Chapter 3

Efficiency-Fairness Trade-off Based Resource Allocation Approaches for Centralized Network

This chapter focuses on utility function based resource allocation for centralized wireless network for the users, engaged in best effort type of applications. In the beginning of the chapter, the essentiality and adequacy of utility function based allocation for cooperative scenario is advocated. Then, types and properties of utility function to cater different traffic is presented followed by system model and optimization problem formulation. Total data rate, individual data rate, fairness index and price of fairness are the performance metrics to evaluate and compare the results. Utility functions are designed with the perspective of satisfying various degrees of efficiency and fairness of resource allocation. In addition to utility function based approach, resource constraint based approach and E-F function based approach for performing efficiency-fairness trade-off are also discussed and compared. At last, a generic utility function is developed to satisfy many criteria of resource allocation.
3.1 Motivation and Problem Analysis

In cooperative network, relay helps one or more sources to achieve spatial diversity at the destination. The relay can be deployed by the service provider to enhance the performance of the network. In such network, the resources-source power, relay power, and bandwidth are to be allocated judiciously to each source as well as relay to help each source. The service provider installs relay with the objective of maximizing the sum total data rate of the network from the point of view to maximize its data dependent revenue. All the sources experience different channels with the destination. If equal resources are given to them, they would not be able to achieve equal data rate due to different channels with the destination. The source experiencing good channel achieves higher data rate compared to the one experiencing the bad channel. Equal allocation of the resources would be sub-optimal allocation from the service provider’s point of view as it would not yield maximum revenue. If resources are allocated to maximize total data rate of the network, then the sources with good channel condition would get more resource compared to that with poor channel condition. This approach, in turn, affects quality of service criteria to all the sources in the network as promised by the service provider. If resources are allocated by keeping the sources with worse channel in mind, the target of total data rate of the network cannot be achieved which may result in loss of revenue to the service provider. Moreover, in the present scenario with variety of applications, the demand of the sources in network varies over a wide spectrum of expected data rate and real time constraints. Even if the resources are allocated to provide equal data rate to every source, it could not satisfy them equally because their requirement of data rate is different.

It seems that the issue of optimum resource allocation cannot be handled by the conventional method in all respect. However, it would be a better strategy if the allocation is done to maximize sum of satisfaction of the users in the network. The satisfaction of the source can be represented as Utility. Several utility functions with required properties are developed and efficiency-fairness trade-off offered by them are investigated in this chapter. The developed techniques are such that it would encourage service provider to install relays and provide better services to the users (or sources). The users can be given the choice to transmit cooperatively or non-cooperatively. Higher data rate and better link reliability encourage users to pay more for cooperative mode. As a result, the service provider can increase the revenue. In this way,
the resource allocation techniques discussed in this chapter provide cooperation encouragement also.

Efficiency – fairness trade-off in resource allocation can be achieved by ensuring minimum amount of resource to any one source and by putting restriction on maximum amount of resource that can be allocated to any single source. We have refereed this technique as resource constraint based allocation. This is explored in in section 3.4 of this chapter. Further, an E-F function is developed for the purpose of efficiency-fairness trade-off in section 3.5. Selection of the values of parameter $E$ and $F$ determines the type of allocation viz. fair, efficient and proportionally fair. Resource allocation can also be done to satisfy other criteria also viz. max-min fairness, proportional fairness, weighted fairness etc. A generic function is developed to cater various goals of allocation as per the requirements in section 3.6. This function is capable of allocating the resources to satisfy various criteria of allocation by selecting appropriate value of only one coefficient.

### 3.2 Utility Function

A function which maps physical quantity to the degree of satisfaction of a user is called Utility. The concept of utility is taken from microeconomics (Fudenberg). Utility is a unit less quantity showing the perceived value of the goods or services to the user. In wireless network, the end user viz. source can perceive the quality of the communication. Source cannot realize the amount of power or bandwidth assigned to it, the data rate received by it or the channel condition experienced by the terminal. The user only understands the degree of goodness or badness of the end application. For example, for voice communication, threshold of bit error rate (BER) is $10^{-3}$. With any amount of resources and under any channel, if the source gets sufficient signal to noise ratio (SNR) to obtain desired BER, the user is fully satisfied and utility is maximum. If user cannot listen or understand the conversation due to degraded SNR or increased BER, the utility for him would be lower. Real time applications give binary utility whereas in best effort data network, utility varies over a certain range between maximum and minimum. Utility as a function of SNR for real time and best effort applications are shown in Fig. 3.1.
Utility has been demonstrated as a function of signal to interference ratio (SIR) in (Xiao). The authors have considered that $U(0) = 0$ and $U(\infty) = 1$. As SIR increases, the quality of service (QoS) improves and the user becomes more satisfied. In (Saraydar), the utility function has been defined as number of information bits received successfully per unit of energy. If $L$ bits are transmitted at power $p$ watts at a rate of $R$ bits/sec in a packet size of $M$ bits ($M > L$) and frame success rate of $P_c$, then the utility is given as $Utility = \frac{L\cdot R \cdot P_c}{M \cdot p \cdot \text{bits joules}}$. Utility as a function of resource allocated has been defined in (Kuo).

In (Wang), utility function for source and relay are defined as buyer and seller of cooperation. Utility of source is the improvement in data rate minus the price to be paid for achieving it. Utility of the relay is the revenue earned in cooperation minus the cost of cooperation. Utility as a ratio of information bits to transmitted power has been modified by adding pricing and resource sharing components in (Jiang). Utility function similar to (Wang) has been defined in (Baidas) for the purpose of auctioning based power allocation in cooperative network. In (Q. H. Cao), utility as a function of received SNR has been considered and it has been utilized in bargaining based power allocation. Power allocation to sources and relays are done to maximize the signal to noise ratio in the cooperative mode. These references show that well defined utility function could be employed in cooperative wireless network for resource allocation.
3.2.2 Properties of Utility Function

When utility function is employed for the purpose of resource allocation, more is always preferred to less. Utility function is a twice-differentiable function (Fudenberg). Let the parameter be ‘ρ’. It can be SIR, SNR or the other parameters as discussed in 3.2. The utility function established must have following properties.

Property 1: The utility function is monotonically increasing function of parameter, ρ.

\[ U'(\rho) > 0 \], i.e. utility increases with the increase in ρ.

Property 2: The utility function follows the law of diminishing marginal utility, \( U''(\rho) < 0 \), i.e. the rate of change of utility with parameter ρ reduces with increase in ρ. The utility is a concave function of ρ.

In the following section, we propose new utility function based approach for optimized resource allocation for multi-user network environment.

3.3 Utility Maximizing Resource Allocation in Multi-User Network

The user would be satisfied by the given data rate depending upon the application. Data rate can be increased by increasing the amount of resources allocated to the user in cooperative mode. In our work, we have considered utility as a function of data rate achieved by the user, which in turn, depends on the amount of allocated resources. Three resources are considered for optimum allocation: (i) Source power (ii) Relay power (iii) Bandwidth. Following different utility functions are considered as a function of data rate.
\[ U_1 = \log R \] (3.1)

\[ U_2 = 1 - e^{-aR} \quad a > 0 \] (3.2)

\[ U_3 = \frac{R^{b-1}}{c} \quad b > 0, c < 1 \] (3.3)

\[ U_4 = \frac{R^{(1-d)}}{1-d} \quad d < 1 \] (3.4)

Where, \( U_j, j = \{1, 2, 3, 4\} \) represents utility as a function of data rate \( R \) and \( a, b, c \) and \( d \) are the coefficient determining the shape and range of the utility function.

### Table 3.1 Notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_i^{NC} )</td>
<td>Data rate achieved by source ( i ) without cooperation</td>
</tr>
<tr>
<td>( R_i^C )</td>
<td>Data rate achieved by source ( i ) with cooperation</td>
</tr>
<tr>
<td>( \Gamma_i^{SD} )</td>
<td>Signal to noise ratio of source-destination link of source ( i )</td>
</tr>
<tr>
<td>( \Gamma_i^{SR} )</td>
<td>Signal to noise ratio of source-relay link of source ( i )</td>
</tr>
<tr>
<td>( \Gamma_i^{RD} )</td>
<td>Signal to noise ratio of relay-destination link of source ( i )</td>
</tr>
<tr>
<td>( h_i^{SD} )</td>
<td>Channel gain of source-destination link of source ( i )</td>
</tr>
<tr>
<td>( h_i^{SR} )</td>
<td>Channel gain of source-relay link of source ( i )</td>
</tr>
<tr>
<td>( h_i^{RD} )</td>
<td>Channel gain of relay-destination</td>
</tr>
<tr>
<td>( P_i^S )</td>
<td>Power allocated to source ( i )</td>
</tr>
<tr>
<td>( P_i^R )</td>
<td>Power allocated to relay to cooperate with Source ( i )</td>
</tr>
<tr>
<td>( W_i^S )</td>
<td>Bandwidth allocated to source ( i )</td>
</tr>
<tr>
<td>( P_{max}^S )</td>
<td>Maximum available power with sources</td>
</tr>
<tr>
<td>( P_{max}^R )</td>
<td>Maximum available power with relay</td>
</tr>
<tr>
<td>( W_{max}^S )</td>
<td>Maximum available bandwidth for allocation</td>
</tr>
<tr>
<td>( n_R )</td>
<td>Additive white gaussian noise at relay with variance ( \sigma_R^2 )</td>
</tr>
<tr>
<td>( n_D )</td>
<td>Additive white gaussian noise at destination with variance ( \sigma_D^2 )</td>
</tr>
</tbody>
</table>
3.3.1 System Model

a). System Platform and assumptions: A network with $N$ sources communicating with the help of one relay to a common destination is considered. The relay is installed by the service provider to create cooperative diversity at the destination. It is assumed that destination has perfect channel state information of all source-relay and relay-destination links. The destination calculates optimum source power, relay power and bandwidth for each source to maximize the sum of their utility. The destination, through the reverse control channel, informs all the sources about the power to be transmitted by them and the bandwidth allocation. It is assumed that channels allocated to the sources are free from interference. The channels between nodes are assumed to remain stationary at least for few symbols. At regular interval, the destination modifies the resource allocation, if necessary and informs sources and relay.

The relay follows half-duplex Amplify and Forward protocol. Cooperative communication is divided in two phases. In phase I, sources transmits information signal which is received by the destination as well as the relay. In phase II, relay transmits the amplified version of the source’s signal. The destination combines the signal received in phase I and Phase II. The relay is assumed to amplify the signal of different sources with different power as informed by the destination through control channel.

b). Mathematical Analysis: All the notations used in this section are defined in table 3.1. Let $X_{Si}$ be the signal transmitted by the source $i$, $i=\{1,2,...,N\}$ in Phase 1. Due to the broadcast nature of the wireless channel, the relay and the destination will receive signal with noise. The signal received by the relay is given as

$$Y_{Ri} = \sqrt{P_i^S X_{Si} h_{i}^{SR}} + n_R$$

(All the symbols carry the meaning as shown in Table 3.1)
The signal received at the destination in phase - 1 is given as

\[ Y_{Di}^1 = \sqrt{P_i^S X_{Si} h_{i}^{SD}} + n_D \] (3.6)

The relay is assumed to have perfect channel state information (CSI). Therefore, the relay scales the received signal by the factor which is inversely proportional to the received power to equalize the effect of the channel between the source and the relay. The relay amplifies the signal with gain \( G_{AF} \) which can be given as

\[ G_{AF} = \frac{1}{\sqrt{P_i^S |h_{i}^{SR}|^2 + \sigma_R^2}} \] (3.7)

The relay normalize the signal as

\[ X_{Ri} = G_{AF} * Y_{Ri} \] (3.8)

The relay, then, forwards the signal with power \( P_i^R \) to the destination. The received signal at the destination in phase - 2 can be given as,

\[ Y_{Di}^2 = \sqrt{P_R} h_{RD} X_{Ri} + n_D \] (3.9)

\[ = \sqrt{\frac{P_i^S P_i^R}{P_i^S |h_{i}^{SR}|^2 + \sigma_R^2}} h_{i}^{SR} h_{RD} X_S + \sqrt{\frac{P_i^R}{P_i^S |h_{i}^{SR}|^2 + \sigma_R^2}} h_{RD} n_R + n_D \] (3.10)

The destination combines the signals received during phase 1 and 2 using maximal ratio combining (MRC) technique (Liu). Combining equations (1) and (3) yields

\[ Y_{Di} = \sqrt{\frac{P_i^S h_{i}^{SD}}{\sigma_D^2}} Y_{Di}^1 + \sqrt{\frac{P_i^S P_i^R}{P_i^S |h_{i}^{SR}|^2 + \sigma_R^2}} h_{i}^{SR} h_{RD} \sqrt{\frac{P_i^R}{P_i^S |h_{i}^{SR}|^2 + \sigma_R^2}} h_{RD} n_R + n_D \] (3.11)

The combined signal to noise ratio (SNR) at the output of the MRC, \( \Gamma_{AF} \) is given by,
\[
\Gamma_{AF} = \frac{P_i^S |h_{SD}|^2}{\sigma_R^2} + \frac{P_i^S |h_{SR}|^2}{\sigma_R^2} \frac{\sigma_R^2 \sigma_D^2}{\sigma_R^2 + \sigma_D^2 + 1} = \Gamma_{SD} + \frac{\Gamma_{SR} \Gamma_{RD}}{\Gamma_{SR} + \Gamma_{RD} + 1} \quad (3.12)
\]

Maximum achievable transmission rate for direct link can be given as,

\[
R_{NC}^i = \log_2 (1 + \Gamma_{SD}) \quad (3.13)
\]

Assuming perfect cooperation using AF protocol at the relay, achievable data rate can be computed as,

\[
R_{C}^i = \log_2 \left( 1 + \Gamma_{SD} + \frac{\Gamma_{SR} \Gamma_{RD}}{\Gamma_{SR} + \Gamma_{RD} + 1} \right) \quad (3.14)
\]

Assuming source assigns half of the time slot or frequency spectrum to the relay for retransmission, the maximum achievable transmission rate for half-duplex relaying can be computed as

\[
R_{C}^i = \frac{1}{2} \log_2 \left( 1 + \Gamma_{SD} + \frac{\Gamma_{SR} \Gamma_{RD}}{\Gamma_{SR} + \Gamma_{RD} + 1} \right) \quad (3.15)
\]

### 3.3.2 Resource Optimization Problem Formulation

In this section, the problem formulation of optimised utility based resource allocation is presented. Following resources are considered foe developing the objective function - source power, relay power and bandwidth. Multiple sources seek help of single relay for communicating with common destination. Source achieves data rate by utilizing the allocated resources as shown in (3.15). Utility function depicted in (3.1-3.4) converts data rate achieved by the user in a unit-less number – utility, which indicates satisfaction achieved by the user after getting the data rate \(R_{C}^i\) as per (3.15). The resources are allocated to maximize the sum of the utility of all the sources in the network.
The optimization problem formed by the destination is stated as,

$$\max_{[W_i, P_i^S, P_i^R]} \sum_i U(R_i^C) \quad [A]$$

subject to

$$\sum_i P_i^S \leq P_{max}^S, \quad P_i^S > 0 \quad (i)$$
$$\sum_i P_i^R \leq P_{max}^R, \quad P_i^R > 0 \quad (ii)$$
$$\sum_i W_i \leq W_{max}, W_i > 0 \quad (iii)$$

In place of maximizing the total data rate of all the users, the sum of the utility of all the user is maximized. Constraints (i-iii) indicate that each node is assigned minimum non-zero resource and the sum of resources allocated to sources and relay are limited to the upper bound of maximum source power $P_{max}^S$, relay power $P_{max}^R$, and bandwidth $W_{max}$.

### 3.3.3 Performance metrics

Three performance metrics are considered for evaluating different the utility functions, namely achievable data rate, fairness index, and price of fairness. These are described in the following subsections.

#### 3.3.3.1 Achievable Total Data Rate

The efficiency of resource allocation is demonstrated by total data rate achievable by the all the sources served by a relay by utilizing given resources. For AF transmission protocol, the data rate achievable is given as (3.15).
3.3.3.2 *Fairness Index (Jain’s Fairness Index)* (Jain)

In (Joe-Wong) (Lan) (Sediq), the fairness of resource allocation has been measured by Jain’s fairness index. Jain’s fairness index $F(x)$ is defined as

$$F(x) = \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n \times \sum_{i=1}^{n} x_i^2}$$

(3.16)

where, $n$ is the number of users and $x_i$ is the benefit obtained by user $i$ in resource allocation. When any one user gets all the resources, the fairness seems to be completely absent. In that case, the minimum value of fairness index would be $(1/n)$. When all the users get equal amount benefit, the ratio becomes 1. We have computed the fairness index using (3.16).

This index is applicable to any resource sharing or allocation problem. It is independent of the amount of the resource. Its value is upper bounded by 1, which indicates the highest degree of fairness of allocation. The minimum value of the index is inverse of the number of candidates contesting for resources. A value of fairness index of 0.1 for resource allocation among 10 participants indicates that the allocation is unfair to the 9 out of 10 participants. As per (Jain), this index exhibits the following properties:

1. Population size independence
2. Scale and metric independence
3. Boundedness
4. Continuity

3.3.3.3 *Price of Fairness (PoF)*

Any attempt to increase fairness index results in decrease in efficiency. The amount of data rate to be sacrificed for achieving higher fairness index is defined as Price of Fairness (PoF). It can also be referred to as loss in efficiency. Mathematically PoF is expressed as

$$PoF = \frac{R_e - R_f}{R_f}$$

(3.17)
where, $R_e$ is maximum total data rate with efficient allocation and $R_f$ is the total data rate with fair allocation. Though efficiency and fairness seem to be difficult to obtain at the same time, the utility functions (1-4) considered in section 3.3 can achieve reasonable fairness with nominal price of fairness.

Efficient allocation considers the maximum outcome obtained with the help of resource. Hence, it allocates resource to maximize the outcome. The efficient allocation may not be fair as it does not allocate any resource to the participant who is not able to contribute in maximizing the outcome using the allocated resource. Therefore, any attempt to improve fairness of allocation imposes penalty on the efficiency of the allocation. Price of fairness parameter indicates the loss in efficiency from its maximum value as a consequence of performing fair allocation. In other words, price of fairness can be defined as a loss of efficiency. The price of fairness value 0 indicates that the allocation is efficient. Higher value of price of fairness indicates fairness of allocation.

These two parameters fairness index and price of fairness together are the indicators of the efficiency-fairness trade-off.

3.3.4 Performance Evaluation and Discussion

The performance of resource allocation technique with various utility functions is evaluated by extensive simulation. The simulation environment with assumptions, results and discussion on the same is presented in the following subsections. Efficiency-fairness trade-off involving each utility function is also demonstrated with the help of simulations.

3.3.4.1 Simulation Model

Wireless nodes in need of cooperation are considered as source nodes. The relay node is either installed by service provider to facilitate cooperation in centrally controlled network. Source nodes communicate with common destination. A multi-user network considered in simulation consists of 4 source nodes ($S_1$ to $S_4$) communicating with a common destination node ($D$) with
the help of a relay node (R) as shown in Fig 3.2. Path loss channel model with exponent 3 is assumed for the sake of simplicity. However, the proposed technique is applicable to fading channel with Rayleigh distribution as well. Channel bandwidth, source power and relay power under consideration for allocation are normalized to 1 so that the allocation indicates the percentage of the total resource assigned to a particular node to satisfy various criteria of efficiency and fairness. The distance between the nodes is assumed in the range of few tens of meters. The central controller at destination determines the amount of resources to be used by each node and informs all through reverse control channel at regular interval. The distances between the nodes are considered such that path loss is minimum for user 4 and gradually increased for user 3, 2 and 1 respectively. User 1 faces the worst channel. The destination uses the channel knowledge between each pair of nodes and appropriate utility function to determine optimum allocation of the source power, relay power and bandwidth for each source. The destination is assumed to employ combining of direct signal from source and relayed signal from the relay using maximal ratio combining technique. It is further assumed that the relays follow Amplify and forward protocol of cooperative communication. The data rate with cooperation is calculated by using the (3.15).

![Fig 3.2 Simulation model](image)

3.3.4.2 Simulation Results and Discussion

Utility functions from (3.1) to (3.4) are applied to the optimization problem [A] and the modified optimization problem is shown in Table 3.2 marked as A-I to A-V.
Table 3.2 Modified optimization problems used in simulation

<table>
<thead>
<tr>
<th>Objective</th>
<th>Utility Function</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximization of total data rate</td>
<td>$U = \sum_{i=1}^{N} R_i$</td>
<td>[A-I]</td>
</tr>
<tr>
<td>Maximization of sum of log of data rate</td>
<td>$U = \sum_{i=1}^{N} \log R_i$</td>
<td>[A-II]</td>
</tr>
<tr>
<td>Maximization of sum of utility function (3.2)</td>
<td>$U = \sum_{i=1}^{N} 1 - \exp^{-aR_i}$</td>
<td>[A-III]</td>
</tr>
<tr>
<td>Maximization of sum of utility function (3.3)</td>
<td>$U = \sum_{i=1}^{N} R_i^b - 1/c$</td>
<td>[A-IV]</td>
</tr>
<tr>
<td>Maximization of sum of utility function (3.4)</td>
<td>$U = \sum_{i=1}^{N} \frac{(1-d)}{(1-d)}$</td>
<td>[A-V]</td>
</tr>
</tbody>
</table>

In A-I, utility is represented by total data rate. Maximization of utility results in maximization of the sum of the data rate of network. In this case, the resources are allocated to maximize the total data rate of all the sources. In this system model, source 4 experiences the best channel, followed by source 3 and source 2. Source 1 has the worst channel. Resource allocation by A-I is shown as “Max Total” in Fig 3.3 to Fig 3.9. Maximum amount of source power, relay power and bandwidth are assigned to source 4 because of the best channel. Hence, the given resources result in the highest data rate. Total data rate achieved in this case is 0.379 units. As the network of 4 sources is under consideration, the fairness index for this type of allocation would be 0.251 (i.e. $I/N$) and price of fairness would be zero as per the definition of PoF given in (3.17). Problem A-II, log utility function, is proportionally fair. It results in total data rate of 0.304, fairness index of 0.962 and price of fairness 0.247. The attempt to increasing fairness index from 0.251 in A-I to 0.962 in A-II, incurs reduction in total data rate from 0.379 units to 0.304 units. A more detailed discussion on proportional fairness is given in section 3.7.

Optimization problems A-III, A-IV and A-V consist of coefficients $a$, $b$, $c$ and $d$. The value of these coefficients result in efficiency-fairness trade-off. The trade-off provided by optimization problem A-III is depicted in Table 3.3. It can be seen from Table 3.3 that as the value of coefficient $a$, increases, more emphasis is given to fairness. However, the price of fairness also increases with $a$. The attempt to achieve high fairness incurs 25.2% to 27.1% penalty in data rate. Allocation of source power, relay power and bandwidth is shown in Fig 3.3, Fig 3.4 and Fig 3.5, respectively.
Table 3.3 Trade-off in A-III for different values of coefficient $a$

<table>
<thead>
<tr>
<th>$A$</th>
<th>Total data rate units</th>
<th>Fairness Index</th>
<th>Price of fairness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.363</td>
<td>0.443</td>
<td>0.044</td>
</tr>
<tr>
<td>2</td>
<td>0.349</td>
<td>0.577</td>
<td>0.085</td>
</tr>
<tr>
<td>5</td>
<td>0.324</td>
<td>0.806</td>
<td>0.170</td>
</tr>
<tr>
<td>15</td>
<td>0.302</td>
<td>0.969</td>
<td>0.252</td>
</tr>
<tr>
<td>25</td>
<td>0.298</td>
<td>0.988</td>
<td>0.271</td>
</tr>
</tbody>
</table>

Comparing with A-I

<table>
<thead>
<tr>
<th></th>
<th>Total data rate units</th>
<th>Fairness Index</th>
<th>Price of fairness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.379</td>
<td>0.251</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Fig 3.3 Allocation of source power for different values of $a$ in Problem A-III
Fig 3.4 Allocation of relay power for different values of $a$ in Problem A-III

Fig 3.5 Allocation of bandwidth for different values of $a$ in Problem A-III
When $a = 1$, source 3 is able to get 35% source power, 34% relay power and 32% bandwidth. As $a = 15$, all the four sources are allocated the resources in a fair way approaching towards the equal share of resources. As $a$ increase further, it is clearly visible from Fig 3.3 to Fig 3.5, that the resources are allocated quite fairly.

Optimization problem A-IV consists of two variable $b$ and $c$. Total data rate, Fairness index and Price of fairness obtained for different values of $b$ and $c$ are as shown in Table 3.4. The allocation of source power, relay power bandwidth to all the 4 users considering different set of values for coefficient b and c are plotted in Fig 3.6 to Fig. 3.8 respectively. It is evident from the above table that for $b > c$, the performance of this utility function is same as that of A-I. For $b < c$, the allocation shows trade-off between fairness and efficiency. For smaller value of $b$, fairness index as high as 0.958 can be achieved with total data rate of 0.304 units. Therefore, price of fairness of 0.244 indicates 24.4% loss in total data rate. When $b = 0.7$, moderate fairness index of 0.747 can be achieved with 15% loss in total data rate. It is further noted that resource allocation is very fair for $b = 0.05$ and $c = 0.9$ as all the four sources are getting 24-26% of source power, 22-28% of relay power and 19-32% of bandwidth compared to almost 100% to source 4 in A-I. For $b = 1.1$ and $c = 0.9$, resource allocation is done in the same way as that in A-I. Allocation of resources for above mentioned cases is demonstrated in Fig 3.6, Fig 3.7 and Fig 3.8. It may be concluded that by appropriately selecting the values of coefficients, desired trade-off can be achieved in the network.

<table>
<thead>
<tr>
<th>$b, c$</th>
<th>Total data rate</th>
<th>Fairness Index</th>
<th>Price of fairness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>units</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05, 0.9</td>
<td>0.304</td>
<td>0.958</td>
<td>0.244</td>
</tr>
<tr>
<td>0.3, 0.9</td>
<td>0.308</td>
<td>0.929</td>
<td>0.227</td>
</tr>
<tr>
<td>0.5, 0.9</td>
<td>0.314</td>
<td>0.881</td>
<td>0.204</td>
</tr>
<tr>
<td>0.7, 0.9</td>
<td>0.328</td>
<td>0.747</td>
<td>0.153</td>
</tr>
<tr>
<td>1.1, 0.9</td>
<td>0.379</td>
<td>0.251</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Comparing with A-I

|         | 0.379 | 0.251 | 0.000 |

Table 3.4 Trade-off in A-IV for different values of coefficient $b$ and $c$
Fig 3.6 Allocation of source power for different values of $b$ and $c$ in Problem A-IV

Fig 3.7 Allocation of relay power for different values of $b$ and $c$ in Problem A-IV
Fig 3.8 Allocation of relay power for different values of $b$ and $c$ in Problem A-IV

Optimization problem A-V consists of coefficient $d$. The performance metrics for different values of $d$ is as shown in Table 3.5.

Table 3.5 Trade-off in A-V for different values of coefficient $d$

<table>
<thead>
<tr>
<th>$d$</th>
<th>Total data rate units</th>
<th>Fairness Index</th>
<th>Price of fairness</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.363</td>
<td>0.406</td>
<td>0.043</td>
</tr>
<tr>
<td>0.3</td>
<td>0.329</td>
<td>0.743</td>
<td>0.152</td>
</tr>
<tr>
<td>0.5</td>
<td>0.315</td>
<td>0.876</td>
<td>0.202</td>
</tr>
<tr>
<td>0.7</td>
<td>0.308</td>
<td>0.929</td>
<td>0.227</td>
</tr>
<tr>
<td>0.9</td>
<td>0.305</td>
<td>0.954</td>
<td>0.242</td>
</tr>
<tr>
<td>Comparing with A-I</td>
<td>0.379</td>
<td>0.251</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Table 3.5 shows that small value of $d$ puts more emphasis on efficiency. For $d = 0.1$ incurs 4.3% loss in efficiency for improving fairness index from 0.25 to 0.4. As $d$ approaches 1, fairness becomes prominent with 24.2% loss in efficiency for $d = 0.9$. Allocation of source power, relay power and bandwidth for problem A-V is shown in Fig 3.9, Fig 3.10 and Fig 3.11. For $d = 0.1$, source 4 gets 72-75% resources, source 3 gets 21-23 % resources, source 2 gets 4-5% of resources and source1 gets only 1% of resources. This condition is reflected by fairness index 0.406 in Table 3.5. As $d$ increase, allocation introduces more and more fairness at the penalty of price of fairness. For $d = 0.9$, 24-26% of source power, 22-28% of relay power and 18-32% of bandwidth are allocated to each source. It results in 24.2% reduction in total data rate of the network. Though the performance of this utility function is in line with that of A-III, it is commonly employed for resource allocation problems (Mo) (Pratt) (Masato) (Srikant) (Borst) as a special case of this utility function shows proportional fairness. Proportional fairness utility function allocates the resources in proportion with demand or channel condition of the sources. Therefore, it is commonly used for solving resource allocation problem. This function is explored in more detail in section 3.7.

Fig 3.9 Allocation of source power for different values of $d$ in Problem A-V
The simulation results discussed above very clearly exhibit that the utility function based resource allocation is capable of providing desired degree of efficiency and fairness trade-off.
for resource allocation in cooperative communication network. To obtain judicious efficiency and fairness trade-off during resource allocation, in our approach, we have introduced coefficients \(a, b, c\) and \(d\) in different utility functions. By appropriately setting the values of these coefficients, desired trade-off can be achieved. It is verified by simulation results.

In the next section we propose another resource allocation technique based on resource constrained approach which further provides desired efficiency – Fairness trade off.

### 3.4 Resource Constraint Based Approach

In order to achieve efficiency-fairness trade-off, restriction can be put on minimum and maximum resources which can be assigned to single source. In multi-source wireless network, each source faces different channel. The efficiency perspective is to assign more resources to the source with good channel condition to maximize sum data rate of the network. But this perspective is very much ‘unfair’ to the source with bad channel condition. As a trade-off, an approach can be employed to assign certain minimum resource to each source so that even the worst channel user would not be deprived of resources completely. Remaining resources are then distributed among the sources to maximize sum data rate of the network. Maximum amount of resources that can be given to any one source is also restricted. Allocation of resources for satisfying desired fairness - efficiency trade-off can be achieved by this mechanism. As per our knowledge, this approach is not employed for resource allocation in cooperative network in literature.

#### 3.4.1 Resource Constrained Allocation Mechanism

Source power, relay power and bandwidth are the three resources which are allocated by the controller at the destination. The concept is explored with the help of allocation of one resource, in general. The same concept then can be extended to all the three resources. Consider multiple units of a resource \(R\) to be distributed among \(N\) sources. The equal share given to each of them would be \(R/N\). Minimum and maximum resource assigned to any one source will be \(A\) times \(R/N\) and \(B\) times \(R/N\), respectively within the constraint of limited total resource. If \(A = B = 1\),
all users will be assigned equal share $R/N$. When $A << 1$, small portion of equal resource $R/N$ is ensured to the each source and the remaining portion of the resource is distributed among all the sources to maximize total data rate of the network to achieve efficiency. Maximum amount of resource given to any one source is $B*(R/N)$. Higher value of $B$ yields better efficiency. The term $A$ is identified as fairness parameter with $0 \leq A \leq 1$ and the term $B$ is introduced as efficiency parameter with $1 \leq B \leq B_{max}$. The value of $B$ which results in the highest efficiency for a given value of $A$ is derived in next subsection.

3.4.1.1 Determination of $B_{max}$

Let Maximum available resource be $X_{AV}$, where $X$ – source power, relay power and bandwidth. $X_{eq}$ be the equal allocation of the resource and $N$ be the number of sources. Minimum resource $X_{min}$ to be allocated to each source is $A * X_{eq}$. Resource remaining after allocation of minimum resource

$$X_{rem} = X_{AV} - (N * A * X_{eq})$$

(3.18)

Maximum resource given to any one source would be possible when $B$ possess the highest value. To calculate the maximum value of $B$, consider the case when all the remaining resource is being allocated to any one source. That source has all the remaining resource in addition to its share of minimum resource. Maximum resource with any one source can be given as

$$X_{max} = X_{rem} + (A * X_{eq})$$

(3.19)

which is equal to $B * X_{eq}$.

Combining (3.18) and (3.19)

$$B * X_{eq} = X_{AV} - (N * A * X_{eq}) + (A * X_{eq})$$

$$\therefore B = \left( X_{AV} + (A * X_{eq}) * (1 - N) \right) / X_{eq}$$

(3.20)
**Numerical Example**

No. of sources, \( N = 4 \)
Resource available for allocation = 1 unit

\[ X_{eq} = 0.25 \quad X_{min} = A \cdot X_{eq}, \quad X_{max} = B \cdot X_{eq} \]

For \( A = 0.5 \), \( X_{min} = 0.5 \cdot 0.25 = 0.125 \)

\[ X_{rem} = 1 - (0.125 \cdot 4) = 0.5 \]

\[ X_{max} = 0.5 + 0.125 = 0.625 \text{ which is equal to } B_{max} \cdot X_{eq} \]

\[ \therefore B_{max} = \frac{0.625}{0.25} = 2.5 \]

Table: 3.6 shows \( B_{max} \) for \( 0 \leq A \leq 1 \) for \( N=4 \).

<table>
<thead>
<tr>
<th>( A )</th>
<th>( B_{max} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>0.1</td>
<td>3.7</td>
</tr>
<tr>
<td>0.3</td>
<td>3.1</td>
</tr>
<tr>
<td>0.5</td>
<td>2.5</td>
</tr>
<tr>
<td>0.7</td>
<td>1.9</td>
</tr>
<tr>
<td>0.9</td>
<td>1.3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.6 demonstrates that for smaller value of \( A \), the value of \( B_{max} \) is large. Therefore, any one source can be assigned large quantity of given resource to yield efficiency. For larger \( A \), most part of the resource is distributed equally among the sources which leads to higher fairness index. For \( A = B = 1 \), leads to exactly equal allocation of resource to all the sources.
3.4.2 Optimization problem formulation

The optimization problem for resource constraint based approach is formulated in this sub-section. The objective is to maximize the sum of the data rate of all the sources $R_i^C$ in the network. $R_i^C$ is derived in (3.15).

\[
\max_{\{W, P_s^i, P_r^i\}} \sum_i R_i^C \tag{B}
\]

subject to

\[
\sum_i P_s^i \leq P_s^{\text{max}}, \quad P_s^i \geq A \cdot P_{eq}^s, \quad P_s^i \leq B \cdot P_{eq}^s \quad \ldots (i)
\]

\[
\sum_i P_r^i \leq P_r^{\text{max}}, \quad P_r^i \geq A \cdot P_{eq}^r, \quad P_r^i \leq B \cdot P_{eq}^r \quad \ldots (ii)
\]

\[
\sum_i W_i \leq W_{\text{max}}, \quad W_i \geq A \cdot W_{eq}, \quad W_i \leq B \cdot W_{eq} \quad \ldots (iii)
\]

\[0 \leq A \leq 1, \quad 1 \leq B \leq B_{\text{max}} \quad \ldots (iv)\]

where $P_{eq}^s$, $P_{eq}^r$ and $W_{eq}$ are equal allocation of source power, relay power and bandwidth to all the sources, respectively. The constraints show that the source power, relay power and bandwidth are upper bounded by $P_s^{\text{max}}$, $P_r^{\text{max}}$ and $W_{\text{max}}$, respectively. Each source must be assigned minimum $A \cdot X_{eq}$ resource i.e. $A$ times the equal allocation and remaining resources are to distributed among all users such that maximum resource given to any user is $B \cdot X_{eq}$ i.e $B$ times the equal allocation. By selecting appropriate values of $A$ and $B$, desired degree of efficiency and fairness can be achieved.

3.4.3 Performance Evaluation and Discussion

The simulation model considered in section 3.3.4.1 is used here also. The minimum value of parameter $A$ is assumed as 0.2 to start with. The fairness parameter $A$ is varied from 0.2 to 1 and the efficiency parameter is varied from 1 to 2.5. The price of fairness is calculated by comparing total data rate with maximum total data rate which is obtained in Optimization

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problem A-I in section 3.2.2. Fig 3.12 shows fairness index and price of fairness for various combinations of A and B. For B = 1, fairness index of 0.963 is guaranteed as maximum amount of resource allocated to any single source is same as equal share. In this case, all the sources can achieve data rate as per their channel condition with equal amount of resources. As efficiency parameter B increases, fairness index reduces for given value of parameter A. By keeping B constant and increasing A results in more fairness. For B = 2, fairness index is 0.559 for A = 0.2 and 0.962 for A = 1. For higher value of A, the change in B does not result in significant change in fairness index as high value of A indicates higher portion of resources are distributed equally and very little is left to increase efficiency. Higher value of fairness index results in higher loss in efficiency in terms of higher value of price of fairness. For B = 2.5, price of fairness varies from 0.074 to 0.258 as A varies from 0.2 to 1. Table 3.7 summarizes the values of performance metrics obtained for B = 1 and B = 2 with A varies from 0.2 to 1.

![Fig 3.12 Fairness index and price of fairness for 0.2 ≤ A ≤ 1 and 1 ≤ B ≤ 2.5](image)

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Fig 3.13 Data rate achieved by sources 1 to 4 for $0.2 \leq A \leq 1$ and $B = 1, 1.5, 2, 2.5$

Fig 3.13 shows data rate achieved by an individual source with different combinations of $A$ and $B$. When $B = 1$, irrespective of $A$, each source is given equal share of resource. They can achieve data rate as per their channel condition. In this simulation model, source 4 experiences the best channel followed by source 3, 2 and 1. As a result, source 4 achieves the highest data rate and source 1 achieves the lowest. When $B = 1.5$, and $A = 0.2$, efficiency is given more emphasis by allocating very little amount of resources equally and remaining resources are distributed to increase efficiency. As parameter $A$ further increases and reaches $A = 1$, the effect of parameter $B$ diminishes. All the sources get data rate as per the case of $B = 1$. Similar scenario can be seen for $B = 2$ and $B = 2.5$. Highest data rate of 0.353 units can be achieved by source 4 when $B = 2.5$ and $A = 0.2$, as shown in Table 3.7
Table 3.7 Comparison of trade-off for $A = 0.2, 0.4, 0.6, 0.8, 1$ and $B = 1, 2$

<table>
<thead>
<tr>
<th></th>
<th>$B=1$</th>
<th>$B=2.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A=0.2$</td>
<td>$A=0.4$</td>
</tr>
<tr>
<td>Total data rate</td>
<td>0.302</td>
<td>0.302</td>
</tr>
<tr>
<td>Fairness Index</td>
<td>0.963</td>
<td>0.963</td>
</tr>
<tr>
<td>Price of Fairness</td>
<td>0.255</td>
<td>0.255</td>
</tr>
</tbody>
</table>

This approach of resource allocation is capable of making desired trade-off between efficiency and fairness depending upon the class of service of the sources. It enables the service provider to perform priority based allocation also by selecting appropriate values of $A$ and $B$.

We have considered $0 < F < 1$. Table 3.8 shows type of the allocation achieved by setting the values of parameters $E$ and $F$. If $F = (1 / (1+E))$ in (3.21), the allocation is called proportional fair as per the definition given in (3.23). For $F < (1 / (1+E))$, the fairness index will be higher which leads to higher value of price of fairness and lower value of total data rate. This scenario is depicted in Fig (3.16) and Fig (3.17).

### 3.5 E-F Function Based Approach

In this section, a function reflecting efficiency and fairness as its components is presented for resource allocation. This function is mentioned in (Joe-Wong) for CPU and memory allocation in data centres. It consists of two parameters $F$ and $E$ such that $F \in \mathbb{R}, E \in \mathbb{R}$. Parameter $F$ determines fairness and $E$ decides efficiency.
\[ \varphi_{EF}^R = \text{sign} \left( 1 - F \right) \left\{ \sum_{i=1}^{N} \left( \frac{R_i^C}{\sum_{j=1}^{N} R_j^C} \right)^{1-F} \right\}^{\frac{1}{E}} \left( \sum_{i=1}^{N} R_i^C \right)^E, \quad F \in \mathbb{R}, E \in \mathbb{R} \] (3.21)

where, \( R_i^C \) is the data rate achieved by user source \( i \) in cooperative mode. \( i = \{1, 2..., N\} \) set of sources, \( F \) is fairness function and \( E \) determines efficiency and \( \varphi_{EF}^R \) is the function which performs efficiency-fairness trade-off depending upon the values of \( E \) and \( F \). The value of \( F \) determines the type and degree of fairness. Type of the fairness is determined by the value of \( F \) as max-min fairness, proportional fairness, and \( \alpha \)-fairness. In this section, we have applied this function for source power, relay power and bandwidth allocation in cooperative network. The limiting case of value of \( F \in (0,1) \) is considered which gives \( \alpha \)-fairness. The relation between the value of \( E \) and \( F \) is given in Table 3.8. To reflect all three scenario, the range of \( E \) is chosen from 1 to 2.

<table>
<thead>
<tr>
<th>( F )</th>
<th>Relation of ( E ) and ( F ) determines type of allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F &lt; \frac{1}{1+E} )</td>
<td>More fairness</td>
</tr>
<tr>
<td>( F = \frac{1}{1+E} )</td>
<td>Proportional Fairness</td>
</tr>
<tr>
<td>( F &gt; \frac{1}{1+E} )</td>
<td>More efficiency</td>
</tr>
</tbody>
</table>

For example, for \( E = 1 \), as per \( F = \frac{1}{1+E} \), \( F \) would be 0.5. The combination of \( E = 1 \) and \( F < 0.5 \) puts more emphasis on fairness, \( F > 0.5 \) gives more efficiency and \( F = 0.5 \) is proportional fair. (Proportional fairness is discussed in more detail in section 3.7 in detail). In this function, smaller value of \( F \) ensures more fairness and large \( E \) ensures efficiency.

### 3.5.1 Optimization Problem Formulation

The optimization problem is formed as shown below.
\[
\max_{\{W_i, p_i^S, p_i^R\}} \sum_i \varphi_{\text{EF}}^R
\]

subject to
\[
\sum_i p_i^S \leq p_{\text{max}}^S, p_i^S > 0 \quad \ldots (i)
\]
\[
\sum_i p_i^R \leq p_{\text{max}}^R, p_i^R > 0 \quad \ldots (ii)
\]
\[
\sum_i W_i \leq W_{\text{max}}, W_i > 0 \quad \ldots (iii)
\]

The source power, relay power and bandwidth are allocated to maximize the \(E-F\) function with appropriate weight to fairness and efficiency. Constraints \((i-iii)\) indicate that each node is assigned minimum non-zero resource and the sum of resources allocated to sources and relay are limited to the upper bound of maximum source power \(p_{\text{max}}^S\), relay power \(p_{\text{max}}^R\), bandwidth \(W_{\text{max}}\).

### 3.5.2 Performance Evaluation and Discussion

Same simulation model and assumptions as section 3.3 are applied for this simulation. Simulation is carried out to find total data rate, fairness index and price of fairness for all combinations of \(E\) and \(F\) factors. Fig 3.14 and Fig 3.15 show total data rate and fairness index, respectively for \(0.1 \leq FF \leq 0.9\) and \(0.1 \leq EF \leq 1.9\), where, \(FF\) is the fairness factor and \(EF\) is the efficiency factor. It show that small \(FF\) and \(EF\) results in less data rate and more fairness.
Fig 3.14 Total data rate for $0.1 \leq F \leq 0.9$ and $0.1 \leq E \leq 1.9$

Fig 3.15 Fairness index for $0.1 \leq F \leq 0.9$ and $0.1 \leq E \leq 1.9$
Fairness index as high as 0.994 is achieved with $FF = 0.1$ and $EF = 0.1$. As $E$ is increased by keeping $FF$ constant at 0.1, data rate increases and fairness index reduces to 0.975. Increase in $EF$ results in more efficiency. $EF=1.9$ and $FF=0.9$ results in the highest data rate of 0.351 units. For the given value of $FF$, data rate increases as $EF$ increases and fairness index reduces. For given value of $EF$, data rate increases and fairness reduces with increase in $FF$. Comparison of combination of two extreme cases of $EF$ and $FF$ is depicted in Table 3.9.

Table 3.9 E-F function based allocation: efficiency-fairness trade-off

<table>
<thead>
<tr>
<th></th>
<th>$EF = 0.1$</th>
<th>$EF = 0.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$FF = 0.1$</td>
<td>$FF = 0.9$</td>
</tr>
<tr>
<td>Total data rate</td>
<td>0.295</td>
<td>0.300</td>
</tr>
<tr>
<td>Fairness Index</td>
<td>0.994</td>
<td>0.975</td>
</tr>
<tr>
<td>Price of Fairness</td>
<td>0.284</td>
<td>0.262</td>
</tr>
</tbody>
</table>

Fairness index varies from 0.459 to 0.994 with corresponding price of fairness from 0.284 to 0.078. The price of fairness is calculated by comparing total data rate with maximum total data rate which is obtained in Optimization problem A-I in section 3.2.2.

As shown in Table 3.8, the relation of factors $E$ and $F$ represents three types of allocation: Efficient, fair and proportional fair. For example, for $EF = 0.7$, $FF = 0.588$ results in proportional fair. $FF > 0.588$ gives higher data rate and $FF < 0.588$ gives more fairness. The regions of type of allocation are depicted in Fig 3.16 and Fig 3.17 from data rate and fairness index perspective, respectively. For $EF = 1.3$, $FF = 0.434$ results in proportional fairness and for $EF = 1.9$, $FF = 0.345$ results in proportional fairness. These two cases are shown with dashed line in Fig 3.16 and Fig 3.17. The proposed $E$-$F$ function is capable of providing efficiency-fairness trade-off for resource allocation by selecting appropriate value of parameters $E$ and $F$. 

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Fig 3.16 Total data rate as a function of FF: Regions of allocation

Fig 3.17 Fairness index as a function of FF: Regions of allocation
3.6 Comparison of Resource Allocation Approaches

Two approaches of resource allocation are presented in section 3.4 and one approach is discussed in section 3.3. The comparison of three approaches is presented in Table 3.10.

Optimization problem A-I maximizes the efficiency without any consideration of fairness, which is indicated by 0.254 \((\approx I/N)\) fairness index and 0 price of fairness. Optimization problem A-II considers only fairness with 0.962 fairness index and 24.7% loss in total data rate. A-I and A-II do not possess any coefficient to do trade-off between efficiency and fairness. In A-III, one coefficient can be adjusted so the total data rate varies from 0.298 to 0.363 and fairness index varies from 0.988 to 0.443. Price of fairness indicates 4% to 27.1% penalty in total data rate. Two coefficients are adjusted in A-IV for achieving any desired fairness index with corresponding penalty of price of fairness.

Table 3.10 Comparison of approaches of section 3.3 and 3.4

<table>
<thead>
<tr>
<th></th>
<th>Utility (Table 3.2)</th>
<th>Resource constraint</th>
<th>E-F function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum total data rate</td>
<td>A-I 0.379</td>
<td>A-II 0.304</td>
<td>A-III 0.363</td>
</tr>
<tr>
<td>Minimum total data rate</td>
<td>-</td>
<td>-</td>
<td>0.298</td>
</tr>
<tr>
<td>Maximum fairness index</td>
<td>-</td>
<td>0.962</td>
<td>0.988</td>
</tr>
<tr>
<td>Minimum fairness index</td>
<td>0.254</td>
<td>-</td>
<td>0.443</td>
</tr>
<tr>
<td>Maximum Price of fairness</td>
<td>-</td>
<td>0.247</td>
<td>0.271</td>
</tr>
<tr>
<td>Minimum price of fairness</td>
<td>0.000</td>
<td>-</td>
<td>0.044</td>
</tr>
</tbody>
</table>
The value of $b = 1.1$ and $c = 0.9$, lead to the similar result as A-I with 0.958 fairness index and zero price of fairness. A-V gives fairness index from 0.406 to 0.958 with the corresponding penalty of 4.3% to 24.2%. Two approaches of this section have minimum penalty of 7.4% and 7.8% compared to 4.3% and 4.4% of A-III and A-V. E-F function based approach reaches the highest among all fairness index of 0.994 with 28.4% loss in total data rate. It can be concluded from these results that the range of trade-off of utility based allocation is wider compared to remaining two approaches. It can be employed to yield desired compromise between efficiency and fairness. However, the selection of the coefficient needs additional efforts. Simplicity is the main attribute of resource constraint based approaches it involves optimization of data rate only. E-F function based approach is capable of giving excellent fairness index. Therefore, it can be employed in a network where fairness is essential.

In centrally controlled network, the controller chooses the value of coefficients by considering the loss in data rate and degree of fairness of services. In the following section, A-V utility function is explored further and is presented as a generic utility function which can satisfy many criteria of resource allocation like (a). Maximizing sum total data rate (b). Achieving proportional fairness (c). Reducing delay to minimum (d). Priority based allocation (e). Max-min fairness (f). Any desired trade-off between efficiency and fairness; by choosing only one coefficient.

### 3.7 Generic Utility Function Based Approach

A generic utility function which can readily be used to achieve different attributes of resource allocation is presented in this section. One utility function is used to achieve maximum data rate, proportional fairness, equal data rate, minimum delay and min-max fairness by simply changing one coefficient in a generic utility function. If it is decided to allocate the resources ‘fairly’, it gives rise to a crucial question of ‘what do we mean by fair?’ There are many approaches in the literature to define fairness like min-max fair, equal share fair, proportional
fair, weighted proportional fair (Masato) (Borst) (Srikant). One thing common in all type of
fairness based allocation is that it would lower the sum total data rate. In this section, we have
evolved a single utility function which can allocate resources to achieve different criteria by
setting proper value of the constant in utility function. Our utility function is capable of
performing resource allocation for (a). Maximizing sum total data rate (b). Achieving
proportional fairness (c). Reducing delay to minimum (d). Priority based allocation (e). Max-
min fairness (f). Any desired trade-off between efficiency and fairness.

3.7.1 Generic Utility Function

A single function which can allocate resources to satisfy different criteria such as proportional
fairness, minimum potential delay fairness and max-min fairness as well as maximum
efficiency and equal data rate extremes of the resource allocation by selecting suitable value of
$L$ in (3.22) given below. No other utility function is capable of providing these many criteria of
resource allocation (Masato) (Borst) (Srikant).

The proposed generic utility function is presented as

$$U_{Gi} = \omega_i \cdot R_i^C(1-L)/(1-L) \quad L > 0, \quad L \neq 1 \quad (3.22)$$

where, $\omega$ is the weight or priority given to a particular source in case of weighted or priority
based resource allocation and $L$ is the coefficient to select the type of allocation. $R_i^C$ is the data
rate achieved by source $i$ in cooperative mode.

3.7.2 Optimization Problem Formulation

In this sub-section, the problem formulation of generic utility function based resource allocation
is presented. The resources under consideration are source power, relay power and bandwidth.
Multiple sources seek help of single relay to communicate with common destination. Utility
function depicted in (3.22) converts data rate achieved by the source in utility which indicates
satisfaction achieved by the source after getting that data rate. $R_i^C$ is the data rate of a source $i$
with cooperation as calculated in (3.15).
The optimization problem is stated as

$$\max_{\{W_i, P_i^S, P_i^R\}} \sum_i \omega_i \cdot R_i^C(1-L)/(1-L) \quad L > 0, \quad L \neq 1 \quad [D]$$

subject to

$$\sum_i P_i^S \leq P_{\text{max}}^S, P_i^S > 0 \quad ...(i)$$

$$\sum_i P_i^R \leq P_{\text{max}}^R, P_i^R > 0 \quad ... (ii)$$

$$\sum_i W_i \leq W_{\text{max}}, W_i > 0 \quad ...(iii)$$

Constraints indicate that each node is assigned minimum non-zero resource and the sum of resources allocated to sources and relay are limited to the upper bound of maximum source power $P_{\text{max}}^S$, relay power $P_{\text{max}}^R$, bandwidth $W_{\text{max}}$.

3.7.3 Types of Fairness

3.7.3.1 Proportional fairness

Let $\{\hat{X}_i\}$ be the resource allocation vector according to proportional fairness and $\{X_i\}$ be the vector of any other allocation. $\{\hat{X}_i\}$ is proved to be proportional fair, if it satisfy the inequality (3.23).

$$\sum_i \frac{x_i - \hat{x}_i}{x_i} \leq 0 \quad (3.23)$$

This inequality states that if resource allocation deviates from proportional fair allocation $\{\hat{X}_i\}$ to any other feasible allocation $\{X_i\}$, then the sum of the proportional changes in each user’s share is less than or equal to 0. Proportional fairness can be attained when $L \to 1$. The function depicted in (3.22) becomes indeterminate for $L = 1$. To evaluate this function for $\to 1$, it is modified as (3.24) considering $\omega = 1$. 

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\[ U_g = \frac{(R_i^{C(1-L)} - 1)}{(1 - L)} \]  

(3.24)

Functions depicted in (3.22) and (3.24) are going to apply for optimization. As far as optimization is concern, additive constant terms in objective functions do not affect optimal decisions.

Using L’ Hospital’s rule on (3.24)

\[
\lim_{L \to 1} \frac{(R_i^{C(1-L)} - 1)}{(1 - L)} = \log(R_i^C)
\]

(3.25)

For \( L \to 1 \), the utility function of (3.22) reduces to log utility function which is inherently proportional fair (Masato) (Borst) (Srikant) (Joe-Wong).

3.7.3.2 Weighted Proportional Fairness

In multi-user network, different users’ class of service can be different. The service provider can earn higher revenue by allocating priority to such users by providing weighing factor or priority factor in the utility factor. For weighted proportional fairness, the inequality (3.23) can be modified as

\[
\sum_i \omega_i \frac{x_i - \bar{x}_i}{x_i} \leq 0
\]

(3.26)

where, \( \omega_i \) is the weight or priority given to a particular user.

3.7.3.3 Max-Min Fairness

If an allocation attempts to maximize the minimum resource allocated in the network, it is referred to as max-min fairness. It gives maximum protection to the source who suffers from the weak channel. Once the allocation is done using Max-min approach, then it is not possible to increase the resources given to any source without decreasing the resource of the source.
whose data rate is minimum among all. In any set of allocation \( \{X_i\} \) is proved to be max-min fair \( \{X_i^*\} \), if it proves that “If \( X_s > X_s^* \) for any source \( s \) in the network, then there exists another source \( p \) such that \( X_p^* \leq X_s^* \) and \( X_p < X_p^* \).” When \( \omega_i = 1 \) and \( L \to \infty \) in utility function (3.22), it corresponds to max-min fairness (Joe-Wong).

### 3.7.3.4 Minimum Delay Fairness

This criteria of fairness deals with the time in which the user can transmit the desired amount of data. Let the vector of data to be sent be \( \{d_i\} \) and data rate achieved by the user be \( \{R_i\} \). Then \( \{d_i/R_i\} \) would be the vector of time taken by each user to complete the data transfer. The objective of resource allocation is to minimize the total delay. i.e. \( \min \sum_i \{d_i/R_i\} \), where \( \sum_i R_i = R_i^c \). It is equivalent to \( \max - \sum_i \{d_i/R_i\} \). If data to be sent is normalized to 1, then it would be \( \max \ (-\sum_i \{1/R_i^c\}) \).

Let \( \omega_i = 1 \) and \( L = 2 \) in (3.22)

\[
U_{Gi} = R_i^{c(1-2)/(1-2)} = -1/R_i^c
\]  

(3.27)

Replacing this utility in objective function \([D]\) yields

\[
\max_{\{W_i P_i^p P_i^R\}} \sum_i - (1/R_i^c)
\]  

(3.28)

which is same as

\[
\min_{\{W_i P_i^p P_i^R\}} \sum_i (1/R_i^c)
\]  

(3.29)

Particular source can be given priority to enable it to send data quickly than others. It is done by adding weighing factor \( \omega_i \). It is, then, referred to as weighted minimum delay fairness.
3.7.3.5 Desired Efficiency-Fairness Trade-off

Moreover, the same utility function can be applied to achieve efficiency, neglecting fairness by choosing $L \to 0$. As $L$ increases from 0, efficiency starts reducing at the cost of fairness and fairness reaches proportional fairness when $L \to 1$. When $L = 2$, it becomes minimum delay fair and for higher $L$, reduction in efficiency becomes significant. Higher $L$ gradually results in allocation such that the data rate of all the users become similar but the sum total data rate of the network goes down. In this way, our proposed utility function can cover the whole spectrum of efficiency-fairness trade-off including both the extremes- perfect efficiency and perfect fairness.

3.7.4 Performance Evaluation and Discussion

Same simulation model and assumptions as section 3.3 are applied for performance evaluation. Simulation is carried out to find total data rate, fairness index and price of fairness for all the cases of allocation.

Table 3.11 Types of resource allocation

<table>
<thead>
<tr>
<th>$L$</th>
<th>$\omega$</th>
<th>Type of allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L = 0$</td>
<td>$\omega = 1$</td>
<td>Maximizing sum of data rates</td>
</tr>
<tr>
<td></td>
<td>$0 &lt; \omega &lt; 1$</td>
<td>Priority based</td>
</tr>
<tr>
<td>$0.1 \leq L \leq 0.9$</td>
<td>1</td>
<td>Efficiency – Fairness trade-off</td>
</tr>
<tr>
<td></td>
<td>$0 &lt; \omega &lt; 1$</td>
<td>Priority based Efficiency – Fairness trade-off</td>
</tr>
<tr>
<td>$L \to 1$</td>
<td>1</td>
<td>Proportional fair</td>
</tr>
<tr>
<td>$L = 2$</td>
<td>1</td>
<td>Minimum delay</td>
</tr>
<tr>
<td></td>
<td>$0 &lt; \omega &lt; 1$</td>
<td>Priority based minimum delay</td>
</tr>
<tr>
<td>$L &gt; 2$</td>
<td>1</td>
<td>Max-min</td>
</tr>
</tbody>
</table>
3.7.4.1 Case – I  Maximizing Sum of Data Rates

When the value of \( L = 0 \) and \( \omega = 1 \), the utility maximization reduces to maximization of the sum total data rate of the network. As source 4 faces the best channel, the destination assigns maximum resources to source 4 and other sources are deprived of the resources which is apparent from Fig 3.22, Fig. 3.23, and Fig. 3.24. Fig 3.18 shows the maximum data rate is achieved by source 4 with maximum resources assigned to it. Fig 3.19 shows the total data rate achieved by the network of four sources. It is evident that total data rate of the network is dominated by the data rate achieved by source 4 in this case. Fig 3.20 shows that fairness index of this case is 0.251 which is the lowest among all allocations.

3.7.4.2 Case – II Efficiency-Fairness Trade-off at \( L = 0.1 \)

As \( L \) rises from 0, the objective function maximizes the sum of the utility of all the users. The value of \( L \) is small so still more resources are assigned to source 4 but the other sources are also given some portion of the resource which makes their data rate higher compared to case - I. As a consequence, total data rate reduces from 0.379 to 0.363 units, fairness index improves from 0.251 to 0.404 and the loss in efficiency, depicted by price of fairness becomes 0.043 (Fig 3.19, Fig. 3.20 and Fig. 3.21 respectively). It can be seen from Fig 3.22, Fig 3.23 and Fig 3.24 that still nearly 72-75% of the total resources are allocated to source 4 and remaining resources are shared among remaining three sources.

3.7.4.3 Case – III Efficiency-Fairness Trade-off at \( L = 0.5 \)

To give more emphasis on fairness, the value of \( L \) is increased further. It can be seen from Fig 3.18 that the difference in the data rates achieved by source 4 and source 1 becomes smaller. This allocation results in further reduction in total data rate and hence increase in price of fairness but fairness index improves to 0.876 from 0.406. The price of fairness increases from 0.043 to 0.202. By keeping \( 0.1 < L < 1 \), any trade-off between efficiency and fairness can be achieved.
3.7.4.4 Case – IV Proportional Fairness at $L \rightarrow 1$

As $L$ approaches 1, the fairness achieved is referred to as proportional fairness. All the sources experience different channel condition. In this case, all the sources would get resources in proportion to their relative channel condition. The resources are allocated to all the source are such that the data rates achieved by each source maintain their mutual relation with each other. As per the definition of proportional fairness in (3.23), Table 3.12 proves that this allocation is proportional fair. Proportional fair allocation means the summation of difference in data rate achieved with any other allocation and proportional fair allocation divided by proportional allocation is less than or equal to 0. In Table.3.12, last row shows that as the allocation done with $L \rightarrow 1$ is proportional fair. In this allocation, fairness index reaches 0.963 with price of fairness 0.248.

![Fig 3.18 Data rate of each source under different cases](image)
Fig 3.19 Total data rate under different cases

Fig 3.20 Fairness index under different cases
Fig 3.21 Price of fairness under different cases

Fig 3.22 Allocation of source power to sources under different cases
Fig 3.23 Allocation of relay power to sources under different cases

Fig 3.24 Allocation of bandwidth to sources under different cases
Table 3.12 Proof of proportional fairness

<table>
<thead>
<tr>
<th>Source</th>
<th>(\frac{(R_{L=0} - R_{L-1})}{R_{L-1}})</th>
<th>(\frac{(R_{L=0.1} - R_{L-1})}{R_{L-1}})</th>
<th>(\frac{(R_{L=0.5} - R_{L-1})}{R_{L-1}})</th>
<th>(\frac{(R_{L=2} - R_{L-1})}{R_{L-1}})</th>
<th>(\frac{(R_{L=5} - R_{L-1})}{R_{L-1}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source 1</td>
<td>-0.996</td>
<td>-0.970</td>
<td>-0.262</td>
<td>0.201</td>
<td>0.295</td>
</tr>
<tr>
<td>Source 2</td>
<td>-0.996</td>
<td>-0.817</td>
<td>-0.097</td>
<td>-0.021</td>
<td>-0.043</td>
</tr>
<tr>
<td>Source 3</td>
<td>-0.996</td>
<td>-0.110</td>
<td>0.096</td>
<td>-0.162</td>
<td>-0.261</td>
</tr>
<tr>
<td>Source 4</td>
<td>2.964</td>
<td>1.884</td>
<td>0.259</td>
<td>-0.054</td>
<td>-0.322</td>
</tr>
<tr>
<td>(\sum_{i=1,2,3,4} \frac{(R_{L=k} - R_{L-1})}{R_{L-1}})</td>
<td><strong>-0.024</strong></td>
<td><strong>-0.014</strong></td>
<td><strong>-0.004</strong></td>
<td><strong>-0.036</strong></td>
<td><strong>-0.332</strong></td>
</tr>
</tbody>
</table>

\(k \in \{0, 0.1, 0.5, 2, 5\}\)

3.7.4.5 Case – V Minimum Potential Delay at \(L=2\)

For \(L = 2\), the allocation tries to minimize the time taken to transmit the data of fixed size with minimum delay. The sources with poor channel are now assigned more resource compared to sources with good channel. Sources 1 and 2 are assigned more than 50% of the source power and relay power compared to 5-6% in case of \(L = 0.1\). As a result, the fairness achieved by this technique is excellent 0.9831 but with heavy price of fairness of 0.286.

3.7.4.6 Case – VI Max-Min Fairness at \(L>>2\)

For \(L >> 2\), (here \(L=5\) is considered in simulation) data rate achieved by source 1 is maximum as compared to source 4. The source with poor channel is given more protection. The data rate achieved by all four sources become nearly equal, which is indicated by fairness index of 0.997 but price of fairness becomes 0.432, which indicates 43.2% loss in efficiency. In this allocation, the source with minimum data rate is given resources to maximize it data rate.

In above mention all the cases, weighing factor is assumed to be 1. All allocations can be priority based by adding weighing factor \(\omega_i\) to each source in the network. Case – I to VI show that proposed generic utility function can be employed to achieve desired type of resource
allocation in the cooperative network. The generic utility function can attain different goals of resource allocation like full efficiency, proportional fairness, max-min fairness, minimum delay fairness. It is also possible to achieve any desired efficiency-fairness trade-off using the same utility function. Simulation results show that fairness index from 0.251 to 0.997 can be achieved with price of fairness ranging from 0 to 0.432. The priority base allocation can also be done by the generic utility function be putting weighing factor in utility function of each source.

3.8 Conclusion

Resource allocation techniques for centrally control multi-source cooperative network are developed in this chapter. The service provider installs one or more relays to generate cooperative diversity in the network. The goal of the service provider is to maximize the revenue as well as to provide satisfactory services in the network. Utility based resource allocation maps data rate of the source in utility and allocate the resources to maximize sum of the utility. Utility functions suitable for data network are developed. Simulation results have shown that limited resource are allocated to achieve different goals of the resource allocation like full efficiency with 0.251 fairness index to 0.998 fairness index with 27% loss in data rate. Resource constraint approach put restriction of data rate and achieve the trade-off with the help of two parameters $A$ and $B$. $B=1$ leads to equal resource allocation and hence fulfil the requirements of fairness, whereas $B=2.5$ leads to efficient allocation for smaller $A$. In E-F function based approach, three types of allocations are obtained, namely efficiency, fair and proportional fair. The condition $F = \frac{1}{1+E}$, leads to proportional fairness, $F < \frac{1}{1+E}$ leads to more fairness and $F > \frac{1}{1+E}$ leads to efficiency. Generic utility function developed in this chapter is proved to be capable of satisfying multiple criteria of resource allocation be selecting only one coefficient. Proportional fair technique of allocation is shown to allocate resource in proportion of their channel gains or demands. Max-min fair technique gives more protection to source with poor channel and allocates more resources, which results in fairness index of 0.997 but loss in efficiency becomes 43.2%.