2 \cos 2\alpha}.$$ \hspace{1cm} (6.19)

Using Eqs. (6.16) and (6.17), the parameters $\mu_1$ and $\mu_2$ defined in Eqs. (6.14) and (6.15) can be written in terms of a single Majorana phase $\alpha$ or $\beta$. We express them as functions of $\beta$:

$$\mu_1 = -(M + m_3) \cos 2\beta,$$

$$\mu_2 = -(M + m_3) \sin 2\beta.$$ \hspace{1cm} (6.20)
Next, we examine the conditions $m_{e\mu} = 0$ and $m_{e\tau} = 0$. For vanishing $\theta_{13}$, $m_{e\mu}$ or $m_{e\tau}$ can only be zero if $m_1 = m_2$ and $\alpha = 90^\circ$ which implies a lower bound on $\theta_{13}$ [4]. Therefore, the angle $\theta_{13}$ will be bounded from below for neutrino mass matrices of class A.

The element $m_{e\mu}$ vanishes if $\text{Re}(m_{e\mu}) = 0$ and $\text{Im}(m_{e\mu}) = 0$ which yields the following two relationships for $s_{13}$:

$$s_{13} = -\frac{c_{12}c_{23}s_{12}}{s_{23}} \frac{m_1 - m_2 \cos 2\alpha}{\mu_1 \cos \delta - \mu_2 \sin \delta}$$

(6.21)

and

$$s_{13} = \frac{c_{12}c_{23}s_{12}}{s_{23}} \frac{m_2 \sin 2\alpha}{\mu_1 \sin \delta + \mu_2 \cos \delta}$$

(6.22)

where the parameters $\mu_1$ and $\mu_2$ are given by Eqs. (6.14) and (6.15). The two values of $s_{13}$ will be equal if

$$\sin \delta = \frac{\mu_1 m_2 \sin 2\alpha - \mu_2 (m_2 \cos 2\alpha - m_1)}{\mu_3 \sqrt{\mu_1^2 + \mu_2^2}},$$

$$\cos \delta = \frac{\mu_2 m_2 \sin 2\alpha + \mu_1 (m_2 \cos 2\alpha - m_1)}{\mu_3 \sqrt{\mu_1^2 + \mu_2^2}}$$

(6.23)

and $s_{13}$ is given by

$$s_{13} = \frac{c_{12}c_{23}s_{12}}{s_{23}} \frac{\mu_3}{\sqrt{\mu_1^2 + \mu_2^2}}$$

(6.24)

where

$$\mu_3 = \sqrt{m_1^2 + m_2^2 - 2m_1 m_2 \cos 2\alpha}.$$  

(6.25)

Therefore, $\delta$ and $s_{13}$ become functions of $m_1$, $\alpha$ and $\beta$ for vanishing $m_{e\mu}$. Similar expressions for $\delta$ and $s_{13}$ can be obtained for vanishing $m_{e\tau}$ from Eqs. (6.23) and (6.24) by applying the transformations $\theta_{23} \to \frac{\pi}{2} - \theta_{23}$ and $\delta \to \delta + \pi$. Therefore, $m_{e\tau}$ vanishes if

$$\sin \delta = -\frac{\mu_1 m_2 \sin 2\alpha - \mu_2 (m_2 \cos 2\alpha - m_1)}{\mu_3 \sqrt{\mu_1^2 + \mu_2^2}},$$

$$\cos \delta = -\frac{\mu_2 m_2 \sin 2\alpha + \mu_1 (m_2 \cos 2\alpha - m_1)}{\mu_3 \sqrt{\mu_1^2 + \mu_2^2}}$$

(6.26)
and $s_{13}$ is, now, given by
\[ s_{13} = \frac{c_{12}s_{23}s_{12}}{c_{23}} \frac{\mu_3}{\sqrt{\mu_1^2 + \mu_2^2}}. \] (6.27)

The transformation $\delta \to \delta + \pi$ keeps $\tan \delta$ unchanged. Equations (6.26) and (6.27) can, also, be derived directly in the same way as we derived Eqs. (6.23) and (6.24).

Now, we examine the implications of vanishing $m_{ee}$ and $m_{e\mu}$ for the neutrino mass matrices of type $A_1$. Since, $\theta_{13}$ must vanish for $\alpha = 90^0$ if $m_{ee} = 0$, and a vanishing $\theta_{13}$ is not allowed by the condition $m_{e\mu} = 0$, the points $\alpha = 90^0$ and $\theta_{13} = 0$ are not allowed for the neutrino mass matrices of type $A_1$.

Substituting the values of $\mu_1$ and $\mu_2$ from Eqs. (6.20) in Eqs. (6.23), we obtain
\[ \sin \delta = -\frac{m_2 \sin 2(\alpha - \beta) + m_1 \sin 2\beta}{\mu_3}, \]
\[ \cos \delta = -\frac{m_2 \cos 2(\alpha - \beta) - m_1 \cos 2\beta}{\mu_3}. \] (6.28)

Eliminating $\beta$ from the Eqs. (6.28) by using the Eqs. (6.16) and (6.17), we obtain
\[ \sin \delta = \frac{m_1 m_2 \sin 2\alpha}{M \mu_3} \] (6.29)
and
\[ \cos \delta = \frac{s_{12}^2 m_2^2 - c_{12}^2 m_1^2 + m_1 m_2 (c_{12}^2 - s_{12}^2) \cos 2\alpha}{M \mu_3}. \] (6.30)

The Dirac-type CP-violating phase $\delta$ can, also, be written in terms of $\beta$ as
\[ \sin \delta = -\frac{m_1 \sin 2\beta}{s_{12}^2 \mu_3}. \] (6.31)

The value $\alpha = 90^0$ corresponds to $\beta = 0^0$ and $\delta = 0^0$. Since, $\alpha = 90^0$ is not allowed, $\beta = 0^0$ and $\delta = 0^0$ are, also, not allowed. Therefore, two texture zero mass matrices of class $A$ are necessarily CP violating. From Eq. (6.24), we obtain
\[ s_{13} = \frac{c_{12} c_{23} s_{12}}{s_{23}} \frac{\mu_3}{M + \mu_3}. \] (6.32)

For the simultaneous existence of texture zeros at the $ee$ and $\mu\mu$ entries, the two relations for $s_{13}$ calculated from the conditions $m_{ee} = 0$ and $m_{e\mu} = 0$ should be consistent with each other. Equations (6.18) and (6.32) give the same value of $s_{13}$ for
\[ M(M + m_3) \tan^2 \theta_{23} = \mu_3 s_{12}^2 c_{12}^2 \] (6.33)
and the value of \( s_{13} \) is given by

\[
s_{13} = \frac{s_{23}^2 M}{c_{12}^2 s_{23}^2 s_{12}^2 \mu_3}.
\] (6.34)

Eq. (6.33) can be solved to obtain \( \alpha \) as a function of \( m_1 \) for the neutrino mass matrices of type \( A_1 \). With this value of \( \alpha \), one can calculate \( \delta \) and \( \theta_{13} \) from Eqs. (6.29), (6.30) and (6.34). Eq. (6.34) can be used to obtain a lower bound on \( \theta_{13} \):

\[
s_{13} > \frac{s_{23}^2 |m_2 s_{12}^2 - m_1 c_{12}^2|}{c_{12}^2 c_{23}^2 s_{12}^2 (m_1 + m_2)}.
\] (6.35)

Similarly, one can show that for neutrino mass matrices of type \( A_2 \), \( \delta \) and \( s_{13} \) are given by

\[
\sin \delta = \frac{m_1 \sin 2\beta}{s_{12}^2 \mu_3}
\] (6.36)

and

\[
s_{13} = \frac{c_{23}^2 M}{c_{12}^2 c_{23}^2 s_{12}^2 \mu_3}.
\] (6.37)

and the condition for the simultaneous existence of two texture zeros is

\[
M(M + m_3) = \mu_3^2 s_{12}^2 c_{12}^2 \tan^2 \theta_{23}.
\] (6.38)

We note the reciprocity between the values of \( \tan^2 \theta_{23} \) in Eqs. (6.33) and (6.38) for mass matrices of types \( A_1 \) and \( A_2 \), respectively. This reciprocity can be, gainfully, used to examine deviations from maximality for the atmospheric mixing angle for matrices of types \( A_1 \) and \( A_2 \) and, hence, to distinguish between them.

At \( \alpha = 90^0 \), we have \( m_1 = s_{12}^2 \sqrt{\Delta m_{12}^2 / \cos 2\theta_{12}} \) and \( \delta + 2\beta = n\pi \) where \( n \) is even for \( A_1 \) and odd for \( A_2 \) type neutrino mass matrices. Although, \( \alpha = 90^0 \) is not allowed, the relation between \( \beta \) and \( \delta \) remains approximately valid even for small deviations about \( \alpha = 90^0 \).

### 6.3 Results and discussion

The numerical analysis for the neutrino mass matrices of type \( A_1 \) is done in the following manner. The oscillation parameters \( \Delta m_{12}^2, \Delta m_{23}^2, s_{12}^2 \) and \( s_{23}^2 \) are varied
within their experimental ranges given in Eqs. (6.11) and \( \alpha \) is calculated as a function of \( m_1 \) by using Eq. (6.33). For these \( (m_1, \alpha) \) values, \( \beta \) is calculated from Eqs. (6.16) and (6.17), \( \delta \) from Eqs. (6.29) and (6.30) and \( \theta_{13} \) from Eq. (6.34). The CHOOZ bound on \( \theta_{13} \) is used to constrain \( m_1 \) and, hence, all the other neutrino parameters. The neutrino mass matrices of type \( A^2 \) can be studied in an analogous manner. The results have been summarized in Table 6.1 at various confidence levels for \( m_1, \alpha, \beta, \theta_{13} \) and Jarlskog rephasing invariant quantity [16]

\[
J = s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^2 \sin \delta. \tag{6.39}
\]

These quantities are the same for mass matrices of types \( A_1 \) and \( A_2 \). It can be seen from Table 6.1 that the 3 \( \sigma \) lower bound on \( \theta_{13} \) is 3.5°. The range for the Majorana-type CP-violating phase \( \beta \) at 1 \( \sigma \) C.L. is found to be \(-59.4° - 59.4°\). However, if the neutrino oscillation parameters are allowed to vary beyond their present 1.2 \( \sigma \) C.L. ranges, the full range for \( \beta \ (-90° - 90°) \) is allowed. As noted earlier, the neutrino mass matrices of type \( A_1 \) and \( A_2 \) differ in their predictions for \( \delta \) and \( \theta_{23} \). At one standard deviation, the allowed range of \( \delta \) is \((-110.8° - 110.8°)\) for type \( A_1 \) and \((69.2° - 290.8°)\) for type \( A_2 \) and the allowed range of \( \theta_{23} \) is \((44.0° - 48.2°)\) for type \( A_1 \) and \((41.8° - 46.0°)\) for type \( A_2 \) [Table 6.2]. Just like \( \beta \), no constraint on \( \delta \) is obtained above 1.2 \( \sigma \) C.L. The neutrino mass matrices of types \( A_1 \) and \( A_2 \) have some overlap in their predictions regarding \( \delta \) and \( \theta_{23} \) at one standard deviation. Therefore, a precise measurement of the oscillation parameters is necessary to distinguish the neutrino mass matrices of types \( A_1 \) and \( A_2 \).

In Fig. 6.1, we depict the allowed values of \( m_1, \alpha \) and \( \beta \) as the correlation plots at one standard deviation. The upper panel shows \( \alpha \) as a function of \( m_1 \) and the

<table>
<thead>
<tr>
<th>C.L.</th>
<th>( m_1 ) (10(^{-3})eV)</th>
<th>( \alpha ) (deg)</th>
<th>( \beta ) (deg)</th>
<th>( \theta_{13} ) (deg)</th>
<th>( J )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1( \sigma ) C.L.</td>
<td>2.8 - 4.2</td>
<td>83.4 - 96.6</td>
<td>-59.4 - 59.4</td>
<td>5.6 - 6.3</td>
<td>-0.011 - 0.011</td>
</tr>
<tr>
<td>2( \sigma ) C.L.</td>
<td>2.0 - 10.0</td>
<td>76.4 - 103.6</td>
<td>-90 - 90</td>
<td>4.6 - 9.8</td>
<td>-0.021 - 0.021</td>
</tr>
<tr>
<td>3( \sigma ) C.L.</td>
<td>1.6 - 16.2</td>
<td>71.2 - 108.8</td>
<td>-90 - 90</td>
<td>3.5 - 12.4</td>
<td>-0.046 - 0.046</td>
</tr>
</tbody>
</table>

Table 6.1: The predictions for neutrino mass matrices of class A.
The lower panel shows $\beta$ as a function of $\alpha$. The large spread in the $(m_1, \alpha)$ plot is due to the errors in the neutrino oscillation parameters. However, there is a very strong correlation between $\alpha$ and $\beta$ as indicated by Eqs. (6.16) and (6.17). As seen earlier, the points $\alpha = 90^0$ and $\beta = 0^0$ are excluded in Fig. 6.1. The two Majorana phases are strongly correlated with each other. Such a correlation between the Majorana phases was noted earlier [3]. However, full ranges ($-90^0$ - $90^0$) are allowed for the two Majorana phases in that analysis [3]. In contrast, we obtain a very narrow range for the Majorana phase $\alpha$ around $90^0$ [Table 6.1]. Similarly, the Majorana phase $\beta$ is, also, constrained if the oscillation parameters are limited to their 1 $\sigma$ ranges. In Fig. 6.2, we depict the correlation plots of $\alpha$ and $\beta$ with $\delta$ for matrices of types $A_1$ (left panel) and $A_2$ (right panel). We, also, show the correlation plots of $\delta$ and $\theta_{23}$ with one another as well as with $\theta_{13}$ in Fig. 6.2. The Dirac-type CP-violating phase $\delta$ is strongly correlated with the Majorana-type CP-violating phases $\alpha$ and $\beta$ [Cf. Eqs. (6.29) and (6.31)]. It can be seen from ($\alpha, \delta$) and ($\beta, \delta$) correlation plots that there are small deviations in the values of $\alpha$ around $90^0$ and the correlation between $\beta$ and $\delta$ is almost linear. The fact that $\delta + 2\beta \simeq 0^0$ for $A_1$ type mass matrices and $\delta + 2\beta \simeq 180^0$ for $A_2$ type mass matrices is apparent from the ($\beta, \delta$) correlation plots given in the left and right panels in Fig. 6.2, respectively. The ($\beta, \delta$) correlation was noted earlier by Xing [3] in its approximate form in a different parameterization. The ($\delta, \theta_{23}$) plots, clearly, illustrate the point that the neutrino mass matrices of types $A_1$ and $A_2$ have different predictions for these variables [Table 6.2] and only a limited region is allowed on the ($\delta, \theta_{23}$) plane. This is contrary to the analysis done by Xing [3] where no constraints on $\delta$ and $\theta_{23}$ have been obtained. In fact, this feature is crucial for distinguishing mass matrices of types $A_1$ and $A_2$ which were found to be degenerate.

<table>
<thead>
<tr>
<th>1 $\sigma$ predictions</th>
<th>$\delta$</th>
<th>$\theta_{23}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$-110.8^0$ - $110.8^0$</td>
<td>$44.0^0$ - $48.2^0$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$69.2^0$ - $290.8^0$</td>
<td>$41.8^0$ - $46.0^0$</td>
</tr>
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</table>

Table 6.2: The predictions for $\delta$ and $\theta_{23}$.
in the earlier analyses [1, 2, 3]. The constraints on $\delta$ and $\theta_{23}$ are very sensitive to the values of $\theta_{13}$. For the values of $\theta_{13}$ smaller than $1\sigma$ CHOOZ bound, the constraints on $\delta$ and $\theta_{23}$ become stronger which can be seen from the ($\theta_{13}, \theta_{23}$) and ($\theta_{13}, \delta$) plots. For example, if $\theta_{13} < 6^0$, then $\theta_{23} > 46^0$ (above maximal) for type $A_1$ and $\theta_{23} < 44^0$ (below maximal) for type $A_2$. It can, also, be seen from the ($\theta_{13}, \theta_{23}$) correlation plot in Fig. 6.2 that the deviation of $\theta_{23}$ from maximality is larger for smaller values of $\theta_{13}$. Therefore, if future experiments measure $\theta_{13}$ below its present $1\sigma$ bound, the neutrino mass matrices of types $A_1$ and $A_2$ will have different predictions for $\delta$ and $\theta_{23}$ with no overlap. However, it would be difficult to differentiate between matrices of types $A_1$ and $A_2$ if $\theta_{13}$ is found to be above its present $1\sigma$ range. As noted earlier, different quadrants for $S$ are selected for neutrino mass matrices of types $A_1$ and $A_2$. It can, also, be seen in Fig. 6.2 that the points $S = 0, \pi$ are ruled out implying that neutrino mass matrices of class A are necessarily CP-violating. This feature is, also, apparent in Fig. 6.3 where we have plotted $J$ as a function of $m_1$ and in Fig. 6.4 (identical for $A_1$ and $A_2$) which depicts $J$ as a function of $\delta$ for types $A_1$ (left panel) and $A_2$ (right panel). The symmetry argument summarized in Eq. (6.8) is consistent with the numerical results presented in Fig. 6.2.

As mentioned earlier, the accuracies of the oscillation parameters required to distinguish between the theoretical predictions of neutrino mass matrices of types $A_1$ and $A_2$ depend on the upper bound on $\theta_{13}$. With the current precision of the oscillation parameters, the neutrino mass matrices of types $A_1$ and $A_2$ can be distinguished at $1\sigma$, $2\sigma$ and $3\sigma$ C.L. for $\theta_{13} < 6.2^0, 5.6^0$ and $5.0^0$ respectively. For $\theta_{13} > 6.2^0$ the predictions of neutrino mass matrices of types $A_1$ and $A_2$ can not be differentiated with the presently available accuracies of the oscillation parameters. Even with the reduction of the errors in the measurement of $\Delta m^2_{23}$ to 6% expected at MINOS [17] and to 2% expected at K2K [18], no significant improvement in the prospects for distinguishing between neutrino mass matrices of types $A_1$ and $A_2$ results [Cf. Table 6.3]. We have assumed the central value of atmospheric mixing angle $\theta_{23}$ to be $45^0$ for the calculations presented in Table 6.3. If the future experiments measure $\theta_{23}$ to be above or below maximality at a good statistical significance, the mass matrices of
Table 6.3: The upper bounds on $\theta_{13}$ at which the predictions of neutrino mass matrices of $A_1$ and $A_2$ differ at various confidence levels for the present and future neutrino experiments.

types $A_1$ and $A_2$ will be distinguishable easily.

If neutrinoless double beta decay searches give positive results and $m_{ee}$ is measured experimentally or the atmospheric/reactor neutrino oscillation experiments confirm inverted hierarchy, the neutrino mass matrices of class A will be ruled out. A generic prediction of this class of models is a lower bound on $\theta_{13}$ [Table 6.1]. If the forthcoming neutrino oscillation experiments measure $\theta_{13}$ below $3.5^\circ$, the neutrino mass matrices of class A will, again, be ruled out. The forthcoming neutrino experiments will definitely probe this range of $\theta_{13}$ and aim at measuring the Dirac type CP violating phase $\delta$ and the deviations of the atmospheric mixing angle from maximality [19]. The results of these experiments will fall in one of the following four categories of phenomenological interest:

1. $\theta_{23} > 45^\circ$ and $-90^\circ < \delta < 90^\circ$,
2. $\theta_{23} < 45^\circ$ and $90^\circ < \delta < 270^\circ$,
3. $\theta_{23} < 45^\circ$ and $-90^\circ < \delta < 90^\circ$,
4. $\theta_{23} > 45^\circ$ and $90^\circ < \delta < 270^\circ$.

If the experiments confirm the first possibility, the neutrino mass matrices of type $A_1$ may explain the data. If the experiments confirm the second possibility, the mass matrices of type $A_2$ may be allowed. In case, the experiments select the third or fourth possibility, the neutrino mass matrices of class A will be ruled out.

<table>
<thead>
<tr>
<th>C.L.</th>
<th>1 $\sigma$</th>
<th>2 $\sigma$</th>
<th>3 $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present data</td>
<td>$\theta_{13} &lt; 6.2^\circ$</td>
<td>$\theta_{13} &lt; 5.6^\circ$</td>
<td>$\theta_{13} &lt; 5.0^\circ$</td>
</tr>
<tr>
<td>MINOS</td>
<td>$\theta_{13} &lt; 6.5^\circ$</td>
<td>$\theta_{13} &lt; 6.0^\circ$</td>
<td>$\theta_{13} &lt; 5.5^\circ$</td>
</tr>
<tr>
<td>K2K</td>
<td>$\theta_{13} &lt; 6.6^\circ$</td>
<td>$\theta_{13} &lt; 6.2^\circ$</td>
<td>$\theta_{13} &lt; 5.7^\circ$</td>
</tr>
</tbody>
</table>
The four cases given above are degenerate for the present neutrino oscillation data because of the octant degeneracy of $\theta_{23}$ (i.e. if $\theta_{23} < 45^0$ or $\theta_{23} > 45^0$) and the intrinsic degeneracy in the sign of $\cos\delta$. The above two two-fold degeneracies combined with the two-fold degeneracy in the sign of $\Delta m^2_{23}$ gives rise to the eightfold degeneracy in the neutrino parameter space which has been studied extensively in the literature [20, 21, 22, 23]. A specific project named Tokai-to-Kamioka-Korea (T2KK) two-detector complex which will receive neutrino superbeams from J-PARC [23] has been proposed to resolve these degeneracies. The intrinsic degeneracy in the sign of $\cos\delta$ will be resolved by the spectrum information at T2KK while the $\theta_{23}$ octant degeneracy will be resolved by observing the difference in $\Delta m^2_{12}$ oscillation effects between the intermediate and far detectors at T2KK. However, we find that the degeneracies in the neutrino parameter space are inextricably linked to the degeneracies in the neutrino mass matrix and by finding the octant of $\theta_{23}$ and sign of $\cos\delta$, the future experiments will be able to settle the fate of neutrino mass matrices of types $A_1$ and $A_2$ [24].

If the neutrino mass matrices of class A are confirmed experimentally, this would correlate the CP-violation induced by the Dirac phase $\delta$ (which can be observed in the lepton number conserving processes) with the CP-violation induced by Majorana phases (which can be observed in the lepton number violating processes) in a definite way [24].

### 6.4 Conclusions

In conclusion, the degeneracy between $A_1$ and $A_2$ type neutrino mass matrices has been examined in detail. In the earlier analyses [1, 2, 3], textures within a class were found to be experimentally indistinguishable. However, our analysis shows that the knowledge of the deviations of the atmospheric mixing from the maximality and of the quadrant of the Dirac-type CP-violating phase $\delta$ can be used to lift this degeneracy between mass matrices of types $A_1$ and $A_2$. Contrary to the results of earlier
analyses [3], which leave the Dirac phase completely unconstrained, we have obtained interesting constraints not only for the Dirac phase but, also, for the Majorana-type CP-violating phases. Our analysis, also, shows that the prospects for the measurement of $\theta_{13}$ for neutrino mass matrices of class A are quite optimistic. Moreover, the neutrino mass matrices of class A connect the deviation of $\theta_{23}$ from maximality with the quadrant of $\delta$. Similarly, the CP-violation observed in the neutrino oscillation experiments is linked with the CP-violation induced by Majorana phases [24].
Figure 6.1: Correlation plots for neutrino mass matrices of class A at one standard deviation.
Figure 6.2: Correlation plots for neutrino mass matrices of types $A_1$ (left panel) and $A_2$ (right panel).
Figure 6.3: $J$ as a function of $m_1$ for the neutrino mass matrices of class A.

Figure 6.4: $J$ as a function of $\delta$ for the neutrino mass matrices of types $A_1$ (left panel) and $A_2$ (right panel).
Bibliography


