Resonant third harmonic generation of a short pulse laser in plasma by applying a wiggler magnetic field

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A B S T R A C T

We examine the effect of wiggler magnetic field on pulse slippage of short pulse laser-induced third harmonic generation in plasma. The process of third harmonic generation of an intense short pulse laser in plasma is resonantly enhanced by the application of a magnetic wiggler. The laser exerts a ponderomotive force at second harmonic driving density oscillations. The second harmonic oscillations coupled with electron velocity at the laser frequency, produces a non-linear current, driving the third harmonic. Third harmonic pulse generates in the fundamental pulse domain. However, the group velocity of the third harmonic wave is greater than the fundamental wave. Hence, the third harmonic pulse saturates strongly and moves forward from the fundamental pulse at shorter distance than the second harmonic pulse.

1. Introduction

Harmonic generation of electromagnetic radiation in plasma has been an active area of research in recent years [1–5]. The highly non-linear nature of interaction of the laser pulse with plasma implies that harmonic light should be a significant feature of such interaction. Third harmonic generation is a very useful technique that can convert output of infrared lasers to shorter wavelengths in the visible and near ultraviolet. The usual model of third harmonic generation takes the fundamental beam to be Gaussian and has third harmonic generation as sole linearity. Because of the homogeneous nature of the medium, only odd harmonics can be generated in the air with third harmonic generation being dominant. The third harmonic of the fundamental laser wavelengths at 800 nm lies in the ultraviolet region, which makes intense laser filaments appealing for remote sensing applications. Rickes et al. [1] have demonstrated strong enhancement of third harmonic generation in a non-linear medium, prepared in maximum coherence by Stark chirped adiabatic passage. Many mechanisms can generate laser harmonics in plasmas; the main mechanism is in the presence of density gradient in the plasma [2].

Parashar and Pandey [4] have given two schemes for k-matching for second harmonic generation. First, plasma has a density ripple that provides additional momentum required for second harmonic generation, second a wiggler transverse magnetic field introduced in the medium. A wiggler magnetic field provides additional momentum to third harmonic photon, thereby making third harmonic generation a resonant process. Small mismatch can also be beneficial for third harmonic generation [5]. Ferrante and Zarcone [6] have obtained harmonic generation in the skin layer of hot dense plasma and found the explicit dependencies of the third harmonic generation efficiency on the plasma and pump field parameters. Therberge et al. [7] have performed experiments on long range third harmonic generation in air using Ti-Sapphire chirped pulse amplification laser system and observed co-filamentation of high intensity fundamental and third harmonic pulses over long propagation distance using the Lidar technique. Liu et al. [8] have seen harmonic generation in neutral and ionized gases and obtained results of harmonic generation in a simple gas (hydrogen) using 1-ps, 1-μm laser pulses with a range of intensities exceeding from below to far above laser ionization saturation threshold. Akozbek et al. [9] have got third harmonic generation and self-channeling in air using high power femtosecond laser and showed both theoretically and experimentally, that during laser filamentation in air, an intense ultra short third harmonic is generated forming two colored filament. Richard et al. [10] experimentally observed pump pulses and third harmonic generation with Kerr effects and formulate self-consistent and complete set of non-linear Schrödinger equations for a pair of coupled beams-fundamental and its third harmonic.

Shibu and Tripathi [11] have studied phase-matched third harmonic generation of a laser beam propagating through plasma channel and showed that the presence of a background density perturbation can account for phase-matching. Liu et al. [12] tried to measure third harmonic light produced from relativistic harmonic generation. Yang et al. [13] observed strong third harmonic emission with a conversion efficiency higher than 0.1% from plasma channel formed by self-guided femtosecond laser pulses propagating in air. They found an optimized condition under which third harmonic conversion efficiency is maximized. Their experimental results show that radiation of emission in ultraviolet
wavelength range makes a major attribution to third harmonic emission, whereas the effects of self-phase modulation are not important when intense laser pulse interacts with gaseous media. The model has exact solution in which third harmonic beam takes a Gaussian profile. For a phase mismatch, that is zero or negative [14]. The harmonic generation of high power microwave in plasma filled waveguide is studied and the analytical theory is presented. The theoretical and numerical analysis show that the high power micro wave can generate harmonics in the plasma filled waveguide [15]. The third harmonic is produced through the beating of ponderomotive force-induced second harmonic density oscillations [15]. The third harmonic pulse has a higher group velocity of momentum can be provided to the third harmonic photon by the plasma frequency, hence \( \bar{k}_3 > 3\bar{k}_1 \). The difference of momentum can be provided to the third harmonic photon by the magnetic wiggler when \( \bar{k}_1 = 3\bar{k}_1 + \bar{k}_0 \) where \( \bar{k}_0 = (4\pi n_0 e^2 m)^{1/2} \) is the plasma frequency, \( c \) is the velocity of light in vacuum, \( n_0 \) is the electron density and \( e \) and \( m \) are the charge and mass of the electron respectively. The laser imparts an oscillatory velocity to electrons.

\[
\vec{v}_1 = \frac{e\vec{E}_1}{m(c_1 + iv)}.
\]

\( \vec{v}_1 \) and \( \vec{B}_w \) beat to exert a ponderomotive force \( \vec{F}_1 = (i\bar{k}_1 \times \vec{B}_w) \), on all electrons at \( (\omega, \bar{k}_1 + \bar{k}_0) \) giving an oscillatory velocity

### 2. Non-linear current density

Consider the propagation of an intense short laser pulse in plasma in the presence of a wiggler magnetic field \( \vec{B}_w \).

\[
\vec{E}_1 = \hbar A_1(z, t) \exp[-i(\omega_1 t - k_1 z)],
\]

\[
\vec{B}_1 = \frac{c\bar{k}_1 \times \vec{E}_1}{\omega_1},
\]

\[
\vec{B}_w = y\vec{B}_0 \exp(i\bar{k}_0z).
\]

where \( A_1(z, t) = F(z - v_g t), \quad v_g = c(1 - \omega_0^2/\omega_1^2)^{1/2} \). The laser pump and third harmonic electromagnetic wave obey the linear dispersion relation \( \bar{k}^2 \approx (\omega^2/c^2)(1 - \omega_0^2/\omega^2) \). The wave vector \( \bar{k} \) increases more than linearly with frequency \( \omega_0 \), hence \( \bar{k}_1 > 3\bar{k}_1 \). The difference of momentum can be provided to the third harmonic photon by the magnetic wiggler when \( \bar{k}_1 = 3\bar{k}_1 + \bar{k}_0 \) where \( \omega_0 = (4\pi n_0 e^2 m)^{1/2} \) is the plasma frequency, \( c \) is the velocity of light in vacuum, \( n_0 \) is the electron density and \( e \) and \( m \) are the charge and mass of the electron respectively. The laser imparts an oscillatory velocity to electrons.

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**Fig. 1.** Variation of normalized amplitude of third harmonic pulse \( |A_1/A_0| \) denoted by 2 and incident laser pulse \( |A_1/A_0| \) denoted by 1 with normalized propagation distance \( z' = z/c_0 \) for \( (\omega_0^2/\omega_0^2) = 0.8 \), \( |z_0 A_2|/c_0 R/2|z| = 0.05 \) at (a) \( t' = 0 \), (b) \( t' = 2 \), (c) \( t' = 4 \), and (d) \( t' = 6 \).
\[
\tilde{v}_1 = -\frac{e^2 E_1 B_0}{2\omega_0 \omega_1^2 (\omega_1 + i\nu)} \tilde{z}.
\]  
(3)

It also produces density perturbation at \((\omega_1, k_1 + k_0)\), in compliance with the equation of continuity, we get,

\[
n_1 = \frac{(k_1 + k_0)}{\omega_1} \tilde{v}_1
\]  
(4)

\[\tilde{v}_1\] and \(\tilde{B}_1\) also beat to exert a ponderomotive force on electrons at \((2\omega_1, 2k_1)\),

\[
\tilde{F}_2 = -(e/2c)((\tilde{v}_1 \times \tilde{B}_1) - (e/2c)(\tilde{v}_2 \times \tilde{B}_w)) \text{ which gives an oscillatory velocity at } (2\omega_1, 2k_1).
\]

\[\tilde{v}_2 = -\frac{e^2 E_1 k_1}{4m^2 \omega_1^2 (\omega_1 + i\nu)} \tilde{z}.
\]  
(5)

\[\tilde{v}_1\] and \(\tilde{v}_2\) beat with \(\tilde{B}_1\) and \(\tilde{B}_w\), respectively, to produce a transverse second harmonic ponderomotive force at \((2\omega_1, 2k_1 + k_0)\),

\[
\tilde{F}_2 = -(e/2c)((\tilde{v}_2 \times \tilde{B}_1) - (e/2c)(\tilde{v}_2 \times \tilde{B}_w)) \text{ which gives an oscillatory velocity at } (2\omega_1, 2k_1 + k_0),
\]

\[
\tilde{v}_2^{(1)} = \frac{3e^2 Bo E_1^2}{16cm^2 \omega_1^2 (\omega_1 + i\nu)} \tilde{x}.
\]  
(6)

\[\tilde{n}_2\] and \(\tilde{B}_1\) also beat to exert a ponderomotive force on electrons at \((3\omega_1, 3k_1)\),

\[
\tilde{F}_2 = -(e/2c)((\tilde{v}_2 \times \tilde{B}_1)) \text{ giving oscillatory velocity } \tilde{v}_3 \text{ at } (3\omega_1, 3k_1),
\]

\[
\tilde{v}_3 = \frac{e^2 E_1 k_1^2}{24m^2 \omega_1^2 (\omega_1 + i\nu)} \tilde{z}.
\]  
(7)

\[\tilde{v}_2\] and \(\tilde{B}_w\) beat to exert a ponderomotive force \(\tilde{F}_3 = -(e/2c)(\tilde{v}_2 \times \tilde{B}_w)\), on electrons at \((2\omega_1, 2k_1 + k_0)\), giving an oscillatory velocity \(\tilde{v}_2\)

\[
\tilde{v}_2 = \frac{e^2 E_1 k_1}{16cm^2 \omega_1^2 (\omega_1 + i\nu)} \tilde{z}.
\]  
(8)

\[\tilde{v}_2\] and \(\tilde{v}_3\) beat with \(\tilde{B}_1\) and \(\tilde{B}_w\), respectively to produce a transverse third harmonic ponderomotive force at \((3\omega_1, 3k_1 + k_0)\),

\[
\tilde{F}_3 = -(e/2c)(\tilde{v}_3 \times \tilde{B}_1) - (e/2c)(\tilde{v}_3 \times \tilde{B}_w) \text{ which gives non-linear oscillatory velocity at } (3\omega_1, 3k_1 + k_0),
\]

\[
\tilde{v}_3^{(1)} = \frac{e^2 B_2 k_1 E_1^2}{88 cm^2 \omega_1^2 (\omega_1 + i\nu)} \tilde{x}.
\]  
(9)

The non-linear current density at the third harmonic turns out to be

\[
\tilde{j}_3^{(1)} = -n_0 e^2 \tilde{v}_3^{(1)} \frac{1}{2} n_0 e^2 \tilde{v}_2
\]

\[
= \frac{n_0 e^2 Bo E_1}{16cm^2 \omega_1^2 (\omega_1 + i\nu)} \left[ \frac{5k_1}{18\omega_1} + \frac{k_1 + k_0}{\omega_1 + i\nu} \right] \tilde{x}.
\]  
(10)

In weakly collisional plasma, we may ignore \(v\). The wave equation for third harmonic field \(E_3\) is written as

\[
\nabla^2 E_3 = \frac{c^2}{c^2} \frac{\partial^2 E_3}{\partial t^2} - \frac{1}{c^2} \frac{\partial^2 E_3}{\partial z^2}
\]  
(11)

where \(\tilde{j}_3 = \tilde{j}_1 + \tilde{j}_2\) and \(\tilde{j}_3\) is the linear density due to self-consistent field \(E_3\); \(\tilde{j}_1 = -n_0 e^2 \tilde{v}_1\), where is governed by the equation of motion \(\partial^2 \tilde{j}_1/\partial t^2 = -eE_1/m\), giving

\[
\frac{\partial j_1}{\partial t} = \frac{n_0 e^2 E_1}{m}.
\]  
(12)

\[
\frac{\partial A_3}{\partial z} + \frac{1}{\nu_{63}} \frac{\partial A_3}{\partial \eta} = \frac{\omega_3}{2} |F(z - \nu_3 t)|^2,
\]  
(13)

where \(\nu_3 = c(1 - \omega_3^2/90c_1^2)^{1/2}\) and \(\omega_3 = (3\omega_1) e^2 Bo k_1 / (32 m^2 \omega_1 c k_1) \times (23k_1/18) + k_0). Now we specify the temporal profile of laser pulse to be Gaussian, \(F(z - \nu_3 t) = A_0 \exp(- (z - \nu_3 t)^2 / \nu_3^2), \) where \(\tau\) is the laser pulse length. Introducing a new set of variables \(z - \nu_3 t = \zeta, \text{ and } z = \eta,\) we can write

\[
\frac{\partial}{\partial \zeta} = \frac{\partial}{\partial \zeta} + \frac{\partial}{\partial \eta} \text{ and } \frac{\partial}{\partial \zeta} = -\nu_3 \frac{\partial}{\partial \zeta}
\]

and Eq. (13) can be written as

\[
\beta \frac{\partial A_3}{\partial z} + \frac{\partial A_3}{\partial \eta} = \beta_3 A_3^2 \exp\left(-\frac{\zeta^2}{\zeta_0^2}\right).
\]  
(14)

where \(\beta = (1 - \nu_3/t_0^2)\), \(\zeta_0 = \tau v_3 t_0\). The complimentary solution of the above equation is \(A_3 = (\zeta_3 A_0^2 / \beta) \tilde{f}(\zeta - \beta \eta)\) while the particular integral is \(A_3 = (\zeta_3 A_0^2 / \beta) \tilde{f}(\zeta)\) \(f(\zeta) = \int_0^{\infty} \exp(-\zeta^2 / \zeta_0^2) d\zeta\). Hence the complete solution of Eq. (14) can be written as

\[
A_3 = \frac{\zeta_3 \beta A_0^2}{2\beta} \left[ \text{erf}\left(\frac{\zeta}{\zeta_0}\right) - \text{erf}\left(\frac{\zeta - \beta \eta}{\zeta_0}\right) \right],
\]  
(15)

where \(\text{erf}(\psi)\) is the error function of argument \(\psi\). The ratio of amplitudes of third harmonic and the fundamental is
\[ A_3 \frac{A_0}{N_0} = \frac{2z_A^2 A_0 \sqrt{\pi}}{2\beta} \left[ \text{erf}(z' - t') - \text{erf} \left( (1 - \beta)z' - t' \right) \right], \]  

where \( z' = z/\zeta_0 \) and \( t' = \beta t/\zeta_0 \) are dimensionless quantities. For a typical case, plasma irradiated by a 1.06 \( \mu \text{m CO}_2 \) laser (\( \omega_0 = 1.8 \times 10^{14} \text{ rad/s} \)) of intensity \( 10^{15} \text{ W/cm}^2 \), and plasma density \( n_0 = 2 \times 10^{15} \text{ cm}^{-3} \), we have plotted, in Fig. 1, normalized amplitude of the third harmonic with the normalized propagation distance \( z' \) for \( \omega_2/\omega_1 = 0.8 \), \( |z_A| A_0 \sqrt{\pi}/2\beta | \approx 0.05 \). \( \zeta_0 \) and different values of \( \varepsilon \). At \( t = 0 \), the third harmonic pulse of small amplitude generates in the domain of the fundamental laser pulse as depicted in Fig. 1a. The fundamental laser pulse is denoted by 1 and third harmonic pulse by 3. At \( t = 2 \), the amplitude of the third harmonic pulse increases and third harmonic pulse starts slipping out of the domain of the fundamental laser pulse. At \( t = 4 \), the third harmonic pulse moves forward and its amplitude saturates at \( t = 6 \) as depicted in Fig. 1c and d.

Wiggler magnetic field also plays an important role in resonant third harmonic generation of an intense pulse laser in plasma. We also see the effect of wiggler magnetic field on third harmonic generation. If we increase the value of external wiggler magnetic field, one could improve the efficiency of third harmonic generation. In addition to wiggler field, if one applies a strong guide magnetic field, one could improve the efficiency of harmonic generation. In comparison with the second harmonic pulse [3], third harmonic pulse saturates strongly and moves forward from the fundamental pulse at shorter distance than the second harmonic pulse. Enhancement in the efficiency of the third harmonic generation strongly depends on the strength of the wiggler magnetic field.

3. Discussion

The group velocity mismatch between the fundamental laser and the third harmonic is significant in high density plasma. Effect increases with increase in plasma density. Third harmonic group velocity is found to be greater than the fundamental velocity, hence slips out of the fundamental wave. Here we have shown graphically the comparison of the fundamental pulse and third harmonic pulse at different times. One can see that the third harmonic pulse moves forward and saturate for a long distance. Wiggler strength \( > 10 \text{ kG} \) is required to achieve high efficiency of third harmonic generation. In addition to wiggler field, if one applies a strong guide magnetic field, one could improve the efficiency of harmonic generation.

References