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Self-focusing of a Hermite-cosh Gaussian laser beam in a magnetoplasma with ramp density profile

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The early and strong self-focusing of a Hermite-cosh-Gaussian laser beam in magnetoplasma in the presence of density ramp has been observed. Focusing and de-focusing nature of the Hermite-cosh-Gaussian laser beam with decentered parameter and magnetic field has been studied, and strong self-focusing is reported. It is investigated that decentered parameter "b" plays a significant role for the self-focusing of the laser beam and is very sensitive as in case of extraordinary mode. For mode indices, \( m = 0, 1, 2 \), and \( b = 4.00, 3.14, \) and \( 2.05 \), strong self-focusing is observed. Similarly in case of ordinary mode, for \( m = 0, 1, 2 \) and \( b = 4.00, 3.14, 2.049 \), respectively, strong self-focusing is reported. Further, it is seen that extraordinary mode is more prominent toward self-focusing rather than ordinary mode of propagation. For mode indices \( m = 0, 1, 2 \), diffraction term becomes more dominant over nonlinear term for decentered parameter \( b = 0 \). For selective higher values of decentered parameter in case of mode indices \( m = 0, 1, 2 \), self-focusing effect becomes strong for extraordinary mode. Also increase in the value of magnetic field enhances the self-focusing ability of the laser beam, which is very useful in the applications like the generation of inertial fusion energy driven by lasers, laser driven accelerators, and x-ray lasers. © 2013 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4833635]

I. INTRODUCTION

The interaction of light with solids, liquids, gases, and plasmas occurs very frequently in nature. For over more than four decades, the nonlinear interaction of laser beams with matter has been studied intensively by researchers. Development of short-pulse high intensity lasers of the order of \( 10^{20} \) W/cm\(^2\) makes it possible to investigate the nonlinear interaction of strong electromagnetic waves with plasmas. A series of applications of self-focusing of laser beams in plasmas, like optical harmonic generation, laser driven fusion, x-ray lasers and the laser driven accelerators, the production of quasi mono-energetic electron bunches, etc., create a center of attention of many researchers and scientists. These applications need the laser beams to propagate over several Rayleigh lengths in the plasmas without loss of the energy. The propagation of different kinds of laser beams profile like Gaussian beams, Cosh-Gaussian beams, Hermite-cosh-Gaussian beams (HChG), Elliptic Gaussian beams, Bessel beams, Leguerre-Gaussian beams, etc., in the plasmas attracts the attention of the researchers.

Recently, theoretical investigators focus their attention on paraxial wave family of laser beams. Hermite-cosh-Gaussian beam is one of the solutions of paraxial wave equation and such HChG beam can be obtained in the laboratory by the superposition of two decentered Hermite-Gaussian beams as Cosh-Gaussian ones. Propagation of Hermite-cosh-Gaussian beams in plasmas has been studied theoretically earlier by Belalhal and Ibnchaikh and Patil et al. The focusing of HChG laser beams in magnetoplasma by considering ponderomotive nonlinearity has been theoretically examined by Patil et al. and reported the effect of mode index and decentered parameter on the self-focusing of the beams. Gill et al. have recently studied the relativistic self-focusing and self-phase modulation of cosh-Gaussian laser beam in magnetoplasma in the absence of plasma density ramp and reported strong self-focusing of the laser beams. Gupta et al., in 2007, have investigated the addition focusing of a high intensity laser beams in a plasma with a density ramp and a magnetic field and reported the strong self-focusing of the laser beams. Again density ramp has also been applied by Kant et al. to study the Ponderomotive self-focusing of a short laser pulse and self-focusing of Hermite-Gaussian laser beams in plasma. In both the cases, Kant et al. reported strong self-focusing of the laser beams.

The present work is dedicated to the study of self-focusing of Hermite-cosh-Gaussian laser beams in collisionless magento-plasma under plasma density ramp in applied magnetic field. We derive the equations for beam width parameter for Hermite-cosh-Gaussian beam profile propagating in the plasmas in the presence of magnetic field and plasma density ramp, by applying Wentzel-Kramers-Brillouin (WKB) approximation and Paraxial approximation and hence, solve them numerically by using Mathematica software. The sensitiveness of decentered parameter is observed, and a small increase in its value enhances greatly the self-focusing ability of the beam. In order to simplify the mathematical calculation, only the transversal components of laser field are evaluated and longitudinal components are not taken in to consideration in the present paper. However, while dealing with nonlinear phenomenon, longitudinal components should be considered for an exact formulation.

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II. NON-LINEAR DIELECTRIC CONSTANT

The dielectric constant for the non-linear medium (collision-less magnetoplasma) with density ramp profile is obtained by applying the approach by Sodha et al. with

\[ \varepsilon_{\pm} = \varepsilon_{\pm 0} + \phi_{\pm}(E^2), \]

(1)

with \( \varepsilon_{\pm 0} = 1 - \omega_p^2 / \omega(\omega + \omega_c) \), \( \phi_{\pm}(E^2) = \varepsilon_{\pm 0} \beta^2 \), \( \omega_p^2 = 4\pi e^2 \eta / m, \); \( \omega_c = n_0 \hbar / m \), \( \omega_e = eB_0 / mc \), \( \varepsilon_{\pm 2} = 3\pi \omega_p^2 (1 + \omega_c / 2\omega) / 8\pi \eta (\omega + \omega_c) \), and \( \beta = e^2 \lambda / 2m^2 \omega_p^2 \kappa_0 \). Here, \( \varepsilon_{\pm 0} \) and \( \phi_{\pm}(E^2) \) represent the linear and non-linear parts of the dielectric constant, respectively. \( \omega_p \) and \( \omega_c \) are the plasma frequency and electron cyclotron frequency, respectively. \( e, m, \) and \( n_0 \) are the magnitude of the electronic charge, mass, and electron density, "\( M \)" is the mass of scatterer in the plasma, "\( \omega^* \)" is the frequency of laser used, "\( \xi^* = z/R_d \)" is the normalized propagation distance, "\( R_d \)" is the diffraction length, "\( d \)" is a dimensionless adjustable parameter, "\( k_b \)" is the Boltzmann constant, and "\( T_0 \)" is the equilibrium plasma temperature.

III. SELF-FOCUSED EQUATIONS

Consider the Hermite-cosh-Gaussian laser beam is propagating along the \( z \)-direction with the field distribution in the following form:

\[ E(r, z) = \frac{E_0}{f_{\pm}(z)} e^{\frac{z^2}{c^2}} \left( \frac{\sqrt{2} r}{r_0f_{\pm}(z)} \right) \left[ e^{-(\frac{z^2}{c^2})} + e^{-(\frac{z^2}{c^2})} \right]. \]

(2)

"\( E_0 \)" is the amplitude of Hermite-cosh-Gaussian laser beam for the central position at \( r = z = 0 \). "\( f_{\pm}(z) \)" is the dimensionless beam width parameter, \( H_m \) is the Hermite polynomial and "\( m \)" is the mode index associated with Hermite polynomial, and "\( \gamma^* \)" is the initial spot size of the laser beam.

The general form of wave equation for exponentially varying field obtained from Maxwell’s equation is given by

\[ \nabla^2 \tilde{E} - \nabla(\dot{\nabla} \cdot \tilde{E}) = -\frac{\omega_p^2}{c^2} \varepsilon_{\pm} \tilde{E}. \]

(3)

In components form, this equation can be written as

\[ \frac{\partial^2 E_x}{\partial z^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial}{\partial x} \left( \frac{\partial E_x}{\partial y} + \frac{\partial E_y}{\partial z} \right) = -\frac{\omega_p^2}{c^2} (\varepsilon_{\pm} \tilde{E})_x, \]

(4a)

\[ \frac{\partial^2 E_y}{\partial z^2} + \frac{\partial^2 E_x}{\partial y^2} - \frac{\partial}{\partial x} \left( \frac{\partial E_x}{\partial y} + \frac{\partial E_y}{\partial z} \right) = -\frac{\omega_p^2}{c^2} (\varepsilon_{\pm} \tilde{E})_y, \]

(4b)

\[ \frac{\partial^2 E_z}{\partial z^2} + \frac{\partial^2 E_x}{\partial y^2} - \frac{\partial}{\partial x} \left( \frac{\partial E_x}{\partial y} + \frac{\partial E_z}{\partial y} \right) = -\frac{\omega_p^2}{c^2} (\varepsilon_{\pm} \tilde{E})_z. \]

(4c)

In the present case, the variation of magnetic field (\( \vec{B} = B_0 \hat{k}_z \)) is assumed to be very strong along \( z \)-direction of the coordinate system than \( x \)-\( y \) plane. Thus, the propagating wave is considered to be transverse in nature in zero order approximation. This assumption gives rise to the condition \( \nabla \cdot \vec{D} = 0 \). The Eqs. (4a)–(4c) are coupled and one cannot define the propagation vectors which describe the independent propagation of \( E_x, E_y, \) and \( E_z \). To investigate the independent modes of propagation, we multiply \( y \)-component equation by "\( \ddot{y} \)" and adding to \( x \)-component equation and also using the condition \( \nabla \cdot \vec{D} = 0 \), we get equation for extra-ordinary mode of propagation as

\[ \frac{\partial^2 A_1}{\partial z^2} + \frac{1}{2} \left( 1 + \frac{\dot{e}_{\pm 0}}{e_{\pm 0z}} \right) \left( \frac{\partial^2 A_1}{\partial x^2} + \frac{\partial^2 A_1}{\partial y^2} \right) \]

\[ + \frac{1}{2} \left( \frac{e_{\pm 0}}{e_{\pm 0z}} - 1 \right) \left( \frac{\partial^2 A_2}{\partial x^2} - \frac{\partial^2 A_2}{\partial y^2} \right) - i \left( 1 - \frac{\dot{e}_{\pm 0}}{e_{\pm 0z}} \right) \frac{\partial A_2}{\partial x} \frac{\partial A_2}{\partial y} \]

\[ + \frac{\omega_p^2}{c^2} \left[ \dot{e}_{\pm 0} + \dot{e}_{\pm 2} \left( \frac{A_1 A_1^*}{1 - \frac{e_{\pm 0}}{e_{\pm 0z}}} + \frac{A_2 A_2^*}{1 + \frac{e_{\pm 0}}{e_{\pm 0z}}} \right) \right] A_1 = 0, \]

(5)

with \( A_1 = E_x + iE_y, A_2 = E_x - iE_y, \) and \( e_{\pm 0z} = 1 - \omega_p^2 / \omega^2 \). A similar kind of equation can be obtained for ordinary mode. The functions \( A_1 \) and \( A_2 \), having propagation vectors \( K_+ \) and \( K_- \), respectively, represent the extraordinary and ordinary modes of propagation of a magnetoplasma. In order to study the behavior of one of the modes, other mode can be considered to be zero. Let us assume \( A_2 \approx 0 \), \( A_1 = A \exp[i(\omega t - k \cdot z)] \) and thereafter introducing additional eikonal, \( A = A_{\infty}(x, y, z) \exp[-ik_+(z)S(x, y, z)] \), the real and imaginary parts comes out to be:

\[ \frac{\partial^2 A_1}{\partial z^2} + \frac{\partial^2 A_1}{\partial x^2} - \frac{\partial}{\partial x} \left( \frac{\partial A_1}{\partial x} + \frac{\partial A_1}{\partial y} \right) = -\frac{\omega_p^2}{c^2} (\varepsilon_{\pm} \tilde{E})_y, \]

(4b)

\[ \frac{\partial^2 A_1}{\partial z^2} + \frac{\partial^2 A_1}{\partial y^2} - \frac{\partial}{\partial x} \left( \frac{\partial A_1}{\partial y} + \frac{\partial A_1}{\partial z} \right) = -\frac{\omega_p^2}{c^2} (\varepsilon_{\pm} \tilde{E})_z. \]

(4c)

Real part is

\[ \frac{z \omega_p^2 \omega_c \sec^2 \left( \frac{z}{dR_d} \right)}{k_+^2 c^2 dR_d} = 2 \left( \frac{S \omega_p^2 \omega_c \sec^2 \left( \frac{z}{dR_d} \right)}{k_+^2 c^2 dR_d} \right) \]

\[ + \frac{z \omega_p^2 \omega_c \sec^2 \left( \frac{z}{dR_d} \right)}{k_+^2 c^2 dR_d} - \frac{z \omega_p^2 \omega_c \sec^4 \left( \frac{z}{dR_d} \right)}{4k_+^2 c^4 d^3 R_d^3} + \frac{A_0^2}{k_+^2 c^2} \left( 1 - \frac{\omega_{\pm 0}}{\omega} \right)^2 = 0. \]

(6)
Imaginary part is

\[
\frac{\omega^2 \omega_x \sec^2 \left( \frac{z}{dR_d} \right)}{k^4 \epsilon_c^2 dR_d} + \frac{z \omega^2 \omega_x \sec^2 \left( \frac{z}{dR_d} \right) \tan \left( \frac{z}{dR_d} \right)}{k^4 \epsilon_c^2 dR_d} + \frac{z \omega^4 \omega_x^2 \sec^4 \left( \frac{z}{dR_d} \right)}{4k^4 \epsilon_c^4 R_d^2} + \frac{z \omega^2 \omega_x \sec^2 \left( \frac{z}{dR_d} \right) - \frac{1}{k^4 \epsilon_c^2}}{2A_0^2 k^4 \epsilon_c^2 dR_d} \frac{\partial A_0^2}{\partial z} = 0,
\]

(7)

with \( k_+ = (\omega/c) e_{10}^{1/2} \). The solution of Eqs. (6) and (7) for initially Hermite-cosh-Gaussian beam is of the following form:

\[
A_0 = \frac{E_0^2}{f_+^2(z)} e^{\frac{\phi}{f_+}} H_m \left( \frac{\sqrt{2r}}{r_0 f_+(z)} \right) \left[ e^{-2 \left( r/r_0 f_+ \right)^2} + e^{-2 \left( r/r_0 f_+ \right)^2} + 2 e^{-2 \left( r/r_0 f_+ \right)^2} \right],
\]

(8)

and

\[
S = \frac{x^2}{2} \beta(z) + \frac{y^2}{2} \beta(z) + \phi(z),
\]

(9)

where \( \beta(z) = 2/f_+(z) \left( 1 + \frac{e_{10}}{\epsilon_{02}} \right) \frac{\partial f_+(z)}{\partial z} \), is the curvature of the wave front and \( r^2 = (x^2 + y^2) \). "E_0" is the amplitude of Hermite-cosh-Gaussian laser beam for the central position at \( r = z = 0 \), "f_+(z)" is the dimensionless beam width parameter; "r_0" is the spot size of the beam. Thus, Eq. (7) becomes,

For \( m = 0 \):

\[
\left\{ \frac{\xi \omega_x \sec^2 \left( \frac{\xi}{d} \right)}{d} - 2 \left( 1 - \omega_x \tan \left( \frac{\xi}{d} \right) \right) \frac{\partial^2 f_+}{\partial \xi^2} - \frac{\xi \omega_x \sec^2 \left( \frac{\xi}{d} \right)}{d} \left( \frac{\partial f_+}{\partial \xi} \right)^2 \left\}
\]

\[
- \left[ \xi \omega_x \sec^4 \left( \frac{\xi}{d} \right) \omega_0^2 e_{10} \right. \frac{d^2 \omega_0 e_{02}}{d \xi^2} \left( 1 + \frac{e_{10}}{\epsilon_{02}} \right) - \left. \xi \omega_x \sec^4 \left( \frac{\xi}{d} \right) \frac{2 \sec^2 \left( \frac{\xi}{d} \right) \left( 1 - \omega_x \tan \left( \frac{\xi}{d} \right) \right) \omega_0^2 e_{10}}{d^2 \omega_0 e_{02} \left( 1 + \frac{e_{10}}{\epsilon_{02}} \right)} \right] \frac{\partial f_+}{\partial \xi} + \frac{2 \omega_x \sec^2 \left( \frac{\xi}{d} \right) \left( 1 - \omega_x \tan \left( \frac{\xi}{d} \right) \right)}{d \omega_0^2 e_{02} \left( 1 + \frac{e_{10}}{\epsilon_{02}} \right) d \xi_0^2} \right] \left( 1 + \frac{e_{10}}{\epsilon_{02}} \right) \right) \right] \frac{\partial f_+}{\partial \xi} = 0,
\]

(10)

where \( R_{u1}^2 = (1 - \omega_c / \omega)^2 r_0^2 (e_{10} / e_{12} E_0^2) \) and \( R_{d1} = k_+ r_0^2 \).

For \( m = 1 \):

\[
\left\{ \frac{\xi \omega_x \sec^2 \left( \frac{\xi}{d} \right)}{d} - 2 \left( 1 - \omega_x \tan \left( \frac{\xi}{d} \right) \right) \frac{\partial^2 f_+}{\partial \xi^2} - \frac{\xi \omega_x \sec^2 \left( \frac{\xi}{d} \right)}{d} \left( \frac{\partial f_+}{\partial \xi} \right)^2 \left\}
\]

\[
- \left[ \xi \omega_x \sec^4 \left( \frac{\xi}{d} \right) \omega_0^2 e_{10} \right. \frac{d^2 \omega_0 e_{02}}{d \xi^2} \left( 1 + \frac{e_{10}}{\epsilon_{02}} \right) - \left. \xi \omega_x \sec^4 \left( \frac{\xi}{d} \right) \frac{2 \sec^2 \left( \frac{\xi}{d} \right) \left( 1 - \omega_x \tan \left( \frac{\xi}{d} \right) \right) \omega_0^2 e_{10}}{d^2 \omega_0 e_{02} \left( 1 + \frac{e_{10}}{\epsilon_{02}} \right)} \right] \frac{\partial f_+}{\partial \xi} + \frac{2 \omega_x \sec^2 \left( \frac{\xi}{d} \right) \left( 1 - \omega_x \tan \left( \frac{\xi}{d} \right) \right)}{d \omega_0^2 e_{02} \left( 1 + \frac{e_{10}}{\epsilon_{02}} \right) d \xi_0^2} \right] \left( 1 + \frac{e_{10}}{\epsilon_{02}} \right) \right) \right] \frac{\partial f_+}{\partial \xi} = 0.
\]

(11)
For \( m = 2 \):

\[
\left\{ \frac{\zeta \omega_s \sec^2 \left( \frac{\zeta}{d} \right)}{d} \right\}
- 2 \left( 1 - \omega_s \tan \left( \frac{\zeta}{d} \right) \right) f \left( \frac{\zeta}{d} \right) \frac{\partial f_+}{\partial \zeta}^2
- \frac{\zeta \omega_s \sec \left( \frac{\zeta}{d} \right)}{d} \frac{\partial f_+}{\partial \zeta}^2
- \frac{d^2 \omega_s^2 e_{zz}^2}{d^2 \omega_{0zz}^2} \left( 1 + \frac{\epsilon_{+0}}{\epsilon_{0zz}} \right) \left( 1 - \frac{\omega_{0zz}^2}{\omega_{0zz}^2} \right)
- \frac{2 \sec^2 \left( \frac{\zeta}{d} \right)}{d \omega_{0zz}^2} \left( 1 - \omega_s \tan \left( \frac{\zeta}{d} \right) \right) \omega_{0zz}^2 e_{zz}^2
+ \frac{2 \omega_s \sec \left( \frac{\zeta}{d} \right)}{d \omega_{0zz}^2} \left( 1 - \omega_s \tan \left( \frac{\zeta}{d} \right) \right) e_{zz}^2
- 12b^2 \frac{1 + \frac{\epsilon_{+0}}{\epsilon_{0zz}}}{f_+^2} \left( 1 - \frac{\omega_{0zz}^2}{\omega_{0zz}^2} \right) \left( 1 - \frac{\omega_{0zz}^2}{\omega_{0zz}^2} \right)
- \frac{32R_{+1}^2 e_{zz}^2}{R_{+1}^2 f_+^3} \left( 1 - \frac{\omega_{0zz}^2}{\omega_{0zz}^2} \right) \left( 5 - 2b^2 \right) \left( 1 + \frac{\epsilon_{+0}}{\epsilon_{0zz}} \right) = 0.
\]

Equations (10)–(12) are the required equations for beam width parameter for extraordinary modes of propagation of magnetoplasma. Similarly, the equations for beam width parameter for ordinary mode of propagation can be obtained by replacing \( \omega_s \) by \( - \omega_s \) and + sign by - sign in the subscript of \( f \).

Similarly, on solving imaginary part, we get the condition \( \partial f_+ / \partial \zeta = 0 \) and \( f_+ = \) constant for extraordinary and ordinary modes.

IV. RESULTS AND DISCUSSION

The various parameters taken for numerical calculation are: \( \omega = 1.778 \times 10^{14} \text{rad/s}, m = 253 \mu \text{m}, e = 1.6 \times 10^{-19} \text{C}, \)
\( m = 9.1 \times 10^{-33} \text{Kg}, \) and \( c = 3 \times 10^8 \text{m/s}. \) The value of intensity in the present case is \( I_0 = 1.84 \times 10^{14} \text{W/m}^2. \) Figure 1 represents the variation of beam width parameter \( (f_+) \) with the normalized propagation distance \( (\xi) \) for different values of decentered parameter, \( b = 0.00, 3.90, 3.95, \) and 4.00 for \( m = 0. \) The sensitiveness of decentered parameter is also observed in the present case and it supports the previous work of Nanda et al.\(^{38} \) It is clear from the plot that for extraordinary mode, Fig. 1(a), beam width parameter decreases with the increase in the values of decentered parameter. For \( b = 4.00, \) beam width parameter falls abruptly at normalized propagation distance, \( \xi = 0.10 \) and hence self-focusing becomes strong. In case of ordinary mode, same patterns are observed but self-focusing is weak as compare to extraordinary mode of propagation. Gill et al.\(^{8(6)} \) have studied self-focusing and self-phase modulation as well as self-trapping of cosh-Gaussian beam at various values of decentered parameter \( (b) \) and concluded that self-focusing becomes sharper for \( b = 2 \) and occurs at \( \xi = 1.45 \) and the value of beam width parameter is nearly 0.91 (approximately). In another work, Gill et al.\(^{8(7)} \) reported strong self-focusing effect nearly at \( \xi = 0.65. \) Patil et al.\(^{13} \) have reported strong self-focusing for \( m = 0, b = 2 \) nearly at \( \xi = 0.21. \) In the present work, in the presence of density ramp, we report very strong self-focusing which occurs at \( \xi = 0.10 \) for \( b = 4.00. \) In case of ordinary mode, Fig. 1(b), the self-focusing of beam occurs for \( b = 0.00, 3.90, 3.95, \) and 4.00. For \( b = 4.00, \) self-focusing effect is very strong; however, it is weaker as compared to extraordinary mode. In the absence of decentered parameter no self-focusing is observed in both the cases.

FIG. 1. Variation of beam width parameter with normalized propagation distance for (a) extraordinary and (b) ordinary mode for different values of decentered parameter. The other various values are taken as \( m = 0, d = 5, \)
\( \omega_{0zz}/c = 150, \omega_{0zz}/c = 0.01, \omega_s/c = 0.10, \omega_{0zz}/\omega_s = 0.45, \) and \( m_0/M = 0.01. \)
Figure 2 represents the variation of beam width parameter ($f_z$) with the normalized propagation distance ($\zeta$) for different values of decentered parameter, $b = 0.00$, $3.04$, $3.09$, and $3.14$ for $m = 1$. It is clear from the plot in Fig. 2(a) that with the increase in the values of decentered parameter beam width parameter decreases and hence self-focusing effect is observed. However, for $b = 0$, diffraction term becomes more dominant over focusing term and causes the defocusing of beam. Fig. 2(b) describes the same pattern as that in Fig. 2(a); however, self-focusing is weaker in this case. Patil et al.\textsuperscript{15} have reported strong self-focusing for $m = 1$, $b = 2$ nearly at $\zeta = 1.9$ (approximately) and for $b = 0$ and 1, diffraction effect becomes dominant, both for extraordinary and ordinary modes of propagation. In the present work, in the presence of density ramp, we report very strong self-focusing which occurs at $\zeta = 0.10$ for $b = 3.14$ and only for $b = 0$ defocusing of beam occurs while for $b = 1$, 2, and 3, self-focusing of beam occurs and is very strong for $b = 3.14$.

Figure 3 represents the variation of beam width parameter with normalized propagation distance for different values of decentered parameter, $b = 0.00$, $1.95$, $2.00$, and $2.05$ for $m = 2$ for extraordinary mode of propagation. In this case, for $b = 1.95$, $2.00$, and $2.05$, beam exhibits self-focusing effect for both extraordinary and ordinary modes, however, relatively self-focusing is weaker in case of ordinary mode as compared to that in case of extraordinary mode of propagation. While for $b = 0.00$ beam gets defocused both for extraordinary and ordinary mode of propagation. Patil et al.\textsuperscript{15} have reported strong self-focusing for $m = 2$, $b = 0$ and 1 nearly at $\zeta = 2.3$ (approximately) and for $b = 2$, diffraction effect becomes dominant, both for extraordinary and ordinary modes of propagation. In the present work, in the presence of density ramp, we report very strong self-focusing which occurs at $\zeta = 0.10$ for $b = 2.05$, in case of extraordinary mode, Fig.3(a) and for $b = 2.049$, in case of ordinary mode of propagation Fig. 3(b).
Figure 4 represents the variation of beam width parameter with the normalized propagation distance for mode indices \( m = 0, 1, \) and 2. The other various values are taken as \( b = 3.29, \) \( d = 5, \) \( \text{cor} / c = 150, \) \( \text{aE}^2_0 = 0.01, \) \( \omega_r / \omega = 0.1, \) \( \omega_{cp} / \omega = 0.45, \) and \( m_0/M = 0.01. \)

Figure 4 represents the variation of beam width parameter with the normalized propagation distance for mode indices \( m = 0, 1, \) and 2. The other various values are taken as \( b = 3.29, \) \( d = 5, \) \( \text{cor} / c = 150, \) \( \text{aE}^2_0 = 0.01, \) \( \omega_{cp} / \omega = 0.45, \) and \( m_0/M = 0.01. \)

V. CONCLUSION
From the above results, we conclude that the presence of density ramp enhances the self-focusing effect to a greater extent. For \( \omega_r / \omega = 0.15, \) strong self-focusing is observed than for \( \omega_r / \omega = 0.05, \) and \( 0.10. \) Gupta et al. \( ^{16} \) have investigated the addition focusing of a high intensity laser beam in a plasma with density ramp and a magnetic field and reported strong focusing which occurs at \( \zeta = 0.8 \) for magnetic field 45 MG. However, in our case self-focusing occurs at \( \zeta = 0.1 \) for \( \omega_r / \omega = 0.15. \) Similar patterns are observed in case of mode indices \( m = 1 \) and 2, for optimized values of other parameter as taken in case of \( m = 0. \) Whereas, in case of ordinary mode of propagation, Fig. 5(b), beam width parameter increases with the increase in the value of magnetic field parameter and hence decreases self-focusing of laser beams.
distance. For mode indices $m = 0, 1$ and $2$, self-focusing is more commonly observed for higher values of decentered parameter $b$ viz. 4.00, 3.14, and 2.05, respectively, both for ordinary and extraordinary modes of propagation of laser beams. However, for ordinary mode, self-focusing is a little bit weaker than extraordinary mode. Decentered parameter decides the focusing/defocusing nature of Hermite-cosh-Gaussian beam as for $b = 0$, defocusing of beam occurs and for $m = 1$, weaker self-focusing effect is observed. The dependence of beam width parameter on decentered parameter is previously also investigated by Patil et al.\textsuperscript{15} For $b = 3.29$, self-focusing of Hermite-cosh-Gaussian laser beam is found to occur earlier at $\xi = 0.07$ for mode index $m = 2$ for both modes of propagation. Further, sensitiveness of decentered parameter is observed which strongly supports our previous work.\textsuperscript{20} Also with the increase in the value of magnetic field parameter, self-focusing ability of the laser beam increases abruptly. All this happens due to the presence of plasma density ramp and magnetic field. Thus, plasma density ramp plays a very vital role to the self-focusing of the Hermite-cosh-Gaussian laser beam, and it enhances the self-focusing effect. Present study may be useful for the scientist working on laser-induced fusion.

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