Research Publications
Paper-1
A Time Dependent Model for Image Denoising

Santosh Kumar, Mohammad Kalimuddin Ahmad
Department of Mathematics Aligarh Muslim University, Aligarh, India
Email: skykumar87@gmail.com, ahmad_kalimuddin@yahoo.co.in

Received 27 January 2015; accepted 16 February 2015; published 25 February 2015

Copyright © 2015 by authors and Scientific Research Publishing Inc.
This work is licensed under the Creative Commons Attribution International License (CC BY).
http://creativecommons.org/licenses/by/4.0/

Abstract
In this paper, we propose a new time dependent model for solving total variation (TV) minimization problem in image denoising. The main idea is to apply a priori smoothness on the solution image. This is a constrained optimization type of numerical algorithm for removing noise from images. The constraints are imposed using Lagrange's multipliers and the solution is obtained using the gradient projection method. 1D and 2D numerical experimental results by explicit numerical schemes are discussed.

Keywords
Total Variation, Image Denoising, Signal Denoising

1. Introduction
In many image processing problems, a denoising step is required to remove noise or spurious details from corrupted images. The presence of noise in images is unavoidable. It may be introduced at the stage of image formation like image recording, image transmission, etc. These random distortions make it difficult to perform any required image analysis. For example, the feature oriented enhancement introduced in [1] is very effective in restoring blurry images, but it can be "frozen" by an oscillatory noise component. Even a small amount of noise is harmful when high accuracy is required, especially in case of medical images.

In practice, to estimate a true signal in noise, the most frequently used methods are based on the least squares criteria. This procedure is $L^2$-norm dependent. $L^2$-norm based regularization is known to remove high frequency components in denoised images and make them appear smooth.

Most of the classical image deblurring or denoising techniques, due to linear and global approach, are...
Paper-2
Dr Kalimuddin Ahmad <ahmad_kalimuddin@yahoo.co.in>  
Sun, Feb 14, 2016 at 6:14 PM

To: Rashmi Bhardwaj <rashmibha@gmail.com>
Cc: Santosh Kumar <skykumar87@gmail.com>, Abul Hasan <siddiqi.abulhasan@gmail.com>

Dear Prof Rashmi,

Thanks for your mail. Soon we will be sending you the tex file together with the corresponding files in journals format which I suppose is the format of springer journals.

With kind regards,

Kalimuddin Ahmad
Deptt of Maths, AMU

Sent from Yahoo Mail on Android

On Sun, Feb 14, 2016 at 5:31 PM, Rashmi Bhardwaj <rashmibha@gmail.com> wrote:

Dear Dr Ahmad,

On behalf of Indian Journal of Industrial and Applied Mathematics, we would like to confirm that your article titled "An efficient PDE-based model for image restoration" authored by Santosh Kumar and M.K. Ahmad has been accepted by the editorial board which will publish in the forthcoming issue of the Indian Journal of Industrial and Applied Mathematics.

Kindly send the doc file of the paper for further processing.

with best wishes

Prof. (Mrs.) Dr. Rashmi Bhardwaj
Professor & Incharge Mathematics Section,
University School of Basic & Applied Sciences,
B-504, Nonlinear Dynamics Research Lab,
Guru Gobind Singh Indraprastha University,
Sector 16 C, Dwarka, Delhi 110078, India.
+91-9868818880, +91-9013534105
011-25302415
rashmibha@gmail.com  rashmib22@gmail.com
An efficient PDE-based model for image restoration

Santosh Kumar · M.K. Ahmad

Received: date / Accepted: date

Abstract In this paper, we propose a new time dependent model for solving total variation (TV) minimization problem in image restoration. The main idea is to apply a priori smoothness on the solution image. The total variation of the image is minimized subject to constraints involving the point spread function (PSF) of the blurring process and the statistics of the noise. The blurring operator provides useful information in restoration. The constraints are implemented using Lagrange’s multipliers. The solution is obtained using the gradient-projection method of Rosen [16]. We present proof of the existence, uniqueness and stability of the viscosity solution of our model. The results of our model using explicit numerical schemes are compared with other known image restoration models.

Keywords Total variation norm · image restoration · nonlinear diffusion

2010 AMS Subject Classification: 65M06, 76R50, 68U10, 35D40

1 Introduction

Image restoration is a fundamental problem in both image processing and computer vision with numerous applications. An image can be interpreted as a real function defined on \( \Omega \), a bounded and open domain of \( \mathbb{R}^2 \) (for simplicity we will assume \( \Omega \) to be the square domain henceforth). Our problem is to restore an image which is contaminated with noise and blurred in such a way that the process should recover the edges of the images. Formation of a blurred and noisy image is typically modeled as

\[ u_0 = H[u] + n, \]  

where \( u_0 \) is the observed image, \( H \) is the PSF (point spread function) and also known as blur kernel and \( n \) is additive noise. For all practical purposes, \( n \) is taken as Gaussian white noise, i.e. the values \( n_{i,j} \) of \( n \) at the pixels \((i,j)\) are independent random variables, each with a Gaussian distribution of zero mean and variance \( \sigma^2 \). The blur operator \( H \) can be described by a Fredholm first kind integral operator

\[ H[u(x,y)] = \int_{\Omega} u(s,r)h(x, s, y, r) \, ds \, dr, \]  

1.1

M.K. Ahmad
Department of Mathematics
Aligarh Muslim University, Aligarh-202002, India
E-mail: ahmad.kalimuddin@yahoo.co.in

Santosh Kumar
Department of Mathematics
Aligarh Muslim University, Aligarh-202002, India
E-mail: skykumar87@gmail.com
AEAM: Submission Confirmation for AEAM-D-15-00027R1

International Journal of Advances in Engineering Sciences (AEAM)  
<em@editorialmanager.com> 
Reply-To: "International Journal of Advances in Engineering Sciences (AEAM)" <rajeswari.sundaram@springer.com> 
To: Santosh Kumar <skykumar87@gmail.com> 

Ref.: Ms. No. AEAM-D-15-00027R1  
A time dependent model for image restoration with forward - backward Diffusivities 

Dear Mr. Kumar, 

International Journal of Advances in Engineering Sciences and Applied Mathematics has received your revised submission. 

You may check the status of your manuscript by logging onto Editorial Manager at http://aeam.edmgr.com/. 

Kind regards, 

Editorial Office 
International Journal of Advances in Engineering Sciences and Applied Mathematics 

santosh kumar <skykumar87@gmail.com>  
To: ahmad_kalimuddin@yahoo.co.in  

[Quoted text hidden] 

AEAM-D-15-00027_R1 (6).pdf  
2799K
**A time dependent model for image restoration with forward - backward Diffusivities**

---Manuscript Draft---

<table>
<thead>
<tr>
<th>Manuscript Number:</th>
<th>AEAM-D-15-00027R1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Title:</td>
<td>A time dependent model for image restoration with forward - backward Diffusivities</td>
</tr>
<tr>
<td>Article Type:</td>
<td>S.I. : PDE Models and Computation</td>
</tr>
</tbody>
</table>
| Corresponding Author: | Santosh Kumar, M.Sc  
Aligarh Muslim University  
Aligarh, Uttar Pradesh INDIA |
| Corresponding Author Secondary Information: | |
| Corresponding Author's Institution: | Aligarh Muslim University |
| Corresponding Author's Secondary Institution: | |
| First Author:      | Santosh Kumar, M.Sc |
| First Author Secondary Information: | |
| Order of Authors:  | Santosh Kumar, M.Sc  
Md. Kalimuddin Ahmad, Ph.D. |
| Order of Authors Secondary Information: | |
| Funding Information: | |
| Abstract:          | In image processing community, image denoising and deblurring, data compression, edge detection etc. are the most challenging and basic hurdles. Inspired by A. Marquina et al. [9], Y. Shi et al. [15] and M. Welk et al. [19], we address new nonlinear anisotropic diffusion models for image denoising and deblurring. To judge our models, we compared with the existing models of A. Marquina et al. [9], Y. Shi et al. [15] and M. Welk et al. [19]. 2D numerical experimental results by explicit numerical schemes are discussed. |
| Response to Reviewers: | We have carefully gone through the comments of learned reviewers and incorporated / modified the manuscript as per their suggestions.  
I am thankful to reviewers for their fruitful suggestions that resulted into the improvement of our paper. Many typos are corrected and language of paper is improved.  
Para-wise reply:  
1. We have rewritten the abstract.  
2. We have added TV- model in introduction vis-a-vis de-noising / Page-2, Para-2.  
3. TV model is added / Page-2, Para-1 of Section-2.  
4. It is incorporated on Page-3 / lines 9-11.  
5. Selection of parameters is explained on Page-7, Para-5 (last Para).  
6. Yes, I do agree with the suggestions and that can be incorporated in our future work.  
7. We shall take care of this in our future work.  
8. Yes, we have checked and revised the paper as per reviewers comments. |
Copyright Transfer Statement

The copyright to this article is transferred to the Indian Institute of Technology Madras (for U.S. government employees: to the extent transferable) effective if and when the article is accepted for publication. The author warrants that his/her contribution is original and that he/she has full power to make this grant. The author signs for and accepts responsibility for releasing this material on behalf of any and all co-authors. The copyright transfer covers the exclusive right to reproduce and distribute the article, including reprints, translations, photographic reproductions, microform, electronic form (offline, online), or any other reproductions of similar nature.

An author may self-archive an author-created version of his/her article on his/her own website and or in his/her institutional repository. He/she may also deposit this version on his/her funder’s or funder’s designated repository at the funder’s request or as a result of a legal obligation, provided it is not made publicly available until 12 months after official publication. He/she may not use the publisher’s PDF version, which is posted on www.springerlink.com, for the purpose of self-archiving or deposit. Furthermore, the author may only post his/her version provided acknowledgement is given to the original source of publication and a link is inserted to the published article on Springer’s website. The link must be accompanied by the following text: “The original publication is available at www.springerlink.com”.

The author is requested to use the appropriate DOI for the article. Articles disseminated via www.springerlink.com are indexed, abstracted and referenced by many abstracting and information services, bibliographic networks, subscription agencies, library networks, and consortia. After submission of the agreement signed by the corresponding author, changes of authorship or in the order of the authors listed will not be accepted.

I, the undersigned corresponding author, also certify that I/we have no commercial associations (e.g., consultancies, stock ownership, equity interests, patent-licensing arrangements, etc.) that might pose a conflict of interest in connection with the submitted article, except as disclosed on a separate attachment. All funding sources supporting the work and all institutional or corporate affiliations of mine/ours are acknowledged in a footnote. Please mention if a separate attachment is enclosed.

Title of article:
A time dependent model for image restoration with forward-backward diffusions

Author(s):
Santosh Kumar and M. K. Ahmad

Author’s signature:

Date: 13-4-2015

Please sign this form and upload the scanned form at:
www.editorialmanager.com/aeam

http://www.springer.com/journal/12572/
A time dependent model for image restoration with forward - backward Diffusivities

Santosh Kumar · M.K. Ahmad

Received: date / Accepted: date

Abstract In image processing community, image denoising and deblurring, data compression, edge detection etc. are the most challenging and basic hurdles. Inspired by A. Marquina et al. [9], Y. Shi et al. [15] and M. Welk et al. [19], we address new nonlinear anisotropic diffusion models for image denoising and deblurring. To judge our models, we compared with the existing models of A. Marquina et al. [9], Y. Shi et al. [15] and M. Welk et al. [19]. 2D numerical experimental results by explicit numerical schemes are discussed.

Keywords Total variation norm · image restoration · anisotropic diffusion

1 Introduction

Image restoration is a fundamental problem in both image processing and computer vision with numerous application. Given a blurry and noisy image

\[ u_0 : \Omega \rightarrow \mathbb{R}, \]

\[ u_0 = k \ast u + n, \tag{1} \]

where \( \Omega \) is a bounded open set in \( \mathbb{R}^2 \), \( u_0 \) is the observed image, \( u \) is the original image, \( k \) is the PSF (point spread function)-usually called blurred kernel and \( n \) is additive white noise assumed to be close to Gaussian. The values \( n(i,j) \) of

S. Kumar
Department of Mathematics
Aligarh Muslim University
Aligarh, 202002, India
E-mail: skykumar87@gmail.com

M.K. Ahmad
Department of Mathematics
Aligarh Muslim University
Aligarh, 202002, India
E-mail: ahmadkalimuddin@yahoo.co.in
Paper-4
Applied and Computational Mathematics
An International Journal

ACM Home

ISSN: 1683-3511 (print version)

Editors in-Chief
Abbasov, Ali (Baku, Azerbaijan)
Aliev, Fikret (Baku, Azerbaijan)
Larin, Vladimir (Kiev, Ukraine)

Managing Editor
Gasimov, Yusif (Baku, Azerbaijan)

Journals Impact Factor is 0.452 (Thomson Reuter Agency Journal Citation Report - 2014).

Applied and Computational Mathematics an International Journal was founded in 2002 by the Ministry of Communications and Information Technology (Azerbaijan), Azerbaijan National Academy of Sciences and Institute of Applied Mathematics of Baku State University. This is a three-annual Journal that publishes high-quality peer-reviewed research papers on the board ranges of applied and computational mathematics.


Impact Factor: 0.452 (2014)*
5-year Impact Factor: 0.592
* Journal Citation Reports®, Thomson Reuters

Source Normalized Impact per Paper (SNIP): 1.207

ISSN: 1683-3511 (print version)
My Articles

1

Full Name

PDE-based nonlinear diffusion model for image denoising

Send  Delete  Edit  Status
PDE-BASED NONLINEAR DIFFUSION MODEL FOR IMAGE DENOISING

SANTOSH KUMAR\(^1\) AND M.K. AHMAD\(^2\)

ABSTRACT. In this paper, we propose a new model for image denoising. Second order partial differential equations have been studied as a useful tool for image denoising. Scale space and edge detection using anisotropic by Parona and Malik has an edge preserving property but sometimes it gives undesirable blurred effect. We prove the existence and uniqueness theorem of our proposed model. The results of our model using explicit numerical schemes are compared with other known image restoration models.

Keywords: Second order PDE’s, image denoising, nonlinear diffusion.

AMS Subject Classification: 35K51, 35K55, 35K99.

1. INTRODUCTION

Image denoising is a fundamental problem in both image processing and computer vision with numerous applications. The total variation models \([7, 19, 20]\) and anisotropic diffusion models \([6, 18, 22, 23, 24]\) have been studied as a useful tool to the problem of image denoising and image reconstruction. These partial differential equation based image enhancement techniques have been able to achieve a good edge preservation. An image can be interpreted as a real function defined on \(\Omega\), a bounded and open domain of \(\mathbb{R}^2\) (for simplicity we will assume \(\Omega\) to be the square domain henceforth). Formation of a noisy image is typically modeled as

\[ u_0(x, y) = u(x, y) + n(x, y), \]

where \(u(x, y)\) denote the desired clean image, \(u_0(x, y)\) denote the pixel values of a noisy image for \(x, y \in \Omega\), \(\Omega\) is a bounded open subset of \(\mathbb{R}^2\) and \(n(x, y)\) is additive white noise assumed to be close to Gaussian. The values \(n(i, j)\) of \(n\) at the pixels \((i, j)\) are independent random variables, each with a Gaussian distribution of zero mean and variance \(\sigma^2\).

We propose the following second order - version of the nonlinear diffusion model which is a synthesis of ideas from Catté et al. \([6]\). Our model is given by:

\[
\frac{\partial u}{\partial t} = \nabla \cdot (g_1(|\nabla G_\sigma \ast u|) \nabla u) + \nabla \cdot (g_2(|\nabla G_\sigma \ast u|) \nabla u) - \lambda (u - u_0) \text{ on } \Omega \times (0, T),
\]

\[
u(x, 0) = u_0(x) \text{ in } \Omega,
\]

\[
\frac{\partial u}{\partial n} = 0 \text{ on } \partial \Omega \times (0, T),
\]

where \(g_1\) and \(g_2\) are decreasing function tending to zero at infinity with \(g_1(0) = M_i > 0\) and \(t \to g_i(\sqrt{t})\) is smooth, and \(G_\sigma(x)\) is the Gaussian kernel, namely,

\[
G_\sigma(x) = \frac{1}{2\pi\sigma^2} e^{-\frac{|x|^2}{2\sigma^2}}.
\]

\(^1\) Department of Mathematics Aligarh Muslim University, Aligarh-202002, India

\(^2\) e-mail: skykumar87@gmail.com

\(^2\) e-mail: ahmad.kalimuddin@yahoo.co.in

Manuscript received xx.
Paper-5
Michael P. Bekakos <mbekakos@ee.duth.gr>
To: Santosh Kumar <skykumar87@gmail.com>

Santosh Kumar:

Thank you for submitting the manuscript, "An efficient PDE-based nonlinear anisotropic diffusion model for image denoising" to Neural, Parallel, and Scientific Computations. With the online journal management system that we are using, you will be able to track its progress through the editorial process by logging in to the journal web site:

Manuscript URL:
Username: sk_rs-kumar

If you have any questions, please contact me. Thank you for considering this journal as a venue for your work.

Michael P. Bekakos
Neural, Parallel, and Scientific Computations

santosh kumar <skykumar87@gmail.com>
To: Sarfaraz Mohammad <sarfarazm820@gmail.com>

[Quoted text hidden]

santosh kumar <skykumar87@gmail.com>
To: ahmad_kalimuddin@yahoo.co.in

[Quoted text hidden]
Active Submissions

Start a New Submission

CLICK HERE to go to step one of the five-step submission process.

Refbacks

There are currently no refbacks.

ISSN: 1061-5369
An efficient PDE-based nonlinear anisotropic diffusion model for image denoising

Santosh Kumar, Mohd. Sarfaraz and M.K. Ahmad
Department of Mathematics
Aligarh Muslim University
Aligarh, 202002, India
E-mails: skykumar87@gmail, sarfarazm820@gmail.com, ahmad.kalimuddin@yahoo.co.in

Abstract

In this paper, we propose a new nonlinear anisotropic diffusion model for image denoising. The main idea is to apply a priori smoothness on the solution image. We present proof of the viscosity solution of our model. The results of our model using explicit numerical schemes are compared with other known image denoising models.

Keywords: Image denoising, nonlinear diffusion

1 Introduction

The nonlinear diffusion techniques and PDE-based variational models are very popular in image denoising and restoration. The nonlinear diffusion method for image denoising and edge detection was first introduced by Perona and Malik [13]. This method is based on a diffusion process governed by a partial differential equation (PDE), where diffusion amount depends on the gradient of images.

Mathematically, $u_0 : \Omega \rightarrow \mathbb{R}$ represents a noisy version of a true image, and it is obtained by the following imaging process

$$u_0(x) = u(x) + n(x),$$

(1.1)

where $u(x)$ denotes the desired clean image, $u_0(x)$ denotes the pixel values of a noisy image for $x \in \Omega$, $\Omega \subset \mathbb{R}^2$ is a bounded domain, usually a rectangle and $n(x)$ is additive white noise assumed to be close to Gaussian. The values $n(i, j)$ of $n$ at the pixels $(i, j)$ are independent random variables, each with a Gaussian distribution of zero mean and variance $\sigma^2$. 
Paper-6
Dear Dr. Kumar,

Thank you for approving the changes and returning your submission entitled "Denoising method based on wavelet coefficients via diffusion equation".

You will be able to check on the progress of your paper by logging on to Editorial Manager as an author. The URL is http://istt.edmgr.com/.

Thank you for submitting your work to this journal.

Kind regards,
Editorial Office
Iranian Journal of Science and Technology, Transactions A: Science
Denoising method based on wavelet coefficients via diffusion equation

Abstract:
Image denoising is still a crucial problem for the image processing community as well as mathematicians alike. This paper proposes a denoising technique based on wavelet coefficients via diffusion equation. The present work compares different denoising processes by diffusion models. The data is initially subjected to synthetic noisy data with various levels of standard deviation $\sigma^2$. To quantify the results, we use peak signal to noise ratio (PSNR) as a metric.

Corresponding Author:
Santosh Kumar, Ph.D
Aligarh Muslim University, Aligarh
Aligarh, Uttar-Pradesh INDIA

First Author:
Santosh Kumar, Ph.D

Order of Authors:
Santosh Kumar, Ph.D
Mohd. Sarfaraz, M.Sc.
Md. Kalimuddin Ahmad, Ph.D

Suggested Reviewers:
Khalil Ahmad, Ph.D
Professor, Jamia Millia Islamia, New Delhi, India
khalil_ahmad49@yahoo.com
Working in the same field.

Rathish Kumar, Ph.D
Professor, Indian Institute of Technology Kanpur
bvrk@iitk.ac.in
Working in same field.

S. Sundar, Ph.D
Professor, Indian Institute of Technology Madras
snt@iitm.ac.in
Working in the same field.

Dhanesh Patel, Ph.D
Professor, Maharaja Sayajirao University of Baroda
pdhanesh@yahoo.com
Working in the same field.
Denoising method based on wavelet coefficients via diffusion equation

Received: date / Accepted: date

Abstract Image denoising is still a crucial problem for image processing community as well as mathematicians alike. This paper proposes a denoising technique based on wavelet coefficients via diffusion equation. The present work compares different denoising processes by diffusion models. The data is initially subjected to synthetic noisy data with various levels of standard deviation $\sigma^2$. To quantify the results, we use peak signal to noise ratio (PSNR) as metric.

Keywords Discrete wavelet transform · image denoising · anisotropic diffusion

1 Introduction

Interest in digital image processing methods stems from two principal application areas:

(a) Improvement of pictorial information for human interpretation
(b) Processing of image data for storage, transmission, and representation for autonomous machine perception.

The crucial point in these approaches is to distinguish between important features that should be kept or even enhanced. Mathematically, different ways have been used to model how a smooth image should look like, i.e., a certain smoothness can be formalised in terms of differentiability orders and small modulus of derivatives. This idea leads to regularisation methods and related partial differential equations. Another kind of smoothness assumption is to discard certain wavelet coefficients, leading to the popular wavelet shrinkage method.

For denoising case, the underlying model for the image degradation is an additive noise model. Thus in discrete setting, we have three images $u_0, u, n \in$