Chapter 5

A time dependent model for image restoration with forward-backward diffusivities

5.1 Introduction

Image restoration is a fundamental problem in both image processing and computer vision with numerous application. Given a blurry and noisy image \( u_0 : \Omega \rightarrow \mathbb{R} \),

\[
    u_0 = k * u + n,
\]

(5.1.1)

where \( \Omega \) is a bounded open set in \( \mathbb{R}^2 \), \( u_0 \) is the observed image, \( u \) is the original image, \( k \) is the point spread function (PSF) usually called blurred kernel and \( n \) is additive white noise assumed to be close to Gaussian. The values \( n(i, j) \) of \( n \) at the pixels \( (i, j) \) are independent random variables, each with a Gaussian distribution of zero mean and variance \( \sigma^2 \).

Consider the degradation model (5.1.1), taking the Fourier transform we arrive at

\[
    \hat{u}_0 = \hat{u} \cdot \hat{k} + \hat{n}.
\]

(5.1.2)

To recover \( u \), we need to deconvolve, i.e., we have to divide (5.1.2) by \( \hat{k} \) and then apply the inverse Fourier transform. This procedure is ill-posed. If \( k \) is smooth, high frequencies tend quickly to zero, implying that those frequencies in \( \hat{u}_0 \) get amplified, and the above model, in spite of its simplicity, is far from efficient.

The Total variation (TV) model is proposed by many researchers but the first
approach in this regard recalls the name Rudin et al. [61]. In 1992, they proposed a constrained optimization type numerical scheme for image denoising. The solution of the imposed problem is obtained by gradient projection method. In 1994, Rudin and Osher [60] proposed another model for image restoration. The researchers looked towards the idea of total variation and have given improved and fast versions of the TV technique [15, 17, 47, 60, 61, 76]. Total variation denoising is a popular method and is considered as a bottom line for edge preserving in image restoration. A large statement is that this method is able to restore sharp edges but at the same time, might met up with some staircasing (i.e., spurious edges) in plane regions. In spite of the fact that the variational problem is convex, the Euler-Lagrange equation is non linear and ill-conditioned. Linear semi-implicit fixed point procedures devised by Vogel and Oman [76], and interior-point primal dual implicit quadratic methods by Chain, Golub and Mulet [15], were introduced to solve the models. Deconvolution with total variation regularisation by variational approaches is studied in [7, 16, 47, 85] which deals with both classes of problems arising in non blind and blind deconvolution.

The rest of this Chapter is organized as follows: Section 5.2 reviews the major PDE based image deblurring and denoising models. Section 5.3 describes the choices of the diffusivity functions. In Section 5.4, we discuss our models and existing models applied on various types of gray scale images. To quantify results, the experimental values in terms of improvement in signal to noise ratio (ISNR) are given in Tables 5.4.1-5.4.4. Section 5.5 concludes the chapter.

5.2 Variational/ PDE-based deconvolution models

In this Section, we present the image restoration models [60, 61] and our new nonlinear diffusion models. The total variational functional introduced by Rudin et al. [61] and Rudin and Osher [60] is given as:

$$\min_u \int_{\Omega} |\nabla u| \, dx \, dy,$$

Subject to $||k * u - u_0||_{L^2}^2 = |\Omega|\sigma^2.$  (5.2.1)
A time dependent model for image restoration with forward - backward Diffusivities

The equivalent unconstrained minimization problem of (5.2.1) can be written as:

\[ \min_u \int_\Omega \left( |\nabla u| + \frac{\lambda}{2} \int_\Omega (k \ast u - u_0)^2 \right) \, dx \, dy. \]  

(5.2.2)

The Euler-Lagrange equation is given by

\[ 0 = -\nabla \cdot \left( \frac{\nabla u}{|\nabla u|} \right) + \lambda k \ast (k \ast u - u_0). \]  

(5.2.3)

Since (5.2.3) is not well defined at points where \( \nabla u = 0 \), due to the presence of the term \( \frac{1}{|\nabla u|} \), it is common to slightly perturb the TV algorithm to become

\[ \int_\Omega |\nabla u|_\beta \, dx \, dy = \int_\Omega \sqrt{u_x^2 + u_y^2 + \beta} \, dx \, dy, \]  

(5.2.4)

where \( \beta \) is a small positive parameter [17].

Usually, time dependent approximations to the ill-conditioned Euler-Lagrange equation (5.2.3) are inefficient. This is because a very small time step is required when a simple explicit scheme is used.

Rudin et al. [60] introduced a time dependent model for image restoration which is given by

\[ u_t = \nabla \cdot \left( \frac{\nabla u}{|\nabla u|} \right) - \lambda k \ast (k \ast u - u_0), \]  

(5.2.5)

with \( u(x, y, 0) \) given as initial data (the original blurred and noisy image \( u_0 \) is taken as initial guess) and homogeneous Neumann boundary conditions \( \frac{\partial u}{\partial n} = 0 \) on the boundary of the domain. The total variation norm does not penalize discontinuities in \( u \), and thus allows us to recover the edges of the original image. To overcome the computational difficulties of total variation (TV) restoration problems, Vogel et al. [76] devised linear semi implicit fixed point procedures and Chan et al. [15] gave a primal-dual implicit quadratic methods. These methods give good results when treating pure denoising problems, but methods become very ill-conditioned for the deblurring and denoising case where the computational cost is very high and parameter dependent. As \( t \) increases, we approach a restored version image, and the effect of the evolution should be edge enhancement and smoothing at small scales in order to remove the noise. This solution procedure is a parabolic equation with time as an evolution parameter and
resembles the gradient-projection method of Rosen [59].

Sometimes the model (5.2.5) converges very slowly to its steady state since the parabolic term is singular for small gradients. At the same time, the Courant-Friedrichs-Lewy (CFL) restriction for keeping stability must be noticed. In order to regularize the parabolic term we multiply the whole Euler-Lagrange equation (5.2.3) by the magnitude of the gradient and time evolution model (see the reference [47]) is given by

\[ u_t = |\nabla u| \nabla \cdot \left( \frac{\nabla u}{|\nabla u|} \right) - |\nabla u| \lambda k * (k * u - u_0), \]  

(5.2.6)

with \( u(x, y, 0) \) given as initial data (the original blurred and noisy image \( u_0 \) is taken as initial guess) and homogeneous Neumann boundary conditions \( \frac{\partial u}{\partial n} = 0 \).

Applying a priori smoothness on the solution image, a time dependent model becomes,

\[ u_t = \nabla G_{\sigma} * u \nabla - \frac{\nabla G_{\sigma} * u}{|\nabla G_{\sigma} * u|} \lambda k * (k * G_{\sigma} * u - u_0), \]  

(5.2.7)

with \( u(x, y, 0) \) given as initial data and homogeneous Neumann boundary conditions as above. It should be noticed that (5.2.7) only replaces \( u \) in (5.2.6) by its estimate \( G_{\sigma} * u \).

Witkin [83] noticed that the convolution of the signal with Gaussian at each scale was equivalent to solving the heat equation with the signal as initial datum. The term \( (G_{\sigma} * \nabla u)(x, y, t) = (\nabla G_{\sigma} * u)(x, y, t) \), which appears inside the divergence term of (5.2.7), is simply the gradient of the solution at time \( \sigma \) of the heat equation with \( u(x, y, 0) \) as initial datum. Then the restoration analysis associated with \( u_0 \) consists in solving the problem

\[ \frac{\partial u(x, y, t)}{\partial t} = \Delta u(x, y, t), \quad u(x, y, 0) = u_0(x, y). \]

The solution of this equation at time \( t \) is given by

\[ u(x, y, t) = G_{\sigma} * u_0, \]

where \( G_{\sigma} \) is the Gaussian function.

In order to preserve the notion of scale in the gradient estimate, it is convenient
A time dependent model for image restoration with forward - backward Diffusivities

that this kernel $G_\sigma$ depends on a scale parameter [45]. In fact, the function $G_\sigma$ can be considered as “low-pass filter” or any smoothing kernel, i.e., a denoising technique is used before solving the nonlinear diffusion problem [3, 13].

The use of nonconvex regularization functionals in image restoration has been investigated by Welk et al. [82]. In general, variational deblurring and denoising of an image can be achieved by minimizing the energy functional

$$E(u) = \int_{\Omega} \psi(|\nabla u|^2) \, dx \, dy + \frac{\lambda}{2} \int_{\Omega} (k * u - u_0)^2 \, dx \, dy.$$  \hspace{1cm} (5.2.8)

The first integral is the smoothness term or regulariser and the second integral, the data term is the squared error of the reconstruction of the blurred image from the deblurred image. This data term arises naturally in the deconvolution context and is also used in the variational blind models in [7, 85].

The Euler-Lagrange equation associated with (5.2.8) with homogeneous Neumann boundary conditions, given by

$$\begin{cases} 
  0 = - \text{div}(\psi'(|\nabla u|^2)\nabla u) + \lambda k * (k * u - u_0), & x, y \in \Omega, \\
  \frac{\partial u}{\partial n} = 0, & x, y \in \partial\Omega,
\end{cases}$$  \hspace{1cm} (5.2.9)

where $\partial\Omega$ is the boundary of $\Omega$ and $\vec{n}$ is the outward normal to $\partial\Omega$.

A gradient descent leading for $t \to \infty$ to a minimizer of $E$ is given by

$$u_t = \text{div}(g(|\nabla u|^2)\nabla u) - \lambda k * (k * u - u_0),$$  \hspace{1cm} (5.2.10)

with homogeneous Neumann boundary conditions. It is also known as diffusion-reaction equation where the diffusion term with diffusivity $g(s^2) = \psi'(s^2)$ is related to the regulariser in the energy functional.

The model (5.2.10) converges very slowly to its steady state in explicit schemes for image restoration, for details we refer [46]. So we multiply the whole Euler-Lagrange equation (5.2.9) by the magnitude of the gradient and our nonlinear anisotropic diffusion model takes the following form:

$$u_t = |\nabla u|\text{div}(g(|\nabla u|^2)\nabla u) - |\nabla u|\lambda k * (k * u - u_0).$$  \hspace{1cm} (5.2.11)
Applying a priori smoothness on the solution image, our nonlinear anisotropic diffusion model becomes,

\[ u_t = |\nabla G_\sigma * u| \text{div}(g(|\nabla G_\sigma * u|^2) \nabla G_\sigma * u) - |\nabla G_\sigma * u| \lambda k * (k * G_\sigma * u - u_0). \tag{5.2.12} \]

It should be noticed that (5.2.12) only replaces \( u \) in (5.2.11) by its estimate \( G_\sigma * u \).

We still write \( G_\sigma * u \) as \( u \). Let \( u_{ij}^n \) be the approximation to the value \( u(x_i, y_j, t_n) \), where

\[ x_i = i \Delta x, \quad y_j = j \Delta x, \quad i, j = 1, 2, \ldots, N, \]

\[ t_n = n \Delta t, \quad n \geq 1, \]

where \( \Delta x, \Delta y \) and \( \Delta t \) are the spatial step sizes and the time step size respectively.

The explicit partial derivatives of models (5.2.6) and (5.2.7) can be expressed as:

\[ u_t = u_{xx}(u_x^2 + \beta) - 2u_{xy} u_x u_y + u_{yy}(u_y^2 + \beta) \]

\[ \frac{(u_x^2 + u_y^2 + \beta)}{(u_x^2 + u_y^2 + \beta)} - \sqrt{u_x^2 + u_y^2 + \beta} \lambda k * (k * u - u_0). \tag{5.2.13} \]

We define the derivative terms as,

\[ u_{x}^{ij} = \frac{u_{i+1,j}^n - u_{i-1,j}^n}{2 \Delta x}; \quad u_{y}^{ij} = \frac{u_{i,j+1}^n - u_{i,j-1}^n}{2 \Delta x}; \]

\[ u_{xx}^{ij} = \frac{u_{i+1,j+1}^n - 2u_{i,j+1}^n + u_{i-1,j+1}^n}{\Delta x^2}; \quad u_{yy}^{ij} = \frac{u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n}{\Delta x^2}; \]

\[ u_{xy}^{ij} = \frac{u_{i+1,j+1}^n - u_{i-1,j+1}^n - u_{i+1,j-1}^n + u_{i-1,j-1}^n}{4 \Delta x \Delta x}; \quad u_{ij}^t = \frac{u_{i,j}^{n+1} - u_{i,j}^n}{\Delta t}. \]

We let,

\[ r_{ij}^n = u_{ij}^{xx}((u_i^y)^2 + \beta) - 2u_{ij}^{xy} u_{ij}^x u_{ij}^y + u_{ij}^{yy}((u_i^x)^2 + \beta), \tag{5.2.14} \]

and

\[ p_{ij}^n = ((u_{ij}^x)^2 + (u_{ij}^y)^2 + \beta). \tag{5.2.15} \]

Then (5.2.13) reads as follows:

\[ u_{ij}^t = \frac{r_{ij}^n}{p_{ij}^n} - \sqrt{((u_{ij}^x)^2 + (u_{ij}^y)^2 + \beta)} \lambda k * (k * u_{ij}^n - u_{ij}^0), \tag{5.2.16} \]
A time dependent model for image restoration with forward-backward Diffusivities with homogeneous Neumann boundary conditions.

The explicit partial derivatives of models (5.2.11) and (5.2.12) can be expressed as:

\[ u_{ij}^t = \frac{1}{2\Delta x} s^n_{ij} ((g^n_{i+1,j} + g^n_{i,j})(u^n_{i+1,j} - u^n_{i,j}) - (g^n_{i,j} + g^n_{i-1,j})(u^n_{i,j} - u^n_{i-1,j})) \]
\[ + \frac{1}{2\Delta x} s^n_{ij} ((g^n_{i,j+1} + g^n_{i,j})(u^n_{i,j+1} - u^n_{i,j}) - (g^n_{i,j} + g^n_{i,j-1})(u^n_{i,j} - u^n_{i,j-1})) \]
\[ - s^n_{ij} \lambda k * (k * u^n_{ij} - u^0_{ij}), \]  

(5.2.17)

where the diffusivity \( g(|\nabla u|^2) \) is discretised by,

\[ g^n_{ij} = \psi' \left( \left( \frac{u^n_{i+1,j} - u^n_{i-1,j}}{\Delta x} \right)^2 + \left( \frac{u^n_{i,j+1} - u^n_{i,j-1}}{\Delta x} \right)^2 \right), \]

and \( s^n_{ij} = \sqrt{(u^n_{ij})^2 + (u^n_{ij})^2}, \)

with homogeneous Neumann boundary conditions.

The explicit method is stable and convergent for \( \frac{\Delta t}{\Delta x^2} \leq 0.5 \), see [41].

### 5.3 Choice of the diffusivity

In the deconvolution process, the choice of the diffusivity \( g \) is very important. Take the simplest case, the constant diffusivity \( g(s^2) = 1 \). It gives an over-smoothed deblurring result because high gradients at edges of the reconstructed image are penalised over-proportionally. Moreover, in this case the whole deconvolution method is again linear and suffers from the artifacts.

Total variation (TV) diffusivity \( g(s^2) = \frac{1}{|s|} \), in its regularised form \( g(s^2) = \frac{1}{\sqrt{s^2 + \epsilon^2}} \), is a popular choice, see for example references [7, 16, 47]. It enforces piecewise constant results and therefore encourages sharp edges in the image.

The Perona-Malik (PM) diffusivity \( g(s^2) = (1 + \frac{s^2}{\gamma^2})^{-1} \), that is related to the non-convex regulariser \( \psi(s^2) = \gamma^2 \log(1 + \frac{s^2}{\gamma^2}) \), where \( \gamma \) is a contrast parameter determines which steepness edges are enhanced in the gradient descent process, see the references [56, 78]. We have included Total variation and Perona-Malik diffusivity in our experiments.
5.4 Numerical implementation

In this Section, we perform numerical experiments on the gray scale images, Lena and Boat of (256 × 256) pixels. We first scale the intensities of the images into the range between zero and one before we begin our experiments. The Gaussian white noise is added by the normal imnoise function imnoise (I, ‘Gaussian’, M, σ^2), for the mean M and variance σ^2, in Matlab. In our test, we will use the blurred signal to noise ratio (BSNR) is used to measure the ratio of the level of blurred kernel and the level of noise,

\[ \text{BSNR} = 10 \log_{10} \left( \frac{\text{blurred signal variance}}{\text{noise variance}} \right) \text{dB}. \]  

(5.4.1)

Improvement in the signal quality (ISNR) is used to measure the goodness of restored image:

\[ \text{ISNR} = 10 \log_{10} \left( \frac{\sum_{i,j} [u_{ij} - (u_0)_{ij}]^2}{\sum_{i,j} [u_{ij} - (u_{new})_{ij}]^2} \right) \text{dB}, \]  

(5.4.2)

where \( u_{new} \) is the restored image. That is, the value of ISNR is larger, the restored image is better.

Here we use Gaussian kernel, defined as

\[ k_\alpha(x, y) = \frac{1}{2\pi\alpha^2} \exp \left(-\frac{x^2 + y^2}{2\alpha^2}\right). \]  

(5.4.3)

The size \( k_\alpha \) of blurring operator is 5 and 11. We use the blurring parameter \( \alpha = 2 \) and 3 in the blurred kernel (5.4.3). We have taken Lagrange multiplier \( \lambda = 0.85 \), see references [15, 17]. We can choose \( \beta = 10^{-32} \), the smallest positive machine number as set in reference [17]. We have used \( \gamma = 5 \) in PM diffusivity and \( \epsilon = 0.01 \) in TV diffusivity for our experiments. For models (5.2.6) and (5.2.7), we have considered \( \Delta t/\Delta x^2 = 0.01 \) and have taken \( \Delta t/\Delta x^2 = 0.1 \) for models (5.2.11) and (5.2.12).

Figures 5.4.1(b) and 5.4.2(b) represent the blurred and noisy images of Lena and Boat respectively with the size of Gaussian blurred kernel 5, the blurring parameter 2 and additive Gaussian white noise \( \sigma^2 = 0.002 \) wherein their BSNR ≈ 13.35 and 12.19 respectively. Figures 5.4.3(b) and 5.4.4(b) represent the blurred and noisy images of Lena and Boat respectively with the size of Gaussian blurred kernel 11, the blurring parameter 3 and additive Gaussian white noise \( \sigma^2 = 0.002 \) wherein their BSNR ≈ 12.76 and 11.67 respectively.
Figure 5.4.1: (a) Original Lena image, (b) blurred and noisy image with the size of Gaussian blur operator 5, blurring parameter 2 and Gaussian noise $\sigma^2 = 0.002$, (c) and (d) restored image by models (5.2.6) and (5.2.7) respectively, (e) and (f) restored image by models (5.2.11) and (5.2.12) respectively with PM diffusivity, (g) and (h) restored image by models (5.2.11) and (5.2.12) respectively with TV diffusivity.
Figure 5.4.2: (a) Original Boat image, (b) blurred and noisy image with the size of Gaussian blur operator 5, blurring parameter 2 and Gaussian noise $\sigma^2 = 0.002$, (c) and (d) restored image by models (5.2.6) and (5.2.7) respectively, (e) and (f) restored image by models (5.2.11) and (5.2.12) respectively with Perona-Malik diffusivity, (g) and (h) restored image by models (5.2.11) and (5.2.12) respectively with TV diffusivity.
A time dependent model for image restoration with forward - backward Diffusivities

Figure 5.4.3: (a) Original Lena image, (b) blurred and noisy image with $k_\alpha = 11$, $\alpha = 3$ and $\sigma^2 = 0.002$, (c) and (d) restored image by models (5.2.6) and (5.2.7) respectively, (e) and (f) restored image by models (5.2.11) and (5.2.12) respectively with PM diffusivity, (g) and (h) restored image by models (5.2.11) and (5.2.12) respectively with TV diffusivity.

Figure 5.4.4: (a) Original Boat image, (b) blurred and noisy image with $k_\alpha = 11$, $\alpha = 3$ and $\sigma^2 = 0.002$, (c) and (d) restored image by models (5.2.6) and (5.2.7) respectively, (e) and (f) restored image by models (5.2.11) and (5.2.12) respectively with PM diffusivity, (g) and (h) restored image by models (5.2.11) and (5.2.12) respectively with TV diffusivity.
Table 5.4.1: Results obtained by using models (5.2.6), (5.2.7), (5.2.11) and (5.2.12) applied to the image in Figure 5.4.1(b) with the size of Gaussian blur operator 5, blurring parameter 2 and Gaussian noise $\sigma^2 = 0.002$.

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Table 5.4.2: Results obtained by using models (5.2.6), (5.2.7), (5.2.11) and (5.2.12) applied to the image in Figure 5.4.2(b) with the size of Gaussian blur operator 5, blurring parameter 2 and Gaussian noise $\sigma^2 = 0.002$.

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Table 5.4.3: Results obtained by using models (5.2.6), (5.2.7), (5.2.11) and (5.2.12) applied to the image in Figure 5.4.3(b) with the size of Gaussian blur operator 11, blurring parameter 3 and Gaussian noise $\sigma^2 = 0.002$.

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A time dependent model for image restoration with forward - backward Diffusivities

Table 5.4.4: Results obtained by using models (5.2.6), (5.2.7), (5.2.11) and (5.2.12) applied to the image in Figure 5.4.4(b) with the size of Gaussian blur operator 11, blurring parameter 3 and Gaussian noise $\sigma^2 = 0.002$.

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</table>

5.5 Conclusion

We have presented a new nonlinear anisotropic diffusion models for image restoration. The denoising technique is used to every solution image before solving the nonlinear diffusion problem. Our models (5.2.11) and (5.2.12) give better restoration results in comparison with other existing models (5.2.6) and (5.2.7). Nonlinear explicit schemes are used to discretize models (5.2.6), (5.2.7), (5.2.11) and (5.2.12). The models (5.2.11) and (5.2.12) give larger ISNR values than that of models (5.2.6) and (5.2.7), at different iterative numbers.