CHAPTER 7

OBSERVATION OF EARLY AND STRONG RELATIVISTIC SELF-FOCUSING OF COSH-GAUSSIAN LASER BEAM IN COLD QUANTUM PLASMA

7.1 INTRODUCTION

In the year 1962, Askar'yan [1] discovered the self-focusing effect of light. Hora [2], Siegrist [3] etc. have the remarkable contribution in the field of relativistic self-focusing of light. Thereafter, it attracts the attention of researcher and turn out to be most charming field of research. Lot of work has been done on self-focusing of laser beam in plasma[2-4], cluster[5, 6], liquid [7] etc. using various beam profiles like Gaussian beam [4], Hermite-Gaussian beam [15], cosh-Gaussian beam [14], Hermite-cosh-Gaussian beam [16-18, 95, 96, 98] etc. Self-focusing of light has many socially useful applications like x-ray lasers and the laser driven accelerators [8], the generation of inertial fusion energy driven by lasers [9-11] etc. which makes the life of human being quite easier. Short pulse laser having extremely high intensity of the order of $10^{17} - 10^{20} \text{ W/cm}^2$ enabled various high energy related experiments in the field of science and technology.

Self-focusing phenomenon in plasma arises as the laser light propagates through the plasma and modifies the dielectric constant of the plasma. It may be relativistic or ponderomotive or thermal self-focusing in nature. Now a day, propagation of laser beam through cold plasma is widely studied by researcher because the quantum plasma systems have many useful applications. Shukla [101], Misra [102], Bergamin [103] and many other researchers has studied nonlinear interaction in quantum plasma. Quantum plasma has high density and low temperature and now it is possible to produce plasmas with densities near to solid state density. Moreover, quantum plasma systems become more significant because of their relevance to laser-solid interactions, quantum dots [104], astrophysical and cosmological environments [105, 106], nanotechnology [107-109] and fusion-science [110, 111]. In classical plasmas, Boltzmann-Maxwell statistical distribution is widely used while in the quantum plasmas, Fermi-Dirac statistical distribution is used and Wigner formalism is employed rather than classical Vlasov equation [112]. In classical regimes, the de-Broglie wavelength is very small and all
particles are considered as point-like particles, however; if the de-Broglie wavelength becomes of the order of the average interparticle distance, then quantum effect can be considered [113].

In the present case, relativistic self-focusing effect in cold plasma is analyzed. The high intensity laser pulses provide sufficient energy to the constituents like electrons of the plasma which cause an electron oscillatory velocity comparable to the velocity of light. Thus the mass of electron, oscillating at relativistic velocities in laser field, increases by a factor given by $\gamma = 1/\left(1 - v^2/c^2\right)^{1/2}$ and give rise to non-linearity due to which the relativistic self-focusing effect occurs. Earlier, self-focusing in cold plasma has been studied by Jung et al. [79]. The self-focusing of Gaussian laser beam in relativistic cold quantum plasma has been studied by Patil et al. [114] and reported strong self-focusing of the beam with the increase in the value of intensity parameter and relative density parameter due to the generation of quasi-stationary magnetic field. Habibi et al. [115] has studied stationary self-focusing of intense laser beam in cold quantum plasma using ramp density profile.

In the present work, propagation of cosh-Gaussian beam in cold plasma has been studied. We have choose the cosh-Gaussian laser beam profile as cosh-Gaussian laser beam possesses more power than that of Gaussian laser beams having high intensity near the axis of propagation and hence, generates flat top beam profiles [99] which is useful for scribing type of applications in electronics where same intensity of laser beams for long time is required. Zhang et al. [100] has identified a group of virtual sources that generate a cosh-Gaussian wave. Previously, cosh-Gaussian profile has been studied by various authors viz. Gill et al. [14], Nanda et al. [116] etc. We develop the equations for beam width parameter for cosh-Gaussian beam and solve them numerically by applying Wentzel-Kramers-Brillouin (WKB) approximation and Paraxial approximation [4, 90] and observed the early enhancement of self-focusing of the laser beam with normalized propagation distance. This paper is planned as follows: we find the beam width parameter equation in section II, result is discussed in section III and a brief conclusion is given in section IV.
7.2 EVOLUTION OF BEAM WIDTH PARAMETER

The field distribution of cosh-Gaussian laser beam propagating in the plasma along z-axis is of the following form:

\[
E(r, z) = \frac{E_0}{f(z)} e^{\frac{k^2}{4}} \left\{ e^{-\left(\frac{r}{\sqrt{f(z)}} \cdot \frac{b}{2}\right)^2} + e^{-\left(\frac{r}{\sqrt{f(z)}} \cdot \frac{b}{2}\right)^2} \right\} \quad \text{... (7.1)}
\]

here \( E_0 \) is the amplitude of cosh-Gaussian laser beam for the central position at \( r = z = 0 \), \( f(z) \) is the dimensionless beam width parameters, \( r_0' \) is the spot size of the beam and \( b' \) is the decentered parameter of the beam.

The propagating beam imparts an oscillatory velocity, \( v = eE/m_0\omega \), to the electrons. Here \( e, m_0 \) and \( \omega \) are the charge on electron, rest mass of electron and angular frequency of incident laser beam respectively, and \( \gamma = \left(1 + c^2E^2/m_0^2\omega^2\right)^{1/2} \) is the intensity dependent relativistic factor with \( \alpha = e^2/m_0^2\omega^2c^2 \), here \( c \) is the speed of light in vacuum.

The intensity dependent dielectric constant for the non-linear medium is obtained by applying the approach given by Sodha et al. [4]:

\[
\varepsilon = \varepsilon_0 + \phi(EE^*) \quad \text{... (7.2)}
\]

where \( \varepsilon_0 = 1 - \frac{\omega^2}{\omega_p^2} \) is linear part of the dielectric constant with \( \omega_p \) as plasma frequency. For cold plasma the dielectric constant is obtained by applying the approach as applied by Jung and Murakami [79],

\[
\varepsilon_{rel} = 1 - \frac{\omega_p^2}{\gamma^2} \left(1 - \frac{\delta q}{\gamma}\right)^{-1} \quad \text{... (7.3)}
\]

with \( \delta q = 4\pi^4 h^2 / m^2\omega^2\lambda^4 \), where \( h \) is Planck’s constant, \( \lambda \) is the wavelength of incident laser beam. The classical relativistic dielectric constant can be obtained easily by ignoring the quantum effect by setting the value of \( \delta q \) as zero.

For isotropic, non-conducting and non absorbing medium, Maxwell’s equations give the following wave equation

\[
\nabla^2 \vec{E} - \frac{\varepsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} + \vec{\nabla} \left( \frac{\nabla \cdot (\vec{E} \nabla (\varepsilon))}{\varepsilon} \right) = 0 \quad \text{...(7.4)}
\]
For \((1/K^2)\nabla^2 (\ln \varepsilon) << 1\), we get,
\[
\frac{\partial^2 \tilde{E}}{\partial z^2} + \frac{\partial^2 \tilde{E}}{\partial r^2} + \frac{1}{r} \frac{\partial \tilde{E}}{\partial r} + \frac{\omega^2}{c^2} \tilde{E} = 0 \quad \ldots (7.5)
\]
The solution of Eq. (7.5) is of the form \(\tilde{E} = A(r, z) \exp[i(\omega t - kz)]\), with \(k = (\varepsilon_{rel})^{1/2} \omega / c\).
Substituting this value in Eq. (7.5) and neglecting \(\partial^2 A/\partial z^2\), we get a complex differential equation with real and imaginary parts. The real and imaginary parts of this equation are separated by introducing an additional eikonal \(A(r, z) = A_0(r, z) \exp[-ikS(r, z)]\), here \(A_0\) and \(S\) are the real functions of \(r\) and \(z\) respectively.
Real part is
\[
2 \frac{\partial S}{\partial z} + \left( \frac{\partial S}{\partial r} \right)^2 = \frac{1}{2A_0^2 k^2} \frac{\partial^2 A_0^2}{\partial r^2} - \frac{1}{4A_0^4 K^2} \left( \frac{\partial A_0^2}{\partial r} \right)^2 + \frac{1}{2A_0^2 k^2} \frac{\partial A_0^2}{\partial r} + \frac{\phi(A_0^2)}{\varepsilon_0} \quad \ldots (7.6)
\]
Imaginary part is
\[
\frac{\partial A_0^2}{\partial z} + \frac{A_0^2}{r} \frac{\partial S}{\partial r} + A_0^2 \frac{\partial^2 S}{\partial r^2} + \frac{\partial S}{\partial r} \frac{\partial A_0^2}{\partial r} = 0 \quad \ldots (7.7)
\]
The solution of equation (7.6) & (7.7) are of the form,
\[
A_0^2 = \frac{E_0^2}{f^2 (z)} e^{\frac{2b^2}{2}} \left[ e^{-2\left(\frac{r}{\alpha f(z)}\right)^2} + e^{-2\left(\frac{r}{\alpha f(z)}\right)^2} + 2e^{-\left(\frac{2r^2}{\alpha f(z)^2} + \frac{b^2}{2}\right)} \right] \quad \ldots (7.8)
\]
And
\[
S = \frac{r^2}{2} \beta(z) + \phi(z) \quad \ldots (7.9)
\]
with \(\beta(z) = (1/f(z)) df/dz\) where \(\phi(z)\) is an arbitrary function of \(z\).
Using these values in Eq. (7.6), we obtain the equation governing the evolution of beam width parameter,
\[
\frac{d^2 f}{d \xi^2} = \frac{4 - 4b^2}{f^3} \frac{4\alpha E_0^2}{f^3} \left( \frac{\alpha_0 \rho_0}{\omega_0} \right)^2 \frac{\omega}{c} \left( 1 + \frac{4\alpha E_0^2}{f^2} \right)^{\frac{1}{2}} \left[ \left( 1 + \frac{4\alpha E_0^2}{f^2} \right)^{\frac{1}{2}} - \delta q \right]^{-2} e^\frac{b^2}{2} \quad (7.10)
\]
Similarly Eq. (7.7) gives the boundary conditions, \( \xi = 0, f = 1 \) and \( df/d\xi = 0 \).

### 7.3 RESULTS AND DISCUSSION

In the present study numerical calculations has been done by taking the frequency, plasma electron density and spot size of incident laser beam similar as taken by Patil et al.\[114\] and are given by \( \omega = 1.778 \times 10^{20} \text{ rad} / \text{s}, n_e = 4 \times 10^{19} \text{ cm}^{-3} \) and \( r_0 = 20 \mu \text{m} \) respectively. In order to study classical relativistic case, we assume that \( \delta q \to 0 \).

Figure 7.1 depicts the variation of beam width parameter \( f \) with the normalized propagation distance \( \xi \) for classical relativistic and cold quantum cases for decentered parameter \( b = 0 \). From the figure, it is observed that in classical relativistic case the laser beam converges strongly at normalized propagation distance \( \xi = 0.2 \) while in case of cold quantum plasma, it converges strongly earlier at normalized propagation distance \( \xi = 0.12 \). Thus, the focusing length decreases greatly in case of cold quantum plasma than classical relativistic case and hence converging tendency of the laser beam, in the cold quantum case, shifted towards lower value of normalized propagation distance as compared to classical relativistic case. This happens because the quantum contribution adds additional self-focusing effect which is missing in classical relativistic case. The results observed in the present study are in agreement with the previously reported results by Patil et. al.\[114\].

Figure 7.2 depicts the variation of the beam width parameter \( f \) with the normalized propagation distance \( \xi \) at different values of relative density parameter for \( b = 0 \). For relative density parameter, \( \omega_{p0}/\omega = 1 \times 10^{-6} \), the laser beam converges strongly at normalized propagation distance \( \xi = 0.12 \). However; the conversing tendency of the laser beam shifted towards lower value of normalized propagation distance with the increase in the value of relative density parameter. At higher value of relative density parameter, \( \omega_{p0}/\omega = 2 \times 10^{-6} \), earlier and strong self-focusing of laser beam is observed at normalized propagation distance \( \xi = 0.06 \). Further, at certain higher value of relative density parameter, \( \omega_{p0}/\omega = 3 \times 10^{-6} \) strong self-focusing of laser beam is observed at normalized propagation distance \( \xi = 0.04 \). It is obvious from the figure 2 that with the
increase in the value of relative density parameter \( \left( \omega_{p0}/\omega \right) \), self-focusing of cosh-Gaussian beam enhanced and shifted towards lower values of normalized propagation distance. The basic physics behind this is that as the density of the medium enhances, the propagating laser beam in the medium creates more and more relativistic electrons and which results stronger self-focusing effect. Patil et al.[114] has reported strong self-focusing of Gaussian beam at \( \xi = 0.05 \) for \( \omega_p/\omega = 3 \times 10^{-6} \). We have reported strong self-focusing of the laser beam at \( \xi = 0.04 \) for \( \omega_{p0}/\omega = 3 \times 10^{-6} \). In the present study, we observe early and strong self-focusing of cosh-Gaussian laser beam for higher values of relative density parameters.

Figure 7.3 depicts the variation of the beam width parameter \( f \) with the normalized propagation distance \( \xi \) at different values of decentered parameters. It is obvious from the plot that for decentered parameter \( b = 0.0 \), cosh-Gaussian laser beam converges strongly at normalized propagation distance \( \xi = 0.12 \). Similarly, for decentered parameter \( b = 0.9 \), cosh-Gaussian laser beam converges strongly at normalized propagation distance \( \xi = 0.10 \). The early and strong converging of laser beam is observed for decentered parameter \( b = 1.8 \), for which cosh-Gaussian laser beam converges strongly at normalized propagation distance \( \xi = 0.05 \). Thus it is quite obvious from the result obtained that the converging tendency of the cosh-Gaussian laser beam shifted towards lower values of normalized propagation distance for higher values of decentered parameter.

7.4 CONCLUSION

In the present investigation we have studied the relativistic self-focusing of cosh-Gaussian laser beam in cold quantum plasma. We have derived the equation for beam width parameter using WKB approximation and paraxial ray approach. We report early enhancement of self-focusing of cosh-Gaussian laser beam in cold quantum plasma. The comparative study between self-focusing of cosh-Gaussian laser beam in cold quantum case and classical relativistic case has been made for decentered parameter \( b = 0 \) and it is observed that as the beam propagates deeper inside the cold quantum plasma, the self-focusing ability of the laser beam enhances and occurs earlier with normalized
propagation distance due to quantum contribution. Moreover, early and strong self-focusing is observed with the increase in values of the relative density parameter. We conclude that spot size of the cosh-Gaussian laser beam contracts significantly as it propagates deeper inside the cold quantum plasma due to quantum contribution. Also early and strong self-focusing of the laser beam is observed for higher values of decentered parameter. The present study may be helpful to the researchers to select the value of relative density parameter as per their choice to obtained considerable improvement in the focusing quality which may be useful in inertial fusion energy driven by lasers, scribing type of applications in electronics etc.
Figure 7.1: Variation of beam width parameter (f) with normalized propagation distance (ξ) for cold quantum case and classical relativistic case. The various parameters are taken as $aE_0^2 = 0.1$, $\omega_{p0}/\omega = 1 \times 10^{-6}$, $\delta q = 0.00517 \times 10^2$ and $r_0 = 20 \mu m$. 
Figure 7.2: Variation of beam width parameter (f) with normalized propagation distance (ξ) at different values of relative density parameter for b = 0. The other parameters are taken as $\alpha E_0^2 = 0.1$, $\delta \xi = 0.00517 \times 10^2$ and $r_0 = 20 \mu m$. 
Figure 7.3: Variation of beam width parameter ($f$) with normalized propagation distance ($\xi$) at different values of decentered parameter. The other parameters are taken as $\alpha E_0^2 = 0.1$, $\delta q = 0.00517 \times 10^2$ and $r_0 = 20\mu m$. 