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Strong self-focusing of a cosh-Gaussian laser beam in collisionless magneto-plasma under plasma density ramp

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The effect of plasma density ramp on self-focusing of cosh-Gaussian laser beam considering ponderomotive nonlinearity is analyzed using WKB and paraxial approximation. It is noticed that cosh-Gaussian laser beam focused earlier than Gaussian beam. The focusing and de-focusing nature of the cosh-Gaussian laser beam with decentered parameter, intensity parameter, magnetic field, and relative density parameter has been studied and strong self-focusing is reported. It is investigated that decentered parameter “b” plays a significant role for the self-focusing of the laser beam as for b = 2.12, strong self-focusing is seen. Further, it is observed that extraordinary mode is more prominent toward self-focusing rather than ordinary mode of propagation. For b = 2.12, with the increase in the value of magnetic field self-focusing effect, in case of extraordinary mode, becomes very strong under plasma density ramp. Present study may be very useful in the applications like the generation of inertial fusion energy driven by lasers, laser driven accelerators, and x-ray lasers. Moreover, plasma density ramp plays a vital role to enhance the self-focusing effect. © 2014 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4889862]

I. INTRODUCTION

The nonlinear interactions of laser beams with plasmas have been studied intensively for over more than 40 years. Short-pulse high intensity lasers of the order of 10^20 W/cm^2 make it possible to investigate the nonlinear interaction of strong electromagnetic waves with plasmas. The various applications of self-focusing of laser beam in plasmas like optical harmonic generation,3 laser driven fusion,4 x-ray lasers and the laser driven accelerators,5 the production of quasi mono-energetic electron bunches,6 the generation of inertial fusion energy driven by lasers,7 etc., attract the attention of researchers and make self-focusing of laser beams in plasmas as most interesting and fascinating field of research for several decades. These applications need the laser beams to propagate over several Rayleigh lengths in the plasmas without loss of the energy. Investigators choose the propagation of different kind of laser beams profile like Gaussian beams,8 cosh-Gaussian beams,9,10 Hermite-cosh-Gaussian (HChG) beams,11 etc., in the plasmas. Recently, theoretical investigators focus their attention on paraxial wave family of laser beams. Propagation of Hermite-cosh-Gaussian beams in plasmas has been studied theoretically earlier by Belafhal and Ibnchaikh,11 Nanda et al.,12 Nanda and Kant,13 and Patil et al.14,15 The focusing of HChG laser beam in magneto-plasma by considering ponderomotive nonlinearity has been theoretically examined by Patil et al.16 and reported the effect of mode index and decentered parameter on the self-focusing of the beams.

In collisionless plasmas, ponderomotive force on electrons acting in an inhomogeneous electromagnetic field causes self-focusing. This force arises due to the drift of electrons in an inhomogeneous field and the interaction of drift velocity of electron with magnetic field. The ponderomotive force of the incident laser beam pushes the electron out of the region of high intensity and reduces the local concentration of electrons density in plasma. It increases the plasma dielectric function and laser beams become more self-focused in plasma. Gill et al.10 have recently studied the relativistic self-focusing and self-phase modulation of cosh-Gaussian laser beam in magnetoplasma in the absence of plasma density ramp and reported strong self-focusing of the laser beams. Gupta et al.17 have investigated the additional focusing of a high intensity laser beams in a plasma with a density ramp and a magnetic field and reported the strong self-focusing of the laser beams. Again plasma density ramp has also been applied by Kant et al.18,19 to study the Ponderomotive self-focusing of a short laser pulse and self-focusing of Hermite-Gaussian laser beams in plasma. In both the cases, Kant et al.18,19 reported strong self-focusing of the laser beams in the presence of plasma density ramp profile.

The present work is dedicated to the study of self-focusing of cosh-Gaussian laser beams in collisionless magneto-plasma under plasma density ramp in applied magnetic field. The cosh-Gaussian beam has the ability to focus earlier than Gaussian beam as it is obvious from Figure 2. Moreover, a cosh-Gaussian laser beam possesses more power than that of Gaussian laser beams having high intensity near the axis of propagation and hence, generates flat top beam profiles,20 which is useful for scribing type of applications in electronics where same intensity of laser beams for long time is required. On the basis of superposition of beams, a group of virtual sources that generate a cosh-Gaussian wave is identified by Zhang et al.21 Belafhal et al.11 have investigated that for b = 0, the intensity profile of the HChG beam is similar to a Hermite-Gaussian distribution and with increasing b, the cosph function acts to concentrate the energy in the outer lobes of the beam. Moreover, previous works by Gupta et al.17 and Kant et al.18,19 indicate the enhancement of self-
focusing of laser beams due to the presence of plasmas density ramp profile. So, it is quite interesting to apply plasmas density ramp profile in the medium in which coss-Gaussian laser beam is propagating.

In the present paper, we investigate the self-focusing of a coss-Gaussian laser beam in collisionless magneto-plasma under plasma density ramp. We derive the equations for beam width parameter for coss-Gaussian beam profile propagating in the plasmas in the presence of magnetic field and plasma density ramp, by applying WKB and paraxial approximation and solve them numerically by applying initial conditions. We observe the enhancement in the self-focusing of the laser beam as the beam width parameter decreases with the normalized distance for the change in various parameters like intensity of laser beam, relative plasma density, decentered parameter, and magnetic field. The presence of plasma density ramp plays a vital role to affect the self-focusing nature of propagating laser beams in the plasmas. To make the mathematical calculation simpler, only the transversal components of laser field are evaluated and longitudinal components are not taken into consideration in the present paper. However, while dealing with nonlinear phenomenon, longitudinal components should be considered for an exact formulation.

II. BASIC FORMULATION

The field distribution of coss-Gaussian laser beam propagating in the plasma along z-axis is of the following form:

\[ E(r, z) = \frac{E_0}{f_z(z)} e^{i (\sqrt{\omega^2 \text{m}^2} \cdot z}) + e^{i (\sqrt{\omega^2 \text{m}^2} \cdot z})}. \]  

Here, \( E_0 \) is the amplitude of coss-Gaussian laser beam for the central position at \( r = z = 0 \), \( f_z(z) \) is the dimensionless beam width parameters for extraordinary (+ sign) and ordinary (− sign) mode of propagation of magnetoplasma, \( r_0 \) is the spot size of the beam, and \( b \) is the decentered parameter of the beam.

The dielectric constant for the non-linear medium due to ponderomotive non-linearity (collision-less magnetoplasma) is obtained by applying the approach as applied by Sodha et al. \(^8\)

\[ \varepsilon_\pm = \varepsilon_{\pm 0} + \phi_\pm (EE^*) \]  

As relative displacements between ions and electrons of plasma set up a restoring electric field \( E = 4\pi n_0 e^2 d \), which returns the electrons to equilibrium position and hence in this fashion, motion of each electron becomes simple harmonic with plasma frequency \( \omega_p^2 = 4\pi n_0 e^2 / m \). Here, \( n_0, e, d \) and \( m \) are the electron density, electronic charge, displacement of charge layer from original position, and rest mass of electron, respectively. Figure 1 shows a layer of negative charge per unit area on one side of the plasma slab with the stationary ions producing a layer of positive charge on the other side for (a) constant electron density and (b) varying electron density, respectively.

In Figure 1(a), each electron experiences electric force \( F = -4\pi n_0 e^2 d \), in the direction of its equilibrium position.

The equation of motion of electron is of the form, \( m\ddot{d} = (4\pi n_0 e^2)\dot{d} \), with index of refraction given by \( (1 - \omega_p^2/\omega^2) \). If the value of \( \omega_p \) is less than plasma frequency \( \omega_p \), then the refractive index is purely imaginary which gives rise to attenuation and if the value of \( \omega_p \) is greater than \( \omega_p \), then the refractive index is real. We consider the plasma density ramp profile \( n(z) = n_0 tan(z/d_0) \), as previously taken by various authors. \(^17,19-25\) where \( z = \tau/R_d \) is the normalized propagation distance, \( d_0 \) is a dimensionless adjustable parameter, and \( R_d \) is the diffraction length. Now, each electron experiences an electric force \( F = -4\pi n_0 \omega^2 e^2 d \), in the direction of its equilibrium position as depicted in Figure 1(b). The equation of motion of electron, in this case, is of the form, \( m\ddot{d} = (4\pi n_0 e^2)\dot{d} \). So, in the present study, application of plasma density ramp and external static magnetic field modify the index of refraction to, \( \varepsilon_{\pm 0} = 1 - \omega_p^2(\omega/\omega_\pm) \), where \( \omega_\pm = eB_0/mc \) is the electron cyclotron frequency, \( \omega_p(z) = (4\pi e^2 n(z)/m)^{1/2} \) is the plasma frequency which varies along z-axis. Plasma frequency can be written as \( \omega_p(z) = 4\pi e^2 n(z) tan(z/d_0)/m = \omega_{p0} tan(z/d_0) \).

The non-linear part of dielectric constant is given by \( \phi_\pm = \varepsilon_\pm - \varepsilon_\pm 0 (EE^*) \) with \( \varepsilon_\pm 0 = 3\pi \alpha \omega_\pm/d^2 (1 - \omega_\pm/\omega_c) / 2M \omega_\pm(\omega_\pm - \omega_c) \) and \( \alpha = e^2 M / 6m^2 \omega_\pm^2 k B T_0 \), here \( M \) is the mass of scatterer in the plasma, \( \omega_c \) is the frequency of incident laser, \( k_B \) is the Boltzmann constant, and \( T_0 \) is the equilibrium plasma temperature. \( \alpha^2 \) will be defined later on in Eq. (10).

The general form of wave equation for exponentially varying field obtained from Maxwell’s equation by applying the approach as applied by Sodha et al. \(^8\) is given by

\[ \nabla^2 E - \nabla(\nabla \cdot E) = -\frac{\omega^2}{c^2} e.E. \]  

In components form, this equation can be written as

\[ \frac{\partial^2 E_x}{\partial z^2} + \frac{\partial^2 E_x}{\partial y^2} - \frac{\partial}{\partial x} \left( \frac{\partial E_x}{\partial y} + \frac{\partial E_y}{\partial x} \right) = -\frac{\omega^2}{c^2} (e.E)_x \]  

(4a)
\[
\frac{\partial^2 E_x}{\partial z^2} + \frac{\partial^2 E_y}{\partial x^2} - \frac{\partial}{\partial y} \left( \frac{\partial E_x}{\partial x} + \frac{\partial E_z}{\partial z} \right) = -\frac{\omega^2}{c^2} (\varepsilon E)_y \quad (4b)
\]

\[
\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} - \frac{\partial}{\partial z} \left( \frac{\partial E_z}{\partial x} + \frac{\partial E_y}{\partial y} \right) = -\frac{\omega^2}{c^2} (\varepsilon E)_z \quad (4c)
\]

In the present case, the variation of magnetic field is assumed to be very strong along z-direction of the co-ordinate system than x-y plane. This gives the condition \( \nabla \cdot \mathbf{D} = 0 \). Equations (4a)–(4c) are coupled and one cannot define the propagation vectors which describe the independent propagation of \( E_x \), \( E_y \), and \( E_z \). Following Sodha et al., \(^8\) we get

\[
\frac{\partial^2 A_1}{\partial z^2} + \frac{1}{2} \left( 1 + \frac{\varepsilon}{\varepsilon_{zz}} \right) \left( \frac{\partial^2 A_1}{\partial x^2} + \frac{\partial^2 A_1}{\partial y^2} \right) + i \left( \frac{\varepsilon 
abla^2 A_1}{\varepsilon_{zz}} - 1 \right) \left( \frac{\partial A_1}{\partial x} + \frac{\partial A_1}{\partial y} \right) - \frac{\omega^2}{c^2} \left( \varepsilon_{zz} A_1 \right) = 0 \quad (5a)
\]

and

\[
\frac{\partial^2 A_2}{\partial z^2} + \frac{1}{2} \left( 1 + \frac{\varepsilon}{\varepsilon_{zz}} \right) \left( \frac{\partial^2 A_2}{\partial x^2} + \frac{\partial^2 A_2}{\partial y^2} \right) + i \left( \frac{\varepsilon}{\varepsilon_{zz}} \right) \left( \frac{\partial A_2}{\partial x} + \frac{\partial A_2}{\partial y} \right) + \frac{\omega^2}{c^2} \left( \varepsilon_{zz} A_2 \right) = 0 \quad (5b)
\]

with \( A_1 = E_x + iE_y \), \( A_2 = E_x - iE_y \), \( \varepsilon_{zz} = 1 - \varepsilon_0^2/\omega^2 \). The functions \( A_1 \) and \( A_2 \) having propagation vectors \( k_+ \) and \( k_- \), respectively, represents the extraordinary and ordinary modes of propagation of a magnetoplasma. Equations (5a) and (5b) are loosely coupled with each other due to slow variation of field component in x and y-direction as compared to those in z-direction. In order to study the behavior of one of the mode, other mode can be considered to be zero. Let us assume \( A_2 \approx 0 \), then Eq. (5a) becomes

\[
\frac{\partial^2 A_1}{\partial z^2} + \frac{1}{2} \left( 1 + \frac{\varepsilon}{\varepsilon_{zz}} \right) \left( \frac{\partial^2 A_1}{\partial x^2} + \frac{\partial^2 A_1}{\partial y^2} \right) + \frac{\omega^2}{c^2} \left( \varepsilon_{zz} A_1 \right) = 0 \quad (6)
\]

Now Eq. (6) is independent of \( A_2 \) and represents the propagation of extraordinary mode. A similar equation for ordinary mode of propagation can be obtained by considering \( A_1 \approx 0 \) in Eq. (5b). The solution of Eq. (6) is of the form \( A_1 = A \exp i(\omega t - k_x z) + k_+ (\omega/c)^{1/2}(z) \), where \( A \) is a complex function of \( x, y, \) and \( z \). Substituting these values in Eq. (6), omitting the term with \( \partial^2 A/\partial z^2 \) and separating real and imaginary parts by introducing the additional eikonal, \( A = A_0(x, y, z) \exp [-ik_+S(x, y, z)], \) where \( A_0 \) and \( S \) are the real function of \( x, y, \) and \( z \).

Real part is

\[
\left[ \frac{z\omega^2 \omega_x \sec^2 \left( \frac{z}{d_0R_d} \right)}{k_x c^2 d_0 R_d} - 2 \right] \frac{\partial S}{\partial z} - \frac{1}{2} \left( 1 + \frac{\varepsilon}{\varepsilon_{zz}} \right) \left[ \left( \frac{\partial S}{\partial x} \right)^2 + \left( \frac{\partial S}{\partial y} \right)^2 \right] + \frac{1}{4A_0} \left( 1 + \frac{\varepsilon}{\varepsilon_{zz}} \right) \left( \frac{\partial A_0}{\partial k_x} \right)^2 \\
\times \left[ \frac{\partial^2 A_0}{\partial x^2} + \frac{\partial^2 A_0}{\partial y^2} \right] - \frac{1}{2A_0} \left( \frac{\partial A_0}{\partial x} \right)^2 + \left( \frac{\partial A_0}{\partial y} \right)^2 \right] - \frac{S\omega^2 \omega_x \sec^2 \left( \frac{z}{d_0R_d} \right)}{k_x c^2 d_0 R_d} - \frac{S \omega^2 \omega_x \sec^2 \left( \frac{z}{d_0R_d} \right)}{2k_x c^2 d_0 R_d} \\
+ \frac{z\omega^2 \omega_y \sec^2 \left( \frac{z}{d_0R_d} \right)}{k_y c^2 d_0 R_d} - \frac{z\omega^2 \omega_y \sec^2 \left( \frac{z}{d_0R_d} \right)}{4k_y c^2 d_0 R_d^2} + \frac{\omega^2}{k_x c^2} e^{\varepsilon_2} \left( \frac{A_0}{1 - \frac{\omega}{\omega_0}} \right)^2 = 0 \quad (7)
\]

Imaginary part is
\[
\frac{\omega_0^2 \omega_z \sec^2 \left( \frac{z}{d_0 R_d} \right) + \frac{\omega_0^2 \omega_z \sec^2 \left( \frac{z}{d_0 R_d} \right) \tan \left( \frac{z}{d_0 R_d} \right)}{k_0^2 c^2 d_0 R_d} + \frac{\omega_0^4 \omega_z^2 \sec^4 \left( \frac{z}{d_0 R_d} \right)}{4k_0^2 c^2 d_0^2 R_d^2} + \left( \frac{\omega_0^2 \omega_z \sec^2 \left( \frac{z}{d_0 R_d} \right)}{2A_0^2 c^2 d_0 R_d} - \frac{1}{k_0 A_0^2} \right) \frac{\partial A_0^2}{\partial z} \right)
\]
\[
- \frac{1}{2k_0} \left[ 1 + \frac{\varepsilon_0 + \omega_0}{\varepsilon_0 \omega_0} \right] \frac{\partial S}{\partial x} + \frac{\partial S}{\partial y} \right] + \frac{1}{2k_0 A_0} \left[ 1 + \frac{\varepsilon_0 + \omega_0}{\varepsilon_0 \omega_0} \right] \left[ \frac{\partial S}{\partial x} \frac{\partial A_0^2}{\partial x} + \frac{\partial S}{\partial y} \frac{\partial A_0^2}{\partial y} \right] = 0
\]

(8)

with \( \omega_z = \omega_{z0} - \omega(\omega - \omega_z) \).

The solution of Eqs. (7) and (8) for initially cosh-Gaussian beam are of the following form

\[
A_0^2 = \frac{E_0^2}{f_0^2(z)} \exp \left[ \frac{\omega_0^2 \omega_z \sec^2 \left( \frac{z}{d_0} \right) \xi}{f_0^2(z)} \right] \left( e^{-2 \left( \frac{\xi}{f_0^2(z)} \right)^2} + e^{-2 \left( \frac{\xi}{f_0^2(z)} \right)^2} + 2 e^{- \frac{\xi^2}{f_0^2(z)^2}} \right)
\]

(9)

\[
S = \frac{r^2}{2} \beta(z) + \phi(z),
\]

(10)

where \( \beta(z) = 2 \left( \frac{\partial f_+}{\partial z} / \left[ f_+(z) \right] \right) \) is the curvature of the wave front, \( \phi(z) \) is an arbitrary function of \( z \) and \( r^2 = x^2 + y^2 \).

Thus, Eq. (8) becomes

\[
\frac{\xi \omega_z \sec^2 \left( \frac{\xi}{d_0} \right)}{d_0} - 2 \left( 1 - \omega_z \tan \left( \frac{\xi}{d_0} \right) \right) \frac{\partial^2 f_+}{\partial \xi^2} - \frac{\xi \omega_z \sec^2 \left( \frac{\xi}{d_0} \right)}{d_0} \frac{1}{f_+} \left( \frac{\partial f_+}{\partial \xi} \right)^2 - \frac{\xi \omega_z \sec^2 \left( \frac{\xi}{d_0} \right)}{d_0} \frac{1}{f_+} \left( \frac{\partial f_+}{\partial \xi} \right)^2 = \frac{\xi \omega_z \sec^2 \left( \frac{\xi}{d_0} \right)}{d_0} \frac{1}{f_+} \left( \frac{\partial f_+}{\partial \xi} \right)^2
\]

\[
- \frac{\xi \omega_z \sec^2 \left( \frac{\xi}{d_0} \right)}{d_0} \frac{1}{f_+} \left( \frac{\partial f_+}{\partial \xi} \right)^2 = \frac{8R_0^2}{f_+^2(\xi)} \exp \left[ 1 - \frac{\omega_0^2}{\omega_0^2 \tan \left( \frac{\xi}{d_0} \right)} \right] \left( 1 + \frac{\varepsilon_0 + \omega_0}{\varepsilon_0 \omega_0} \right) = 0
\]

(11)

where \( R_0^2 = \left( 1 - \omega_z / \omega_0 \right)^2 \left( \frac{1}{\varepsilon_0 + \omega_0} \right) \) and \( R_0^2 = k_0 R_0^2 \).

Equation (12) is the required equation for beam width parameter for extraordinary modes of propagation of magnetoplasma. Similarly equation for beam width parameter for ordinary mode can be obtained by replacing \( \omega_z \) by \( - \omega_z \) and sign by \( - \) sign in the subscript of \( f \).

However, for initially Gaussian beam, \( A_0^2 = E_0^2 \exp(-r^2/f_0^2(z)) / f_0^2(z) \), the beam width parameter comes out to be

\[
\frac{\xi \omega_z \sec^2 \left( \frac{\xi}{d_0} \right)}{d_0} - 2 \left( 1 - \omega_z \tan \left( \frac{\xi}{d_0} \right) \right) \frac{\partial^2 f_+}{\partial \xi^2} - \frac{\xi \omega_z \sec^2 \left( \frac{\xi}{d_0} \right)}{d_0} \frac{1}{f_+} \left( \frac{\partial f_+}{\partial \xi} \right)^2 - \frac{\xi \omega_z \sec^2 \left( \frac{\xi}{d_0} \right)}{d_0} \frac{1}{f_+} \left( \frac{\partial f_+}{\partial \xi} \right)^2 = \frac{\xi \omega_z \sec^2 \left( \frac{\xi}{d_0} \right)}{d_0} \frac{1}{f_+} \left( \frac{\partial f_+}{\partial \xi} \right)^2
\]

\[
- \frac{\xi \omega_z \sec^2 \left( \frac{\xi}{d_0} \right)}{d_0} \frac{1}{f_+} \left( \frac{\partial f_+}{\partial \xi} \right)^2 = \frac{8R_0^2}{f_+^2(\xi)} \exp \left[ 1 - \frac{\omega_0^2}{\omega_0^2 \tan \left( \frac{\xi}{d_0} \right)} \right] \left( 1 + \frac{\varepsilon_0 + \omega_0}{\varepsilon_0 \omega_0} \right) = 0
\]

(12)

The initial boundary conditions are \( \xi = 0, f_+ (z) = 1 \) and \( df_+ (z) / dz = 0 \) for extraordinary and ordinary modes.

### III. RESULTS AND DISCUSSION

In the present case, we assume the plasma density ramp profile given by the relation \( n(\xi) = n_0 \tan \left( \frac{\xi}{d_0} \right) \) with initial electron density \( n_0 \approx 5 \times 10^{20} \text{cm}^{-3} \), angular frequency of incident laser \( \omega = 1.778 \times 10^{15} \text{rad/s} \) and \( r_0 = 49.44 \mu \text{m} \). The variation
of beam width parameter \( f \) with normalized propagation distance \( \bar{\xi} \) is depicted in Figure 2 for the parameters taken as \( \alpha E_0^2 = 0.5, \ d_0 = 0.9, \ \omega_c/\omega = 0.3, \ \omega r_0/c = 293, \ m_0/M = 0.0006 \) and \( \omega r_0/\omega = 0.77 \). From Figure 2(a), it is obvious that strong self-focusing occurs at normalized distance, \( \bar{\xi} = 0.12 \) for cosh-Gaussian beam while for Gaussian beam, strong self-focusing occurs at \( \bar{\xi} = 0.5 \) for extraordinary mode of propagation. Figure 2(b) depicts the diffraction tendency of Gaussian beam and converging tendency of cosh-Gaussian beam with normalized propagation distance for ordinary mode of propagation; however, extraordinary mode is more prominent toward self-focusing than ordinary mode. The propensity of cosh-Gaussian beam to converge earlier than Gaussian beam leads us to choose the cosh-Gaussian laser beam profile.

Figure 3 represents the variation of beam width parameter \( f_\xi \) with the normalized propagation distance \( \bar{\xi} \) for different values of decentered parameter, \( b = 0, 1, 2 \) and 2.12. It is clear from Fig. 3(a), that for extraordinary mode, beam width parameter decreases earlier with increase in the values of decentered parameter. For \( b = 2.12 \), beam width parameter falls abruptly at normalized propagation distance, \( \bar{\xi} = 0.12 \) and hence self-focusing becomes stronger. In the absence of decentered parameter, strong self-focusing effect is observed at certain higher value of normalized distance, \( \bar{\xi} = 0.3 \). However, for other two values of decentered parameter, \( b = 1 \) and 2, strong self-focusing effect is reported at...
\(\xi = 0.23\) and 0.13. Gill \textit{et al.}\cite{24} have studied self-focusing and self-phase modulation as well as self-trapping of \(\cosh\)-Gaussian beam at various values of decentered parameter (b) and reported sharper self-focusing for \(b = 2\) at \(\xi = 1.45\) and the value of beam width parameter is nearly 0.91 (approximately). However, for \(b = 1\), strong self-focusing occurs at nearly \(\xi = 2.5\). In another work, Gill \textit{et al.}\cite{25} reported strong self-focusing effect nearly at \(\xi = 0.65\), however, for \(b = 2\), they reported self-focusing which occur at \(\xi = 0.07\), but the value of beam width parameter at this normalized distance is very less and is nearly \(f = 0.995\). In the present work, strong converging of \(\cosh\)-Gaussian beam occurs at \(\xi = 0.12\) for \(b = 2.12\) due to the presence of plasma density ramp. In case of ordinary mode, Fig. 3(b), the self-focusing of laser beam occurs for \(b = 0, 1, 2\) and 2.12 and in the absence of decentered parameter self-focusing effect occurs at longer normalized propagation distance. For \(b = 2.12\), self-focusing effect is less strong in this case than in case of extraordinary mode.

Figure 4 represents the variation of beam width parameter with the normalized propagation distance for different values of intensity parameter, \(\alpha E_0^2 = 0.1, 0.3\) and 0.5 corresponding to intensities \(1.23 \times 10^{17}\) \(\text{W/cm}^2\), 3.67 \(\times 10^{17}\) \(\text{W/cm}^2\) and \(6.12 \times 10^{17}\) \(\text{W/cm}^2\), respectively, for \(b = 2.12\). It is obvious from Figure 4(a) that with the increase in the value of intensity parameter, beam width parameter decreases. For extraordinary mode, \(\alpha E_0^2 = 0.5\), the beam width parameter decreases up to its minimum value \(f = 0.09\) at \(\xi = 0.12\). Hence, self-focusing becomes very strong. This happens because at high intensities of incident laser beam, more electrons contribute to self-focusing. Gill \textit{et al.}\cite{25} have studied relativistic self-focusing.
and self-phase modulation of cosh-Gaussian laser beam in magnetoplasma in the absence of plasma density ramp and reported strong self-focusing, for $x[A_{0}\zeta]^2 = 0.5$ which occur nearly at $\zeta = 0.3$. In the present work, the self-focusing of beam is strong and occurs earlier due to the presence of plasma density ramp than the results reported by Gill et al.\textsuperscript{10} in uniform density profile of the plasma. In case of ordinary mode, Figure 4(b), self-focusing occurs for all taken values of intensity parameter and is strong and occurs earlier for higher value of intensity parameter. However, for extraordinary mode, self-focusing occurs earlier than that of ordinary mode for same values of intensity parameter.

Figure 5 represents the variation of beam width parameter with the normalized propagation distance for different values of magnetic field parameter ($\omega_{pe}/\omega_0$), viz., 0.1, 0.2 and 0.3 for extraordinary mode, Figure 5(a) and ordinary mode, Figure 5(b) for $b = 2.12$. It is found that with the increase in the value of magnetic field parameter, ($\omega_{pe}/\omega_0$), beam width parameter decreases and occurs earlier and hence self-focusing becomes strong. However, in case of ordinary mode, beam width parameter decreases with the increase in the value of magnetic field parameter but conversing tendency of cosh-Gaussian beam is shifted towards longer normalized propagation distance. Gill \textit{et al.}\textsuperscript{10} have studied relativistic self-focusing and self-phase modulation of cosh-Gaussian laser beam in magnetoplasma in the absence of plasma density ramp and reported strong self-focusing, for $\omega_{pe}/\omega_0 = 0.06$, which occur nearly at $\zeta = 0.6$. In the present case, strong self-focusing occurs at $\zeta = 0.12$ for extraordinary mode at magnetic field parameter, $\omega_{pe}/\omega_0 = 0.3$ for $b = 2.12$, in the presence of plasma density ramp. Hence, present results in the presence of plasma density ramp may be more useful.

Figure 6 represents the variation of beam width parameter with the normalized propagation distance for different values of relative plasma densities, $\omega_{pe}/\omega_0 = 0.65, 0.70, 0.75$ and 0.77 at $b = 2.12$ for extraordinary mode, Figure 6(a) and ordinary mode, Figure 6(b). It is obvious that for higher value of relative plasma density, the beam width parameter reaches its minimum value $f = 0.09$ at $\zeta = 0.12$ and hence self-focusing effect enhanced. However, for ordinary mode, self-focusing trends are similar to extraordinary mode but occurs at higher value of normalized propagation distance. Singh and Walia\textsuperscript{26} have studied the self-focusing of a laser beam in relativistic plasma and reported the strong self-focusing for relative plasma density, $\omega_{pe}/\omega_0 = 0.7$ at $\zeta = 0.7$ (approximately). In another work, Kant \textit{et al.}\textsuperscript{18} analyzed the ponderomotive self-focusing of a short pulse laser in an underdense plasma under density ramp and reported strong self-focusing for $\omega_{pe}^2/\omega_0^2 = 0.08$ at $\zeta = 0.5$. In our case, the strong self-focusing of beam occurs earlier than the results reported by Refs. 18 and 26 due to the presence of plasma density ramp and cosh-Gaussian beam profile.

**IV. CONCLUSION**

From the above results, we conclude that with the increase in the value of magnetic field for decentered parameter $b = 2.12$, self-focusing of cosh-Gaussian laser beam becomes stronger for extraordinary mode while for ordinary mode, it becomes somewhat weaker and occurs at higher values of normalized propagation distance. Decentered parameter also plays a significant role to decide the early and strong focusing ability of cosh-Gaussian beam as for $b = 0$, focusing of beam occurs at higher value of $\zeta$ for extraordinary and ordinary modes. The self-focusing of the cosh-Gaussian laser beam is found to be very strong and occurs earlier at $\zeta = 0.12$. This happens due to the presence of plasma density ramp and chosen beam profile. Thus, plasma density ramp plays a very vital role to the self-focusing of the cosh-Gaussian laser beam. Also, self-focusing becomes

![Figure 6](image-url)

**FIG. 6.** Variation of beam width parameter ($f_\zeta$) with normalized propagation distance ($\zeta$) for different values of relative plasma density parameter for (a) extraordinary and (b) ordinary mode of propagation. The other parameters are taken as $xE_0^2 = 0.5$, $d_0 = 0.9$, $\omega_{pe}/\omega_0 = 0.3$, $\omega_{pe}/c = 293$, and $m_0/M = 0.0006$. 

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stronger with the increase in value of intensity and magnetic field parameter under the application of plasma density ramp. It is concluded that extraordinary mode is more prominent toward self-focusing rather than ordinary mode of propagation and density ramp enhances the self-focusing effect. Our investigation may be useful for laser induced fusion as well as for scribing type of applications in electronics.