Chapter 7

Summary

This thesis is devoted to the study of amplitude death and the synchronization aspects of interacting nonlinear systems. In Chapter 2, I discussed the phenomena of amplitude death in identical conjugate coupled nonlinear oscillators and in Chapter 3, different synchronization regimes in conjugate coupled chaotic systems were discussed. In Chapter 4, I studied the universal occurrence of phase flip transition and in Chapter 5, an analytical study of the phase flip for the transient dynamics in the AD regime was presented. The main results have been summarized below.

In Chapter 2, effects of coupling systems via conjugate or dissimilar variables were looked into. This type of interaction, which frequently arises in experiments, can give rise to novel phenomena: amplitude death in the absence of delays, and riddling in identical coupled systems. The results appear to apply generally to coupled nonlinear dynamical systems, and have been verified both analytically for coupled Landau-Stuart oscillators and numerically for coupled Lorenz oscillators. Both diffusive and non-diffusive coupling mechanisms were studied. In cases of AD when the coupling is diffusive, the interaction terms can vanish. It is also possible that the coupling term does not vanish and there are new solutions that are created. Apart from simple two oscillator systems, regimes of oscillator death in the case of a globally coupled network of identical Landau Stuart oscillators [65] were also seen, which further underscores the generality of the phenomenon.

In Chapter 3, different synchronization regimes in conjugate coupled systems were discussed. The coupling between identical dynamical systems may naturally occur through conjugate variables and this can give rise to different regimes of synchrony that were explored in this Chapter. With conjugate coupling, complete synchronization occurs in a manner quite distinct from the
situation when the coupling is in similar variables: the coupling function does not vanish on the synchronization manifold, and instead each of the systems is driven to a dynamical state that cannot occur in the absence of coupling. The only completely synchronized solution possible in conjugate coupling is necessarily periodic. When the systems are not identical phase synchronization is possible, while generalized synchrony occurs when the coupling is unidirectional. These results were illustrated in coupled Rössler oscillators for which analytic estimates for the coupling threshold, \( \varepsilon_c \), for a dynamical transition to a phase-locked regime was calculated. These transitions were not easily discerned from other indicators such as the Lyapunov spectrum. Using a combination of numerical and analytical techniques \([130]\), it was shown that generalized synchronization was achieved above a threshold that could be detected via computation of the Lyapunov exponent for the difference dynamics. These results hold for a range of parameter mismatch, so that the phenomena observed here are robust and should be observable in experiments.

In the Chapter 4, I studied the universality of the phase flip transition in different classes of dynamical systems. In this Chapter, previous findings of phase–flip in periodic, chaotic and fixed point regimes were extended to different types of oscillators which include excitable dynamics such as laser or neuronal models, and periodic and chaotic ecological models. The Chapter also presented the results for time-delay coupled Chua oscillators and gave a clear experimental evidence for the phase-flip.

In Chapter 5, phase–flip transition in delay coupled systems was examined and the origin of this dynamical phenomenon was traced to an avoided crossing in the spectrum of Lyapunov exponents as a parameter is varied. A solvable model for the analysis was provided by coupled Landau Stuart limit cycle oscillator system in the region of amplitude death, namely when a fixed point is stabilised. Crossings or avoided crossings in the spectrum of Lyapunov exponents have significance only in very special circumstances. In the case presented, the importance of the crossing comes from the fact that it involves the largest two pairs of Lyapunov exponents. Before the transition, the system is in a regime of in–phase dynamics, and once the exponents cross, the system responds by abruptly going out of phase, although remaining in synchrony. The numerical results are supported by explicit computation of an order parameter that can detect this transition.

Although analytic calculations are more difficult when the dynamics is more complicated, numerical results suggest that the phase flip that has been seen in a variety of systems and in a number of different dynamical
states is always associated with an avoided crossing. When the coupled oscillator dynamics is periodic or chaotic before and after the change in phase (see [113], Fig. 2) we find numerically that there is a crossing involving a Lyapunov exponent that is zero. The associated complete eigenvalue analysis is not possible, but numerical calculations of the Lyapunov exponents and the order parameter $\gamma^2$ support the correspondence between the Lyapunov exponent crossing and the change from in- to anti-phase synchrony.

In Chapter 6, I discussed a set of studies that extended the problems examined in the thesis in two directions. The first has to do with the nature of interactions; how does the dynamics of coupled systems change with the introduction of many-body terms? The second focusses on paying attention to the relative sense of the oscillations in coupled oscillator systems and examines the aging transition in a linear one dimensional chain with nearest neighbour interactions of co and counter-rotating units.

The present set of studies raise several questions which need to be explored in detail. For conjugate coupled systems, the relationship between the conjugate variables and time-delayed variables requires an in-depth analysis. It would be interesting to determine how this delay varies with coupling strength. Further, for diffusively coupled Landau-Stuart oscillators, two distinct regimes of amplitude death are observed (see Fig. 2.2). In the first regime (between points A and B in Fig. 2.2(a)), the origin is stable for a range of coupling values. On further increasing the coupling, the system goes into a regime where there are an infinite stable fixed points (beyond point B). Asymptotically, the system can settle on any of these fixed points depending on the initial conditions. A systematic study of the system in this regime is required to understand the properties of these fixed points. Also a possibility of controlling the system in this regime is important.

Experimental studies aimed at the exploration of different dynamical regimes in conjugate coupled electronic circuits have already been initiated [139]. Implementation of the conjugate coupling scheme in different modeling situations can help to better understand the behaviour of systems where such interactions are common. One such instance is population dynamics of interacting predator–prey systems, and studies of such problems are currently in progress [68].

The phase flip in time-delay coupled systems has been correlated with a degeneracy in the spectrum of the Lyapunov exponents. Avoided crossings in the Lyapunov spectrum have been reported in more general contexts [2] and it would be interesting if we can further study these cases.

In summary, the present set of studies have addressed the instances of
amplitude death and synchronization in conjugate coupled oscillators and phase synchronization in time-delay coupled oscillators. A number of questions and many other directions still remain open and are worthy of further exploration.